Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.4-Cotangent/112-4.4.2.1-a+b-cot-^m-c+d-cot-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [106]. This is test number [112].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (106)	0.00 (0)
Mathematica	99.06 (105)	0.94 (1)
Maple	97.17 (103)	2.83 (3)
Mupad	97.17 (103)	2.83 (3)
Maxima	74.53 (79)	25.47 (27)
Fricas	29.25 (31)	70.75 (75)
Giac	2.83 (3)	97.17 (103)
Sympy	1.89 (2)	98.11 (104)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

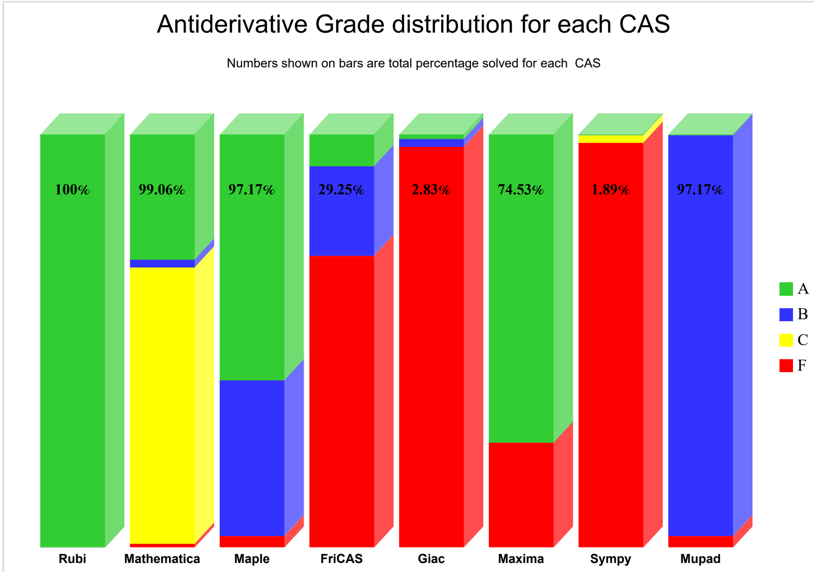
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

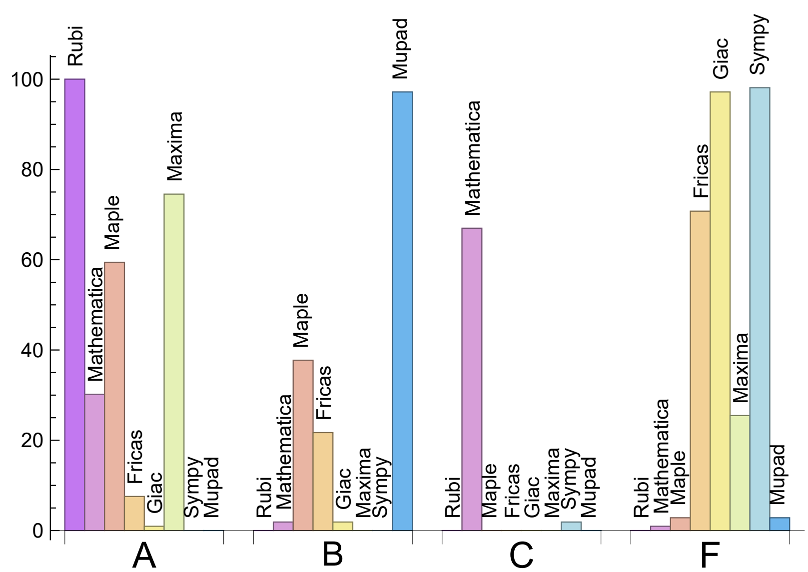
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maxima	74.53	0.00	0.00	25.47
Maple	59.43	37.74	0.00	2.83
Mathematica	30.19	1.89	66.98	0.94
Fricas	7.55	21.70	0.00	70.75
Giac	0.94	1.89	0.00	97.17
Mupad	N/A	97.17	0.00	2.83
Sympy	0.00	0.00	1.89	98.11

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	3	100.00 %	0.00 %	0.00 %
Fricas	75	4.00 %	78.67 %	17.33 %
Giac	103	96.12 %	3.88 %	0.00 %
Maxima	27	92.59 %	3.70 %	3.70 %
Sympy	104	95.19 %	3.85 %	0.96 %
Mupad	3	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

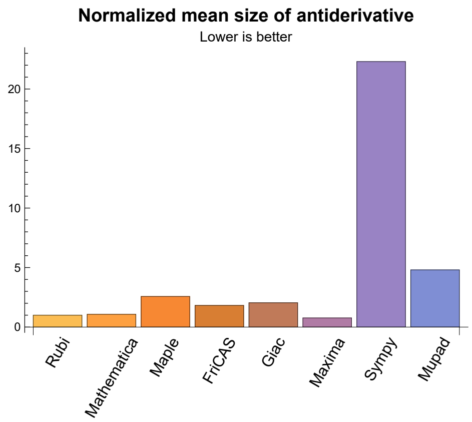
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

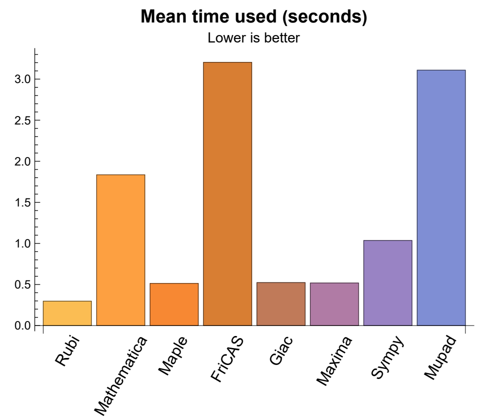
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.30	232.98	1.00	221.50	1.00
Mathematica	1.84	212.27	1.08	193.00	0.86
Maple	0.51	417.96	2.57	326.00	1.24
Maxima	0.52	187.24	0.77	161.00	0.71
Fricas	3.20	210.84	1.82	182.00	1.61
Sympy	1.04	2245.00	22.31	2245.00	22.31
Giac	0.52	249.33	2.05	241.00	2.17
Mupad	3.11	1060.52	4.81	366.00	1.69

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
5
10
15
20
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
  r"""
  Return the tree size of this expression.
  """
  if expr not in SR:
    # deal with lists, tuples, vectors
    return 1 + sum(tree_size(a) for a in expr)
  expr = SR(expr)
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

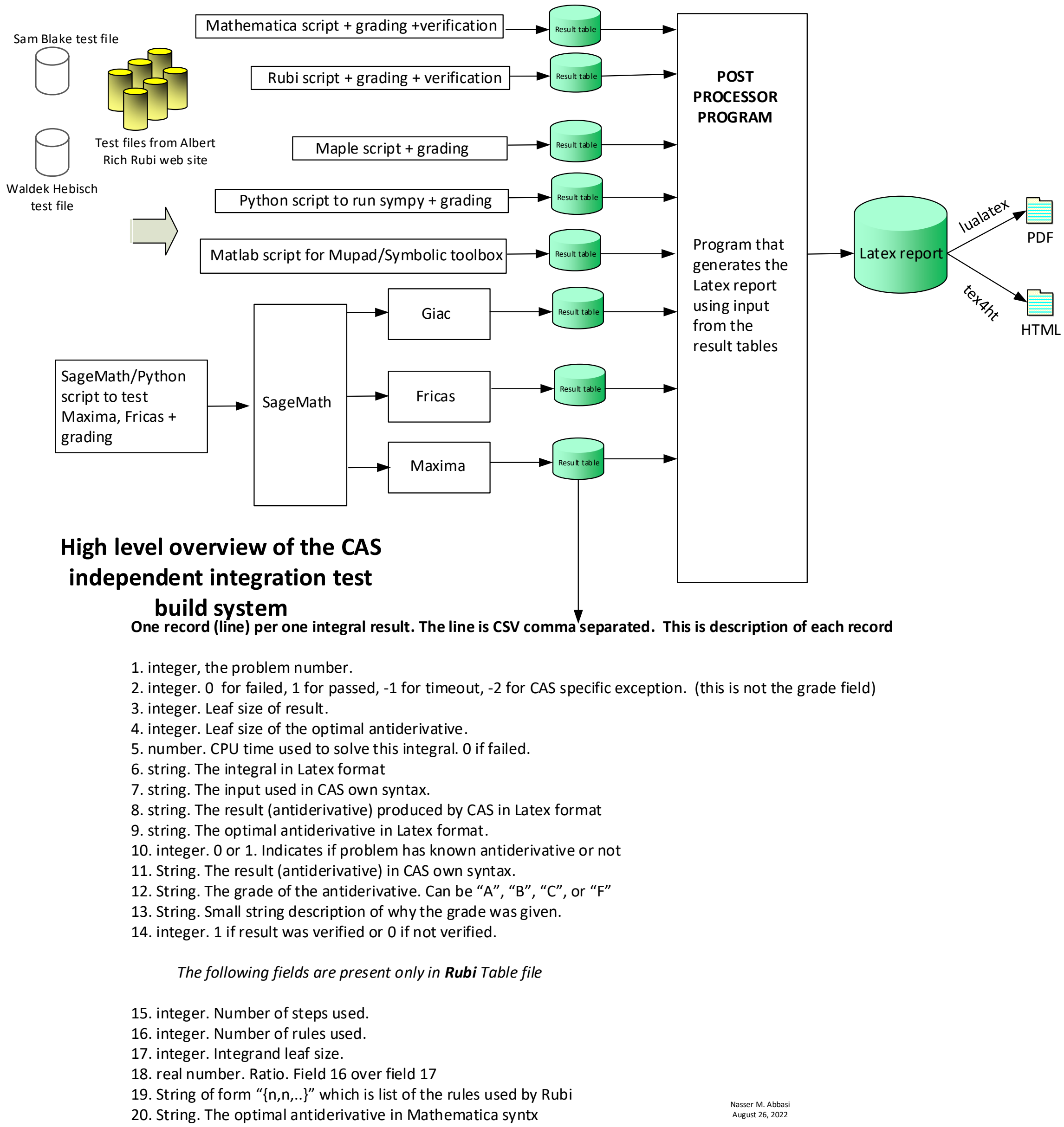
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 8, 10, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 90, 91, 92, 96, 97, 98, 101, 102, 103, 104, 105, 106 }

B grade: { 1, 95 }

C grade: { 2, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 93, 94, 99, 100 }

F grade: { 89 }

2.1.3 Maple

A grade: { 8, 9, 10, 11, 12, 13, 14, 29, 30, 31, 32, 33, 34, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 92, 93, 94, 99 }

B grade: { 2, 3, 4, 5, 6, 7, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 35, 36, 37, 38, 39, 40, 46, 90, 91, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106 }

C grade: { }

F grade: { 1, 88, 89 }

2.1.4 Maxima

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 92, 93, 94 }

B grade: { }

C grade: { }

F grade: { 1, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

2.1.5 FriCAS

A grade: { 15, 16, 17, 18, 19, 36, 38, 92 }

B grade: { 2, 3, 4, 5, 6, 7, 20, 21, 22, 23, 24, 25, 26, 27, 28, 35, 37, 39, 40, 90, 91, 93, 94 }

C grade: { }

F grade: { 1, 8, 9, 10, 11, 12, 13, 14, 29, 30, 31, 32, 33, 34, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

2.1.6 Sympy

A grade: { }

B grade: { }

C grade: { 92, 93 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

2.1.7 Giac

A grade: { 92 }

B grade: { 93, 94 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

2.1.8 Mupad

A grade: { }

B grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

C grade: { }

F grade: { 1, 88, 89 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	117	0	0	0	0	0	-1
N.S.	1	1.00	2.39	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.342	0.807	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	68	319	101	181	0	0	144
N.S.	1	1.00	0.59	2.75	0.87	1.56	0.00	0.00	1.24
time (sec)	N/A	0.114	0.210	0.425	0.534	3.288	0.000	0.000	1.978

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	67	304	82	165	0	0	98
N.S.	1	1.00	0.71	3.23	0.87	1.76	0.00	0.00	1.04
time (sec)	N/A	0.082	0.112	0.366	0.532	3.595	0.000	0.000	1.161

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	154	289	78	117	0	0	128
N.S.	1	1.00	2.17	4.07	1.10	1.65	0.00	0.00	1.80
time (sec)	N/A	0.052	0.304	0.401	0.517	3.515	0.000	0.000	0.777

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	165	273	57	82	0	0	65
N.S.	1	1.00	3.37	5.57	1.16	1.67	0.00	0.00	1.33
time (sec)	N/A	0.033	0.218	0.492	0.513	4.414	0.000	0.000	0.728

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	191	294	78	154	0	0	84
N.S.	1	1.00	2.55	3.92	1.04	2.05	0.00	0.00	1.12
time (sec)	N/A	0.059	0.256	0.337	0.512	4.421	0.000	0.000	0.960

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	203	309	83	171	0	0	103
N.S.	1	1.00	2.05	3.12	0.84	1.73	0.00	0.00	1.04
time (sec)	N/A	0.102	0.412	0.372	0.501	3.191	0.000	0.000	1.483

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	187	187	166	0	0	0	125
N.S.	1	1.00	0.70	0.70	0.62	0.00	0.00	0.00	0.46
time (sec)	N/A	0.229	1.251	0.448	0.527	0.000	0.000	0.000	1.728

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	52	172	146	0	0	0	104
N.S.	1	1.00	0.21	0.70	0.59	0.00	0.00	0.00	0.42
time (sec)	N/A	0.165	0.400	0.438	0.520	0.000	0.000	0.000	0.948

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	175	170	153	0	0	0	104
N.S.	1	1.00	0.72	0.70	0.63	0.00	0.00	0.00	0.43
time (sec)	N/A	0.156	0.458	0.436	0.547	0.000	0.000	0.000	0.697

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	53	155	132	0	0	0	86
N.S.	1	1.00	0.24	0.70	0.59	0.00	0.00	0.00	0.39
time (sec)	N/A	0.140	0.268	0.409	0.517	0.000	0.000	0.000	0.445

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	236	159	139	0	0	0	86
N.S.	1	1.00	1.06	0.72	0.63	0.00	0.00	0.00	0.39
time (sec)	N/A	0.152	1.906	0.418	0.519	0.000	0.000	0.000	0.593

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	233	174	147	0	0	0	99
N.S.	1	1.00	0.94	0.70	0.60	0.00	0.00	0.00	0.40
time (sec)	N/A	0.170	1.348	0.391	0.502	0.000	0.000	0.000	0.701

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	141	174	156	0	0	0	99
N.S.	1	1.00	0.57	0.70	0.63	0.00	0.00	0.00	0.40
time (sec)	N/A	0.177	0.427	0.391	0.522	0.000	0.000	0.000	1.292

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	729	354	127	258	0	0	177
N.S.	1	1.00	3.92	1.90	0.68	1.39	0.00	0.00	0.95
time (sec)	N/A	0.218	6.112	0.481	0.508	4.534	0.000	0.000	2.440

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	332	339	122	248	0	0	143
N.S.	1	1.00	2.08	2.12	0.76	1.55	0.00	0.00	0.89
time (sec)	N/A	0.176	2.908	0.500	0.511	2.945	0.000	0.000	1.617

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	315	323	101	189	0	0	136
N.S.	1	1.00	2.28	2.34	0.73	1.37	0.00	0.00	0.99
time (sec)	N/A	0.144	1.676	0.482	0.510	2.519	0.000	0.000	0.990

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	292	309	96	168	0	0	100
N.S.	1	1.00	2.50	2.64	0.82	1.44	0.00	0.00	0.85
time (sec)	N/A	0.112	5.462	0.451	0.510	2.580	0.000	0.000	0.621

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	311	305	86	178	0	0	119
N.S.	1	1.00	2.73	2.68	0.75	1.56	0.00	0.00	1.04
time (sec)	N/A	0.131	3.100	0.421	0.529	2.727	0.000	0.000	0.576

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	417	303	97	182	0	0	101
N.S.	1	1.00	3.56	2.59	0.83	1.56	0.00	0.00	0.86
time (sec)	N/A	0.129	6.114	0.436	0.530	4.417	0.000	0.000	0.705

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	269	323	101	235	0	0	126
N.S.	1	1.00	1.91	2.29	0.72	1.67	0.00	0.00	0.89
time (sec)	N/A	0.159	3.443	0.437	0.526	3.803	0.000	0.000	1.261

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	174	338	125	252	0	0	129
N.S.	1	1.00	1.05	2.05	0.76	1.53	0.00	0.00	0.78
time (sec)	N/A	0.204	2.123	0.431	0.508	3.910	0.000	0.000	1.924

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	110	312	88	169	0	0	123
N.S.	1	1.00	0.99	2.81	0.79	1.52	0.00	0.00	1.11
time (sec)	N/A	0.298	0.931	0.525	0.499	4.068	0.000	0.000	0.682

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	107	298	83	142	0	0	79
N.S.	1	1.00	1.23	3.43	0.95	1.63	0.00	0.00	0.91
time (sec)	N/A	0.167	3.781	0.516	0.563	2.254	0.000	0.000	0.492

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	98	304	75	140	0	0	102
N.S.	1	1.00	1.13	3.49	0.86	1.61	0.00	0.00	1.17
time (sec)	N/A	0.170	0.280	0.527	0.536	4.790	0.000	0.000	0.368

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	107	304	83	139	0	0	79
N.S.	1	1.00	1.29	3.66	1.00	1.67	0.00	0.00	0.95
time (sec)	N/A	0.152	0.548	0.526	0.511	2.142	0.000	0.000	0.523

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	176	319	88	213	0	0	123
N.S.	1	1.00	1.59	2.87	0.79	1.92	0.00	0.00	1.11
time (sec)	N/A	0.305	2.174	0.501	0.517	2.568	0.000	0.000	0.644

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	131	333	109	229	0	0	132
N.S.	1	1.00	0.97	2.47	0.81	1.70	0.00	0.00	0.98
time (sec)	N/A	0.361	1.408	0.477	0.506	2.479	0.000	0.000	0.929

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	224	191	161	0	0	0	375
N.S.	1	1.00	0.80	0.68	0.57	0.00	0.00	0.00	1.33
time (sec)	N/A	0.372	2.080	0.570	0.512	0.000	0.000	0.000	0.898

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	312	197	161	0	0	0	376
N.S.	1	1.00	1.12	0.71	0.58	0.00	0.00	0.00	1.35
time (sec)	N/A	0.390	3.019	0.523	0.495	0.000	0.000	0.000	0.823

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	207	200	161	0	0	0	366
N.S.	1	1.00	0.74	0.72	0.58	0.00	0.00	0.00	1.32
time (sec)	N/A	0.343	1.380	0.537	0.513	0.000	0.000	0.000	0.722

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	337	197	161	0	0	0	366
N.S.	1	1.00	1.20	0.70	0.57	0.00	0.00	0.00	1.30
time (sec)	N/A	0.374	0.878	0.538	0.540	0.000	0.000	0.000	0.813

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	203	212	174	0	0	0	414
N.S.	1	1.00	0.66	0.69	0.57	0.00	0.00	0.00	1.35
time (sec)	N/A	0.538	1.401	0.526	0.498	0.000	0.000	0.000	0.923

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	467	227	185	0	0	0	425
N.S.	1	1.00	1.41	0.69	0.56	0.00	0.00	0.00	1.28
time (sec)	N/A	0.697	6.355	0.515	0.507	0.000	0.000	0.000	1.226

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	192	349	136	246	0	0	154
N.S.	1	1.00	1.17	2.13	0.83	1.50	0.00	0.00	0.94
time (sec)	N/A	0.402	2.231	0.592	0.509	2.544	0.000	0.000	1.037

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	131	349	127	242	0	0	178
N.S.	1	1.00	0.80	2.13	0.77	1.48	0.00	0.00	1.09
time (sec)	N/A	0.437	2.147	0.585	0.526	2.866	0.000	0.000	0.938

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	181	349	137	246	0	0	151
N.S.	1	1.00	1.12	2.17	0.85	1.53	0.00	0.00	0.94
time (sec)	N/A	0.415	0.857	0.576	0.493	2.801	0.000	0.000	0.898

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	217	349	129	233	0	0	173
N.S.	1	1.00	1.32	2.12	0.78	1.41	0.00	0.00	1.05
time (sec)	N/A	0.432	1.323	0.597	0.537	3.502	0.000	0.000	0.944

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	156	364	146	331	0	0	175
N.S.	1	1.00	0.83	1.93	0.77	1.75	0.00	0.00	0.93
time (sec)	N/A	0.585	1.234	0.588	0.496	2.314	0.000	0.000	1.141

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	167	379	149	340	0	0	193
N.S.	1	1.00	0.78	1.76	0.69	1.58	0.00	0.00	0.90
time (sec)	N/A	0.721	3.306	0.607	0.507	3.391	0.000	0.000	1.385

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	69	197	0	0	0	0	119
N.S.	1	1.00	0.31	0.88	0.00	0.00	0.00	0.00	0.53
time (sec)	N/A	0.199	0.178	0.509	0.000	0.000	0.000	0.000	0.631

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	61	174	0	0	0	0	210
N.S.	1	1.00	0.45	1.29	0.00	0.00	0.00	0.00	1.56
time (sec)	N/A	0.177	0.098	0.275	0.000	0.000	0.000	0.000	0.483

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	96	197	0	0	0	0	254
N.S.	1	1.00	0.69	1.42	0.00	0.00	0.00	0.00	1.83
time (sec)	N/A	0.197	0.336	0.260	0.000	0.000	0.000	0.000	0.996

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	98	290	0	0	0	0	254
N.S.	1	1.00	0.44	1.31	0.00	0.00	0.00	0.00	1.15
time (sec)	N/A	0.151	0.280	0.209	0.000	0.000	0.000	0.000	0.674

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	67	283	0	0	0	0	238
N.S.	1	1.00	0.31	1.32	0.00	0.00	0.00	0.00	1.11
time (sec)	N/A	0.136	0.188	0.259	0.000	0.000	0.000	0.000	0.438

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	51	181	0	0	0	0	230
N.S.	1	1.00	0.42	1.50	0.00	0.00	0.00	0.00	1.90
time (sec)	N/A	0.090	0.082	0.483	0.000	0.000	0.000	0.000	0.410

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	65	173	0	0	0	0	208
N.S.	1	1.00	0.47	1.24	0.00	0.00	0.00	0.00	1.50
time (sec)	N/A	0.154	0.139	0.233	0.000	0.000	0.000	0.000	0.501

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	71	197	0	0	0	0	121
N.S.	1	1.00	0.31	0.87	0.00	0.00	0.00	0.00	0.54
time (sec)	N/A	0.150	0.154	0.255	0.000	0.000	0.000	0.000	0.400

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	75	197	0	0	0	0	242
N.S.	1	1.00	0.52	1.38	0.00	0.00	0.00	0.00	1.69
time (sec)	N/A	0.168	0.428	0.244	0.000	0.000	0.000	0.000	0.800

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	69	283	0	0	0	0	238
N.S.	1	1.00	0.32	1.31	0.00	0.00	0.00	0.00	1.10
time (sec)	N/A	0.137	0.259	0.254	0.000	0.000	0.000	0.000	0.709

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	68	306	153	0	0	0	153
N.S.	1	1.00	0.28	1.24	0.62	0.00	0.00	0.00	0.62
time (sec)	N/A	0.155	0.147	0.423	0.555	0.000	0.000	0.000	1.397

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	155	290	141	0	0	0	128
N.S.	1	1.00	0.69	1.28	0.62	0.00	0.00	0.00	0.57
time (sec)	N/A	0.128	0.306	0.413	0.512	0.000	0.000	0.000	0.726

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	166	273	130	0	0	0	118
N.S.	1	1.00	0.80	1.31	0.62	0.00	0.00	0.00	0.57
time (sec)	N/A	0.105	0.214	0.520	0.500	0.000	0.000	0.000	0.647

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	196	295	141	0	0	0	137
N.S.	1	1.00	0.86	1.29	0.62	0.00	0.00	0.00	0.60
time (sec)	N/A	0.147	0.358	0.395	0.506	0.000	0.000	0.000	0.805

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	196	311	154	0	0	0	158
N.S.	1	1.00	0.78	1.23	0.61	0.00	0.00	0.00	0.63
time (sec)	N/A	0.189	0.710	0.404	0.552	0.000	0.000	0.000	1.245

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	224	360	211	0	0	0	1274
N.S.	1	1.00	0.71	1.14	0.67	0.00	0.00	0.00	4.02
time (sec)	N/A	0.234	2.101	0.543	0.509	0.000	0.000	0.000	2.471

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	220	321	192	0	0	0	1157
N.S.	1	1.00	0.76	1.11	0.67	0.00	0.00	0.00	4.02
time (sec)	N/A	0.188	0.569	0.570	0.524	0.000	0.000	0.000	1.212

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	192	306	179	0	0	0	1234
N.S.	1	1.00	0.72	1.15	0.67	0.00	0.00	0.00	4.62
time (sec)	N/A	0.175	0.914	0.474	0.504	0.000	0.000	0.000	1.013

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	218	301	179	0	0	0	1196
N.S.	1	1.00	0.82	1.13	0.67	0.00	0.00	0.00	4.48
time (sec)	N/A	0.193	0.326	0.465	0.504	0.000	0.000	0.000	0.936

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	82	326	193	0	0	0	1214
N.S.	1	1.00	0.28	1.12	0.66	0.00	0.00	0.00	4.17
time (sec)	N/A	0.240	0.324	0.457	0.541	0.000	0.000	0.000	1.512

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	85	347	214	0	0	0	1227
N.S.	1	1.00	0.26	1.08	0.66	0.00	0.00	0.00	3.81
time (sec)	N/A	0.306	0.388	0.473	0.508	0.000	0.000	0.000	2.320

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	251	410	264	0	0	0	2317
N.S.	1	1.00	0.67	1.10	0.71	0.00	0.00	0.00	6.23
time (sec)	N/A	0.397	3.129	0.582	0.517	0.000	0.000	0.000	5.474

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	247	371	244	0	0	0	2071
N.S.	1	1.00	0.72	1.08	0.71	0.00	0.00	0.00	6.06
time (sec)	N/A	0.325	2.655	0.580	0.514	0.000	0.000	0.000	2.546

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	216	337	222	0	0	0	1896
N.S.	1	1.00	0.69	1.08	0.71	0.00	0.00	0.00	6.06
time (sec)	N/A	0.286	1.085	0.508	0.503	0.000	0.000	0.000	1.413

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	193	332	220	0	0	0	1951
N.S.	1	1.00	0.62	1.06	0.70	0.00	0.00	0.00	6.23
time (sec)	N/A	0.289	3.551	0.494	0.508	0.000	0.000	0.000	1.204

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	104	331	223	0	0	0	1946
N.S.	1	1.00	0.33	1.06	0.71	0.00	0.00	0.00	6.22
time (sec)	N/A	0.318	0.382	0.526	0.526	0.000	0.000	0.000	1.671

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	108	359	243	0	0	0	1969
N.S.	1	1.00	0.31	1.05	0.71	0.00	0.00	0.00	5.74
time (sec)	N/A	0.401	0.660	0.492	0.508	0.000	0.000	0.000	3.063

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	116	388	267	0	0	0	1992
N.S.	1	1.00	0.31	1.03	0.71	0.00	0.00	0.00	5.28
time (sec)	N/A	0.467	0.740	0.463	0.523	0.000	0.000	0.000	5.257

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	286	347	191	0	0	0	2500
N.S.	1	1.00	0.88	1.07	0.59	0.00	0.00	0.00	7.69
time (sec)	N/A	0.441	0.889	0.605	0.539	0.000	0.000	0.000	1.973

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	249	326	177	0	0	0	2500
N.S.	1	1.00	0.82	1.08	0.59	0.00	0.00	0.00	8.28
time (sec)	N/A	0.254	0.544	0.610	0.504	0.000	0.000	0.000	1.627

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	226	332	176	0	0	0	2500
N.S.	1	1.00	0.75	1.10	0.58	0.00	0.00	0.00	8.28
time (sec)	N/A	0.243	0.262	0.614	0.510	0.000	0.000	0.000	1.386

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	248	332	176	0	0	0	2500
N.S.	1	1.00	0.82	1.10	0.58	0.00	0.00	0.00	8.28
time (sec)	N/A	0.244	0.245	0.599	0.514	0.000	0.000	0.000	1.768

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	198	354	191	0	0	0	2500
N.S.	1	1.00	0.61	1.09	0.59	0.00	0.00	0.00	7.69
time (sec)	N/A	0.433	0.454	0.546	0.500	0.000	0.000	0.000	1.860

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	109	371	207	0	0	0	2500
N.S.	1	1.00	0.31	1.06	0.59	0.00	0.00	0.00	7.12
time (sec)	N/A	0.594	0.256	0.578	0.520	0.000	0.000	0.000	2.807

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	445	409	304	0	0	0	2500
N.S.	1	1.00	1.02	0.94	0.70	0.00	0.00	0.00	5.72
time (sec)	N/A	0.742	6.160	0.649	0.516	0.000	0.000	0.000	3.970

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	390	387	286	0	0	0	2500
N.S.	1	1.00	0.99	0.98	0.73	0.00	0.00	0.00	6.36
time (sec)	N/A	0.504	2.810	0.625	0.501	0.000	0.000	0.000	3.121

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	322	391	276	0	0	0	2500
N.S.	1	1.00	0.83	1.01	0.71	0.00	0.00	0.00	6.46
time (sec)	N/A	0.448	3.392	0.658	0.507	0.000	0.000	0.000	3.366

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	401	391	278	0	0	0	2500
N.S.	1	1.00	1.04	1.01	0.72	0.00	0.00	0.00	6.48
time (sec)	N/A	0.426	6.096	0.628	0.531	0.000	0.000	0.000	3.084

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	300	396	285	0	0	0	2500
N.S.	1	1.00	0.76	1.01	0.72	0.00	0.00	0.00	6.35
time (sec)	N/A	0.484	2.934	0.654	0.534	0.000	0.000	0.000	8.163

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	244	414	323	0	0	0	2500
N.S.	1	1.00	0.56	0.95	0.74	0.00	0.00	0.00	5.72
time (sec)	N/A	0.741	0.624	0.632	0.508	0.000	0.000	0.000	4.335

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	556	471	442	0	0	0	2500
N.S.	1	1.00	1.05	0.89	0.84	0.00	0.00	0.00	4.73
time (sec)	N/A	1.090	6.257	0.713	0.521	0.000	0.000	0.000	10.399

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	525	460	428	0	0	0	2500
N.S.	1	1.00	1.10	0.97	0.90	0.00	0.00	0.00	5.25
time (sec)	N/A	0.813	6.190	0.656	0.522	0.000	0.000	0.000	7.262

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	488	457	417	0	0	0	2500
N.S.	1	1.00	1.04	0.97	0.89	0.00	0.00	0.00	5.32
time (sec)	N/A	0.835	6.173	0.695	0.531	0.000	0.000	0.000	6.515

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	518	456	410	0	0	0	2500
N.S.	1	1.00	1.12	0.99	0.89	0.00	0.00	0.00	5.42
time (sec)	N/A	0.751	6.150	0.652	0.507	0.000	0.000	0.000	6.213

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	483	460	419	0	0	0	2500
N.S.	1	1.00	1.04	0.99	0.90	0.00	0.00	0.00	5.40
time (sec)	N/A	0.748	6.188	0.742	0.557	0.000	0.000	0.000	6.127

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	411	465	427	0	0	0	2500
N.S.	1	1.00	0.86	0.98	0.90	0.00	0.00	0.00	5.25
time (sec)	N/A	0.820	6.143	0.709	0.521	0.000	0.000	0.000	6.789

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	303	480	472	0	0	0	2500
N.S.	1	1.00	0.57	0.91	0.89	0.00	0.00	0.00	4.73
time (sec)	N/A	1.125	1.899	0.654	0.535	0.000	0.000	0.000	9.999

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	118	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.326	0.541	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	3.296	0.766	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	731	0	159	0	0	1410
N.S.	1	1.00	1.00	16.24	0.00	3.53	0.00	0.00	31.33
time (sec)	N/A	0.051	0.096	0.962	0.000	2.359	0.000	0.000	2.536

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	70	739	0	159	0	0	1410
N.S.	1	1.00	1.56	16.42	0.00	3.53	0.00	0.00	31.33
time (sec)	N/A	0.046	1.427	0.469	0.000	2.534	0.000	0.000	1.403

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	67	91	89	79	524	95	155
N.S.	1	1.00	1.14	1.54	1.51	1.34	8.88	1.61	2.63
time (sec)	N/A	0.056	0.135	0.294	0.538	2.694	0.484	0.469	0.995

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	144	147	185	340	3966	241	268
N.S.	1	1.00	1.30	1.32	1.67	3.06	35.73	2.17	2.41
time (sec)	N/A	0.109	2.016	0.352	0.530	3.316	1.588	0.519	1.460

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	202	216	337	549	0	412	481
N.S.	1	1.00	1.15	1.23	1.93	3.14	0.00	2.35	2.75
time (sec)	N/A	0.192	5.210	0.404	0.536	2.859	0.000	0.581	2.602

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	379	1249	0	0	0	0	2500
N.S.	1	1.00	2.02	6.64	0.00	0.00	0.00	0.00	13.30
time (sec)	N/A	0.315	1.889	0.676	0.000	0.000	0.000	0.000	31.594

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	294	920	0	0	0	0	2823
N.S.	1	1.00	1.96	6.13	0.00	0.00	0.00	0.00	18.82
time (sec)	N/A	0.219	1.008	0.658	0.000	0.000	0.000	0.000	13.758

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	212	632	0	0	0	0	843
N.S.	1	1.00	1.74	5.18	0.00	0.00	0.00	0.00	6.91
time (sec)	N/A	0.150	0.584	0.580	0.000	0.000	0.000	0.000	3.029

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	253	682	0	0	0	0	2500
N.S.	1	1.00	1.68	4.52	0.00	0.00	0.00	0.00	16.56
time (sec)	N/A	0.182	4.128	0.608	0.000	0.000	0.000	0.000	26.556

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	178	390	0	0	0	0	2529
N.S.	1	1.00	0.44	0.96	0.00	0.00	0.00	0.00	6.20
time (sec)	N/A	0.373	2.064	0.626	0.000	0.000	0.000	0.000	11.956

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	158	802	0	0	0	0	583
N.S.	1	1.00	0.37	1.90	0.00	0.00	0.00	0.00	1.38
time (sec)	N/A	0.352	1.069	0.638	0.000	0.000	0.000	0.000	2.571

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	154	1407	0	0	0	0	2909
N.S.	1	1.00	1.51	13.79	0.00	0.00	0.00	0.00	28.52
time (sec)	N/A	0.111	0.538	0.595	0.000	0.000	0.000	0.000	2.290

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	226	2274	0	0	0	0	2500
N.S.	1	1.00	1.64	16.48	0.00	0.00	0.00	0.00	18.12
time (sec)	N/A	0.187	1.769	0.623	0.000	0.000	0.000	0.000	6.466

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	319	3236	0	0	0	0	2500
N.S.	1	1.00	1.72	17.49	0.00	0.00	0.00	0.00	13.51
time (sec)	N/A	0.283	3.661	0.664	0.000	0.000	0.000	0.000	17.933

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	156	783	0	0	0	0	2731
N.S.	1	1.00	1.53	7.68	0.00	0.00	0.00	0.00	26.77
time (sec)	N/A	0.116	0.485	0.655	0.000	0.000	0.000	0.000	2.202

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	216	928	0	0	0	0	2500
N.S.	1	1.00	1.64	7.03	0.00	0.00	0.00	0.00	18.94
time (sec)	N/A	0.165	1.576	0.632	0.000	0.000	0.000	0.000	5.961

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	232	1182	0	0	0	0	2500
N.S.	1	1.00	1.33	6.79	0.00	0.00	0.00	0.00	14.37
time (sec)	N/A	0.263	6.190	0.649	0.000	0.000	0.000	0.000	16.173

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [90] had the largest ratio of [27]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	15	0.133
2	A	5	3	1.00	23	0.130
3	A	4	3	1.00	23	0.130
4	A	3	3	1.00	23	0.130
5	A	2	2	1.00	23	0.087
6	A	3	3	1.00	23	0.130
7	A	4	3	1.00	23	0.130
8	A	16	12	1.00	25	0.480
9	A	15	12	1.00	25	0.480
10	A	15	12	1.00	25	0.480
11	A	14	11	1.00	25	0.440
12	A	13	10	1.00	25	0.400
13	A	14	11	1.00	25	0.440
14	A	14	11	1.00	25	0.440
15	A	7	5	1.00	25	0.200
16	A	6	5	1.00	25	0.200
17	A	5	5	1.00	25	0.200
18	A	4	4	1.00	25	0.160
19	A	4	4	1.00	25	0.160
20	A	4	4	1.00	25	0.160
21	A	5	5	1.00	25	0.200
22	A	6	5	1.00	25	0.200
23	A	7	6	1.00	25	0.240
24	A	6	6	1.00	25	0.240
25	A	6	5	1.00	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	6	6	1.00	25	0.240
27	A	7	6	1.00	25	0.240
28	A	10	10	1.00	25	0.400
29	A	17	14	1.00	25	0.560
30	A	18	15	1.00	25	0.600
31	A	17	14	1.00	25	0.560
32	A	18	15	1.00	25	0.600
33	A	18	15	1.00	25	0.600
34	A	20	16	1.00	25	0.640
35	A	8	8	1.00	25	0.320
36	A	8	7	1.00	25	0.280
37	A	8	8	1.00	25	0.320
38	A	8	7	1.00	25	0.280
39	A	9	8	1.00	25	0.320
40	A	10	8	1.00	25	0.320
41	A	12	9	1.00	13	0.692
42	A	6	5	1.00	11	0.454
43	A	8	7	1.00	13	0.538
44	A	14	9	1.00	11	0.818
45	A	12	8	1.00	13	0.615
46	A	5	4	1.00	11	0.364
47	A	6	5	1.00	13	0.385
48	A	13	10	1.00	11	0.909
49	A	8	7	1.00	13	0.538
50	A	13	9	1.00	11	0.818
51	A	12	8	1.00	23	0.348
52	A	11	8	1.00	23	0.348
53	A	10	7	1.00	23	0.304
54	A	11	8	1.00	23	0.348
55	A	12	8	1.00	23	0.348
56	A	13	9	1.00	25	0.360
57	A	12	9	1.00	25	0.360
58	A	11	8	1.00	25	0.320
59	A	11	8	1.00	25	0.320
60	A	12	9	1.00	25	0.360

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	13	9	1.00	25	0.360
62	A	14	10	1.00	25	0.400
63	A	13	10	1.00	25	0.400
64	A	12	9	1.00	25	0.360
65	A	12	9	1.00	25	0.360
66	A	12	9	1.00	25	0.360
67	A	13	10	1.00	25	0.400
68	A	14	10	1.00	25	0.400
69	A	15	12	1.00	25	0.480
70	A	14	11	1.00	25	0.440
71	A	14	11	1.00	25	0.440
72	A	14	11	1.00	25	0.440
73	A	15	12	1.00	25	0.480
74	A	16	13	1.00	25	0.520
75	A	16	13	1.00	25	0.520
76	A	15	12	1.00	25	0.480
77	A	15	12	1.00	25	0.480
78	A	15	12	1.00	25	0.480
79	A	15	12	1.00	25	0.480
80	A	16	13	1.00	25	0.520
81	A	17	14	1.00	25	0.560
82	A	16	13	1.00	25	0.520
83	A	16	13	1.00	25	0.520
84	A	16	13	1.00	25	0.520
85	A	16	13	1.00	25	0.520
86	A	16	13	1.00	25	0.520
87	A	17	13	1.00	25	0.520
88	A	5	3	1.00	12	0.250
89	A	8	5	1.00	23	0.217
90	A	3	3	1.00	27	0.111
91	A	3	3	1.00	27	0.111
92	A	2	2	1.00	23	0.087
93	A	3	3	1.00	23	0.130
94	A	4	3	1.00	23	0.130
95	A	10	5	1.00	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	9	5	1.00	25	0.200
97	A	8	5	1.00	25	0.200
98	A	10	7	1.00	27	0.259
99	A	13	9	1.00	27	0.333
100	A	13	9	1.00	27	0.333
101	A	7	4	1.00	25	0.160
102	A	8	5	1.00	25	0.200
103	A	9	5	1.00	25	0.200
104	A	7	4	1.00	27	0.148
105	A	8	5	1.00	27	0.185
106	A	9	5	1.00	27	0.185

Chapter 3

Listing of integrals

Local contents

3.1 $\int (a+ia\cot(c+dx))^n\,dx$

Optimal. Leaf size=49

$$\frac{i(a+ia\cot(c+dx))^n\,{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+i\cot(c+dx))\right)}{2dn}$$

```
[Out] 1/2*I*(a+I*a*cot(d*x+c))^n*hypergeom([1, n],[1+n],1/2+1/2*I*cot(d*x+c))/d/n
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3562, 70}

$$\frac{i(a+ia\cot(c+dx))^n\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(i\cot(c+dx)+1)\right)}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Cot[c + d*x])^n,x]
```

```
[Out] ((I/2)*(a + I*a*Cot[c + d*x])^n*Hypergeometric2F1[1, n, 1 + n, (1 + I*Cot[c
 + d*x])/2])/(d*n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\cot(c+dx))^n\,dx &= \frac{(ia)\text{Subst}\left(\int \frac{(a+x)^{-1+n}}{a-x}\,dx, x, ia\cot(c+dx)\right)}{d}\\ &= \frac{i(a+ia\cot(c+dx))^n\,{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+i\cot(c+dx))\right)}{2dn}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 117 vs. $2(49) = 98$.
time = 0.34, size = 117, normalized size = 2.39

$$\frac{i(a+ia\cot(c+dx))^n\left(2(1+n)\,{}_2F_1(1,n;1+n;1+i\cot(c+dx))+(n+in\cot(c+dx))\left({}_2F_1\left(1,1+n;2+n;\frac{1}{2}(1+i\cot(c+dx))\right)-2\,{}_2F_1(1,1+n;2+n;1+i\cot(c+dx))\right)\right)}{4dn(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Cot[c + d*x])^n,x]
```

```
[Out] ((I/4)*(a + I*a*Cot[c + d*x])^n*(2*(1 + n)*Hypergeometric2F1[1, n, 1 + n, 1
 + I*Cot[c + d*x]] + (n + I*n*Cot[c + d*x])*(Hypergeometric2F1[1, 1 + n, 2
+ n, (1 + I*Cot[c + d*x])/2] - 2*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + I*C
ot[c + d*x]])))/(d*n*(1 + n))
```

Maple [F]
time = 0.81, size = 0, normalized size = 0.00

$$\int (a + ia\cot(dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*cot(d*x+c))^n,x)
```

```
[Out] int((a+I*a*cot(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*cot(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*cot(d*x + c) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*cot(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((-2*a/(e^(2*I*d*x + 2*I*c) - 1))^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia\cot(c+dx)+a)^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*cot(d*x+c))**n,x)
```

```
[Out] Integral((I*a*cot(c + d*x) + a)**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*cot(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*cot(d*x + c) + a)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a+a\cot(c+d\,x)\ 1\mathrm{i})^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(c + d*x)*1i)^n,x)
```

```
[Out] int((a + a*cot(c + d*x)*1i)^n, x)
```

3.2 $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx$

Optimal. Leaf size=116

$$-\frac{\sqrt{2}\,ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d}+\frac{2ae^2\sqrt{e\cot(c+dx)}}{d}-\frac{2ae(e\cot(c+dx))^{3/2}}{3d}-\frac{2a(e\cot(c+dx)}{5d}$$

```
[Out] -2/3*a*e*(e*cot(d*x+c))^(3/2)/d-2/5*a*(e*cot(d*x+c))^(5/2)/d-a*e^(5/2)*arct
anh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/
d+2*a*e^2*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3609, 3613, 214}

$$-\frac{\sqrt{2}\,ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,\cot(c+dx)+\sqrt{e}}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d}+\frac{2ae^2\sqrt{e\cot(c+dx)}}{d}-\frac{2ae(e\cot(c+dx))^{3/2}}{3d}-\frac{2a(e\cot(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x]),x]
```

```
[Out] -((Sqrt[2]*a*e^(5/2)*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt
[e*Cot[c + d*x]])])/d) + (2*a*e^2*Sqrt[e*Cot[c + d*x]])/d - (2*a*e*(e*Cot[c
 + d*x])^(3/2))/(3*d) - (2*a*(e*Cot[c + d*x])^(5/2))/(5*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx = -\frac{2a(e\cot(c+dx))^{5/2}}{5d} + \int (e\cot(c+dx))^{3/2}(-ae+ae\cot(c+$$
$$= -\frac{2ae(e\cot(c+dx))^{3/2}}{3d} - \frac{2a(e\cot(c+dx))^{5/2}}{5d} + \int \sqrt{e\cot(c}$$
$$= \frac{2ae^2\sqrt{e\cot(c+dx)}}{d} - \frac{2ae(e\cot(c+dx))^{3/2}}{3d} - \frac{2a(e\cot(c+}{5d}$$
$$= \frac{2ae^2\sqrt{e\cot(c+dx)}}{d} - \frac{2ae(e\cot(c+dx))^{3/2}}{3d} - \frac{2a(e\cot(c+}{5d}$$
$$= -\frac{\sqrt{2}\,ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} + \frac{2ae^2\sqrt{e\cot(c+}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.21, size = 68, normalized size = 0.59

$$-\frac{2ae(e\cot(c+dx))^{3/2}\left(3\cot(c+dx)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\tan^2(c+dx)\right)+5\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(c+dx)\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x]),x]
```

```
[Out] (-2*a*e*(e*Cot[c + d*x])^(3/2)*(3*Cot[c + d*x]*Hypergeometric2F1[-5/4, 1, -
1/4, -Tan[c + d*x]^2] + 5*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2])
)/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(95) = 190.
time = 0.42, size = 319, normalized size = 2.75

method	result
derivativedivides	$-\frac{a\left(\frac{2(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e(e\cot(dx+c))^{\frac{3}{2}}}{3}-2e^2\sqrt{e\cot(dx+c)}+2e^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot}}\right.\right.}{}\right.\right.}{}$

default	$-\frac{a\left(\frac{2(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e(e\cot(dx+c))^{\frac{3}{2}}}{3}-2e^2\sqrt{e\cot(dx+c)}+2e^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(2/5*(e*cot(d*x+c))^(5/2)+2/3*e*(e*cot(d*x+c))^(3/2)-2*e^2*(e*cot(d*
x+c))^(1/2)+2*e^3*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(e
^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2
)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)
^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(
1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.53, size = 101, normalized size = 0.87

$$-\frac{\left(15\left(\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a-\frac{60\,a}{\sqrt{\tan(dx+c)}}+\frac{20\,a}{\tan(dx+c)^{\frac{3}{2}}}+\frac{12\,a}{\tan(dx+c)^{\frac{5}{2}}}\right)e^{\frac{5}{2}}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(15*(sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - s
qrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a - 60*a/sqrt
(tan(d*x + c)) + 20*a/tan(d*x + c)^(3/2) + 12*a/tan(d*x + c)^(5/2))*e^(5/2)
/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(88) = 176$.
time = 3.29, size = 181, normalized size = 1.56

$$\frac{15\left(\sqrt{2}\,a\cos(2\,dx+2\,c)e^{\frac{5}{2}}-\sqrt{2}\,ae^{\frac{5}{2}}\right)\log\left(\left(\sqrt{2}\cos(2\,dx+2\,c)-\sqrt{2}\sin(2\,dx+2\,c)-\sqrt{2}\right)\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}+2\sin(2\,dx+2\,c)+1\right)+4\left(18\,a\cos(2\,dx+2\,c)e^{\frac{5}{2}}+5\,ae^{\frac{5}{2}}\sin(2\,dx+2\,c)-12\,ae^{\frac{5}{2}}\right)\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}}{30\,(d\cos(2\,dx+2\,c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*(sqrt(2)*a*cos(2*d*x + 2*c)*e^(5/2) - sqrt(2)*a*e^(5/2))*log((sqrt
(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) - sqrt(2))*sqrt((cos(2*d*x
+ 2*c) + 1)/sin(2*d*x + 2*c)) + 2*sin(2*d*x + 2*c) + 1) + 4*(18*a*cos(2*d*x
```

```
 + 2*c)*e^(5/2) + 5*a*e^(5/2)*sin(2*d*x + 2*c) - 12*a*e^(5/2))*sqrt((cos(2*
d*x + 2*c) + 1)/sin(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (e\cot(c+dx))^{\frac{5}{2}}\,dx + \int (e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*cot(d*x+c)),x)
```

```
[Out] a*(Integral((e*cot(c + d*x))**(5/2), x) + Integral((e*cot(c + d*x))**(5/2)*
cot(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)*(e*cot(d*x + c))^(5/2), x)
```

Mupad [B]
time = 1.98, size = 144, normalized size = 1.24

$$\frac{2\,a\,e^2\,\sqrt{e\,\cot\left(c+d\,x\right)}}{d}-\frac{2\,a\,e\,{\left(e\,\cot\left(c+d\,x\right)\right)}^{3/2}}{3\,d}-\frac{2\,a\,{\left(e\,\cot\left(c+d\,x\right)\right)}^{5/2}}{5\,d}+\frac{{\left(-1\right)}^{1/4}\,a\,e^{5/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\cot\left(c+d\,x\right)}\,1{}\mathrm{i}}{\sqrt{e}}\right)}{d}-\frac{{\left(-1\right)}^{1/4}\,a\,e^{5/2}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{d}+\frac{{\left(-1\right)}^{1/4}\,a\,e^{5/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,\left(1+1{}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x)),x)
```

```
[Out] (2*a*e^2*(e*cot(c + d*x))^(1/2))/d - (2*a*e*(e*cot(c + d*x))^(3/2))/(3*d) -
 (2*a*(e*cot(c + d*x))^(5/2))/(5*d) + ((-1)^(1/4)*a*e^(5/2)*atan(((-1)^(1/4
)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1 + 1i))/d + ((-1)^(1/4)*a*e^(5/2)*atan
(((-1)^(1/4)*(e*cot(c + d*x))^(1/2)*1i)/e^(1/2)))/d - ((-1)^(1/4)*a*e^(5/2)
*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/d
```

3.3 $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx$

Optimal. Leaf size=94

$$-\frac{\sqrt{2}\,ae^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2a(e\cot(c+dx))^{3/2}}{3d}$$

```
[Out] -2/3*a*(e*cot(d*x+c))^(3/2)/d-a*e^(3/2)*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1
/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d-2*a*e*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3609, 3613, 211}

$$-\frac{\sqrt{2}\,ae^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2a(e\cot(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x]),x]
```

```
[Out] -((Sqrt[2]*a*e^(3/2)*ArcTan[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[
e*Cot[c + d*x]])])/d) - (2*a*e*Sqrt[e*Cot[c + d*x]])/d - (2*a*(e*Cot[c + d*
x])^(3/2))/(3*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = -\frac{2a(e\cot(c+dx))^{3/2}}{3d} + \int \sqrt{e\cot(c+dx)}\,(-ae+ae\cot(c+$$

$$= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2a(e\cot(c+dx))^{3/2}}{3d} + \int \frac{-ae^2-ae^2}{\sqrt{e\cot}}$$

$$= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2a(e\cot(c+dx))^{3/2}}{3d} - \frac{(2a^2e^4)\,\mathrm{Subst}}{}$$

$$= -\frac{\sqrt{2}\,ae^{3/2}\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} - \frac{2ae\sqrt{e\cot(c+d}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 67, normalized size = 0.71

$$-\frac{2ae\sqrt{e\cot(c+dx)}\left(\cot(c+dx)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(c+dx)\right)+3\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x]),x]
```

```
[Out] (-2*a*e*Sqrt[e*Cot[c + d*x]]*(Cot[c + d*x]*Hypergeometric2F1[-3/4, 1, 1/4,
-Tan[c + d*x]^2] + 3*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2]))/(3*
d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(77) = 154.
time = 0.37, size = 304, normalized size = 3.23

method	result
derivativedivides	$-\frac{a\left(\frac{2(e\cot(dx+c))^{\frac{3}{2}}}{3}+2e\sqrt{e\cot(dx+c)}-2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}}\right.\right.}{}\right.\right.}{}$

default	$-\frac{a\left(\frac{2(e\cot(dx+c))^{\frac{3}{2}}}{3}+2e\sqrt{e\cot(dx+c)}-2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(2/3*(e*cot(d*x+c))^(3/2)+2*e*(e*cot(d*x+c))^(1/2)-2*e^2*(1/8/e*(e^2
)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(
1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/
2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(
d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)+1))))
```

Maxima [A]
time = 0.53, size = 82, normalized size = 0.87

$$\frac{\left(3\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)\right)a-\frac{6a}{\sqrt{\tan(dx+c)}}-\frac{2a}{\tan(dx+c)^{\frac{3}{2}}}\right)e^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))))*a - 6*a/sqrt(tan
(d*x + c)) - 2*a/tan(d*x + c)^(3/2))*e^(3/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(74) = 148$.
time = 3.60, size = 165, normalized size = 1.76

$$-\frac{3\sqrt{2}\,a\arctan\left(-\frac{\left(\sqrt{2}\cos(2dx+2c)-\sqrt{2}\sin(2dx+2c)+\sqrt{2}\right)\sqrt{\frac{\cos(2dx+2c)+1}{\sin(2dx+2c)}}}{2(\cos(2dx+2c)+1)}\right)e^{\frac{3}{2}}\sin(2dx+2c)+2\left(a\cos(2dx+2c)e^{\frac{3}{2}}+3ae^{\frac{3}{2}}\sin(2dx+2c)+ae^{\frac{3}{2}}\right)\sqrt{\frac{\cos(2dx+2c)+1}{\sin(2dx+2c)}}}{3d\sin(2dx+2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*sqrt(2)*a*arctan(-1/2*(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x
 + 2*c) + sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))/(cos(2*d*x
```

```
 + 2*c) + 1))*e^(3/2)*sin(2*d*x + 2*c) + 2*(a*cos(2*d*x + 2*c)*e^(3/2) + 3*
a*e^(3/2)*sin(2*d*x + 2*c) + a*e^(3/2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d
*x + 2*c)))/(d*sin(2*d*x + 2*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (e\cot(c+dx))^{\frac{3}{2}}\,dx+\int (e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*cot(d*x+c)),x)
```

```
[Out] a*(Integral((e*cot(c + d*x))**(3/2), x) + Integral((e*cot(c + d*x))**(3/2)*
cot(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)*(e*cot(d*x + c))^(3/2), x)
```

Mupad [B]
time = 1.16, size = 98, normalized size = 1.04

$$-\frac{2\,a\,{\left(e\,\mathrm{cot}\left(c+d\,x\right)\right)}^{3/2}}{3\,d}-\frac{2\,a\,e\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}{d}+\frac{{\left(-1\right)}^{1/4}\,a\,e^{3/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}{\sqrt{e}}\right)\,\left(1-\mathrm{i}\right)}{d}+\frac{{\left(-1\right)}^{1/4}\,a\,e^{3/2}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}{\sqrt{e}}\right)\,\left(-1-\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x)),x)
```

```
[Out] ((-1)^(1/4)*a*e^(3/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1
- 1i))/d - (2*a*e*(e*cot(c + d*x))^(1/2))/d - (2*a*(e*cot(c + d*x))^(3/2))/
(3*d) - ((-1)^(1/4)*a*e^(3/2)*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(
1/2))*(1 + 1i))/d
```

3.4 $\int \sqrt{e \cot(c+dx)}\,(a+a\cot(c+dx))\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{2}\,a\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{2a\sqrt{e\cot(c+dx)}}{d}$$

```
[Out] a*arctanh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^
(1/2)*e^(1/2)/d-2*a*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3609, 3613, 214}

$$\frac{\sqrt{2}\,a\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\cot(c+dx)+\sqrt{e}}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{2a\sqrt{e\cot(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x]),x]
```

```
[Out] (Sqrt[2]*a*Sqrt[e]*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e
*Cot[c + d*x]])])/d - (2*a*Sqrt[e*Cot[c + d*x]])/d
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))\,dx = -\frac{2a\sqrt{e\cot(c+dx)}}{d}+\int \frac{-ae+ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$$

$$= -\frac{2a\sqrt{e\cot(c+dx)}}{d}-\frac{(2a^2e^2)\,\text{Subst}\left(\int \frac{1}{2a^2e^2-ex^2}\,dx, x, \frac{-ae-a\ldots}{\sqrt{e\,\text{co}\ldots}}\right.}{d}$$

$$= \frac{\sqrt{2}\,a\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{2a\sqrt{e\cot(c+dx)\ldots}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.30, size = 154, normalized size = 2.17

$$-\frac{a\sqrt{e\cot(c+dx)}\left(8\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(c+dx)\right)+\sqrt{2}\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)\sqrt{\tan(c+dx)}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x]),x]
```

```
[Out] -1/4*(a*Sqrt[e*Cot[c + d*x]]*(8*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*
x]^2] + Sqrt[2]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c + d*x]
]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(58) = 116$.
time = 0.40, size = 289, normalized size = 4.07

method	result
derivativedivides	$-\frac{a\left(2\sqrt{e\cot(dx+c)}-2e\left(\frac{(e^2)^{\frac14}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\,\text{arcta}\ldots\right.}{\ldots}\right.\right.}{\ldots}$
default	$-\frac{a\left(2\sqrt{e\cot(dx+c)}-2e\left(\frac{(e^2)^{\frac14}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\,\text{arcta}\ldots\right.}{\ldots}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(2*(e*cot(d*x+c))^(1/2)-2*e*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*
x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)+1))-1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.52, size = 78, normalized size = 1.10

$$\frac{\left(\left(\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)\right)a-\frac{4\,a}{\sqrt{\tan\left(dx+c\right)}}\right)e^{\frac{1}{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2
)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a - 4*a/sqrt(tan(d
*x + c)))*e^(1/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(55) = 110$.
time = 3.51, size = 117, normalized size = 1.65

$$\frac{\sqrt{2}\,ae^{\frac{1}{2}}\log\left(-\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)-\sqrt{2}\sin\left(2\,dx+2\,c\right)-\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}+2\,\sin\left(2\,dx+2\,c\right)+1\right)-4\,a\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\,e^{\frac{1}{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*a*e^(1/2)*log(-(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x +
 2*c) - sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)) + 2*sin(2*d*
x + 2*c) + 1) - 4*a*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*e^(1/2))/
d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{e\cot\left(c+dx\right)}\,dx+\int\sqrt{e\cot\left(c+dx\right)}\cot\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)*(a+a*cot(d*x+c)),x)
```

```
[Out] a*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(sqrt(e*cot(c + d*x))*cot(c
+ d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)*sqrt(e*cot(d*x + c)), x)
```

Mupad [B]
time = 0.78, size = 128, normalized size = 1.80

$$-\frac{2\,a\,\sqrt{e\cot\left(c+d\,x\right)}}{d}-\frac{{\left(-1\right)}^{1/4}\,a\,\sqrt{e}\,\left(\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)-\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\right)}{d}-\frac{{\left(-1\right)}^{1/4}\,a\,\sqrt{e}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,1{}\mathrm{i}}{d}-\frac{{\left(-1\right)}^{1/4}\,a\,\sqrt{e}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x)),x)
```

```
[Out] - (2*a*(e*cot(c + d*x))^(1/2))/d - ((-1)^(1/4)*a*e^(1/2)*atan(((-1)^(1/4)*(
e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*a*e^(1/2)*atanh(((-1)^(
1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*a*e^(1/2)*(atan((
(-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)) - atanh(((-1)^(1/4)*(e*cot(c +
d*x))^(1/2))/e^(1/2))))/d
```

3.5 $\int \frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal. Leaf size=49

$$\frac{\sqrt{2}\,a\text{ArcTan}\left(\frac{\sqrt{e}\,(1-\cot(c+dx))}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}}$$

```
[Out] a*arctan(1/2*(1-cot(d*x+c))*e^(1/2)*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d
/e^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3613, 211}

$$\frac{\sqrt{2}\,a\text{ArcTan}\left(\frac{\sqrt{e}\,(1-\cot(c+dx))}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cot[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (Sqrt[2]*a*ArcTan[(Sqrt[e]*(1 - Cot[c + d*x]))/(Sqrt[2]*Sqrt[e*Cot[c + d*x]
])])/(d*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = -\frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{-2a^2-ex^2}\,dx, x, \frac{a-a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\,a\tan^{-1}\left(\frac{\sqrt{e}\,(1-\cot(c+dx))}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.22, size = 165, normalized size = 3.37

$$\frac{a\left(3\sqrt{2}\left(-2\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)+8\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)\right)}{12d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cot[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (a*(3*Sqrt[2]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) + 8*Hypergeometric
2F1[3/4, 1, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(3/2)))/(12*d*Sqrt[e*Cot[c +
 d*x]]*Sqrt[Tan[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(40) = 80$.
time = 0.49, size = 273, normalized size = 5.57

method	result
derivativedivides	$-\frac{a\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
default	$-\frac{a\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/4/(e^2)^(1/4)*2
^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*
arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Maxima [A]
time = 0.51, size = 57, normalized size = 1.16

$$-\frac{\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)\right)ae^{\left(-\frac{1}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))))*a*e^(-1/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(32) = 64.
time = 4.41, size = 82, normalized size = 1.67

$$\frac{\sqrt{2}\,a\arctan\left(-\frac{\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)-\sqrt{2}\sin\left(2\,dx+2\,c\right)+\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{2\left(\cos\left(2\,dx+2\,c\right)+1\right)}\right)e^{\left(-\frac{1}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*a*arctan(-1/2*(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c)
+ sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))/(cos(2*d*x + 2*c)
+ 1))*e^(-1/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{1}{\sqrt{e\cot\left(c+dx\right)}}\,dx+\int\frac{\cot\left(c+dx\right)}{\sqrt{e\cot\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(cot(c + d*x)/sqrt(e*cot(c
 + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)/sqrt(e*cot(d*x + c)), x)
```

Mupad [B]
time = 0.73, size = 65, normalized size = 1.33

$$\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,\left(-1+1\mathrm{i}\right)}{d\,\sqrt{e}}+\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,\left(1+1\mathrm{i}\right)}{d\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(c + d*x))/(e*cot(c + d*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1 + 1i))/
(d*e^(1/2)) - ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2
))*(1 - 1i))/(d*e^(1/2))
```

3.6 $\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}}+\frac{2a}{de\sqrt{e\cot(c+dx)}}$$

```
[Out] -a*arctanh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2
^(1/2)/d/e^(3/2)+2*a/d/e/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3610, 3613, 214}

$$\frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cot[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] -((Sqrt[2]*a*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c
 + d*x]])])/(d*e^(3/2))) + (2*a)/(d*e*Sqrt[e*Cot[c + d*x]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx &= \frac{2a}{de\sqrt{e\cot(c+dx)}}+\frac{\int \frac{ae-ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^2}\\
&= \frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{2a^2e^2-ex^2}\,dx,x,\frac{ae+ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}\\
&= -\frac{\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}}+\frac{2a}{de\sqrt{e\cot(c+dx)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.26, size = 191, normalized size = 2.55

$$
\frac{a\left(6\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)-6\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)+3\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-3\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+24\sqrt{\tan(c+dx)}+8\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)\right)}{12d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cot[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (a*(6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 6*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]]] + 3*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]] - 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d
*x]] + 24*Sqrt[Tan[c + d*x]] + 8*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*
x]^2]*Tan[c + d*x]^(3/2)))/(12*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(62) = 124.
time = 0.34, size = 294, normalized size = 3.92

method	result
derivativedivides	$-\frac{a\left(-\frac{2}{e\sqrt{e\cot\left(dx+c\right)}}+\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\right.}{4e}\right.}{}$

default	$-\frac{a\left(-\frac{2}{e\sqrt{e\cot(dx+c)}}+\frac{\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\cdots\right.\right.}{4e}}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(-2/e/(e*cot(d*x+c))^(1/2)+2/e*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot
(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)
-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1))-1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.51, size = 78, normalized size = 1.04

$$-\frac{\left(\left(\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a-4a\sqrt{\tan(dx+c)}\right)e^{\left(-\frac{3}{2}\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(
2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a - 4*a*sqrt(tan(
d*x + c)))*e^(-3/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(56) = 112.
time = 4.42, size = 154, normalized size = 2.05

$$\frac{4a\sqrt{\frac{\cos(2dx+2c)+1}{\sin(2dx+2c)}}\sin(2dx+2c)+\left(\sqrt{2}a\cos(2dx+2c)+\sqrt{2}a\right)\log\left(\left(\sqrt{2}\cos(2dx+2c)-\sqrt{2}\sin(2dx+2c)-\sqrt{2}\right)\sqrt{\frac{\cos(2dx+2c)+1}{\sin(2dx+2c)}}+2\sin(2dx+2c)+1\right)}{2\left(d\cos(2dx+2c)e^{\frac{3}{2}}+de^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + (
sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*log((sqrt(2)*cos(2*d*x + 2*c) - sqr
```

```
t(2)*sin(2*d*x + 2*c) - sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*
c)) + 2*sin(2*d*x + 2*c) + 1))/(d*cos(2*d*x + 2*c)*e^(3/2) + d*e^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\cot(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(cot(c + d*x)/(e*cot(c +
 d*x))**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)/(e*cot(d*x + c))^(3/2), x)
```

Mupad [B]
time = 0.96, size = 84, normalized size = 1.12

$$\frac{2\,a}{d\,e\,\sqrt{e\cot(c+d\,x)}} + \frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(1+1\mathrm{i})}{d\,e^{3/2}} + \frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(-1+1\mathrm{i})}{d\,e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(c + d*x))/(e*cot(c + d*x))^(3/2),x)
```

```
[Out] (2*a)/(d*e*(e*cot(c + d*x))^(1/2)) + ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(
c + d*x))^(1/2))/e^(1/2))*(1 + 1i))/(d*e^(3/2)) - ((-1)^(1/4)*a*atanh(((-1)
^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1 - 1i))/(d*e^(3/2))
```

3.7 $\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{\sqrt{2}\,a\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}+\frac{2a}{de^2\sqrt{e\cot(c+dx)}}$$

```
[Out] 2/3*a/d/e/(e*cot(d*x+c))^(3/2)-a*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^
(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d/e^(5/2)+2*a/d/e^2/(e*cot(d*x+c))^(1/2
)
```

Rubi [A]
time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3610, 3613, 211}

$$-\frac{\sqrt{2}\,a\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}}+\frac{2a}{de^2\sqrt{e\cot(c+dx)}}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cot[c + d*x])/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] -((Sqrt[2]*a*ArcTan[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c
+ d*x]])])/(d*e^(5/2))) + (2*a)/(3*d*e*(e*Cot[c + d*x])^(3/2)) + (2*a)/(d*e
^2*Sqrt[e*Cot[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{\int \frac{ae-ae\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{e^2}$$

$$= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2a}{de^2\sqrt{e\cot(c+dx)}} + \frac{\int \frac{-ae^2-ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^4}$$

$$= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2a}{de^2\sqrt{e\cot(c+dx)}} - \frac{(2a^2)\,\mathrm{Subst}\Big(\int \frac{1}{-2a^2e^4-ex^2}\,dx, x, -$$

$$= -\frac{\sqrt{2}\,a\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}} + \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2a}{de^2\sqrt{e\cot(c+$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.41, size = 203, normalized size = 2.05

$$\frac{a\left(6\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-6\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+3\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-3\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+24\sqrt{\tan(c+dx)}+8\tan^{\frac{3}{2}}(c+dx)-8\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)\right)}{12d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cot[c + d*x])/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] (a*(6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 6*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]]] + 3*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]] - 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d
*x]] + 24*Sqrt[Tan[c + d*x]] + 8*Tan[c + d*x]^(3/2) - 8*Hypergeometric2F1[3
/4, 1, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(3/2)))/(12*d*(e*Cot[c + d*x])^(5
/2)*Tan[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(82) = 164$.
time = 0.37, size = 309, normalized size = 3.12

method	result

derivativedivides	$-\frac{a\left(-\frac{2}{e^{2}\sqrt{e\cot\left(dx+c\right)}}-\frac{2}{3e\left(e\cot\left(dx+c\right)\right)^{\frac{3}{2}}}+\frac{-\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}\cdots}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}\cdots}\right)\cdots\right.}{\cdots}}{\cdots}\right)}{\cdots}$
default	$-\frac{a\left(-\frac{2}{e^{2}\sqrt{e\cot\left(dx+c\right)}}-\frac{2}{3e\left(e\cot\left(dx+c\right)\right)^{\frac{3}{2}}}+\frac{-\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}\cdots}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}\cdots}\right)\cdots\right.}{\cdots}}{\cdots}\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(-2/e^2/(e*cot(d*x+c))^(1/2)-2/3/e/(e*cot(d*x+c))^(3/2)+2/e^2*(-1/8/
e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(
-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(e^2)^(1/4)*2^(1/2)*(ln((
e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d
*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/
2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.50, size = 83, normalized size = 0.84

$$\frac{\left(2\left(a+\frac{3a}{\tan(dx+c)}\right)\tan\left(dx+c\right)^{\frac{3}{2}}+3\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)\right)a\right)e^{\left(-\frac{5}{2}\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*(a + 3*a/tan(d*x + c))*tan(d*x + c)^(3/2) + 3*(sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt
(2) - 2/sqrt(tan(d*x + c)))))*a)*e^(-5/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(74) = 148$.

time = 3.19, size = 171, normalized size = 1.73

$$-\frac{3\left(\sqrt{2}\,a\cos\left(2\,dx+2\,c\right)+\sqrt{2}\,a\right)\arctan\left(-\frac{\left(\sqrt{2}\,\cos(2\,dx+2\,c)-\sqrt{2}\,\sin(2\,dx+2\,c)+\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{2\left(\cos(2\,dx+2\,c)+1\right)}\right)+2\left(a\cos\left(2\,dx+2\,c\right)-3\,a\sin\left(2\,dx+2\,c\right)-a\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{3\left(d\cos\left(2\,dx+2\,c\right)e^{\frac{5}{2}}+de^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*arctan(-1/2*(sqrt(2)*cos(2
*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt((cos(2*d*x + 2*c) +
1)/sin(2*d*x + 2*c))/(cos(2*d*x + 2*c) + 1)) + 2*(a*cos(2*d*x + 2*c) - 3*a*
sin(2*d*x + 2*c) - a)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)))/(d*cos
(2*d*x + 2*c)*e^(5/2) + d*e^(5/2))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx+\int\frac{\cot\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))**(5/2),x)
```

```
[Out] a*(Integral((e*cot(c + d*x))**(-5/2), x) + Integral(cot(c + d*x)/(e*cot(c +
 d*x))**(5/2), x))
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)/(e*cot(d*x + c))^(5/2), x)
```

Mupad [B]

time = 1.48, size = 103, normalized size = 1.04

$$\frac{2\,a}{d\,e^2\,\sqrt{e\cot\left(c+d\,x\right)}}+\frac{2\,a}{3\,d\,e\,{\left(e\cot\left(c+d\,x\right)\right)}^{3/2}}+\frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,\left(1-\mathrm{i}\right)}{d\,e^{5/2}}+\frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,\left(-1-\mathrm{i}\right)}{d\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(c + d*x))/(e*cot(c + d*x))^(5/2),x)
```

```
[Out] (2*a)/(d*e^2*(e*cot(c + d*x))^(1/2)) + (2*a)/(3*d*e*(e*cot(c + d*x))^(3/2))
 + ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1 - 1i)
)/(d*e^(5/2)) - ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(
1/2))*(1 + 1i))/(d*e^(5/2))
```

3.8 $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx$

Optimal. Leaf size=269

$$\frac{\sqrt{2}\,a^2e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{\sqrt{2}\,a^2e^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d}$$

```
[Out] -4/5*a^2*(e*cot(d*x+c))^(5/2)/d-2/7*a^2*(e*cot(d*x+c))^(7/2)/d/e+1/2*a^2*e^
(5/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)
-1/2*a^2*e^(5/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2)
)/d*2^(1/2)+a^2*e^(5/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1
/2)/d-a^2*e^(5/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d+
4*a^2*e^2*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.23, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3624, 12, 16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt{2}\,a^2e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{\sqrt{2}\,a^2e^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{d}+\frac{a^2e^{5/2}\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,d}-\frac{a^2e^{5/2}\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,d}+\frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d}-\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}-\frac{4a^2(e\cot(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (Sqrt[2]*a^2*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/d
- (Sqrt[2]*a^2*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
d + (4*a^2*e^2*Sqrt[e*Cot[c + d*x]])/d - (4*a^2*(e*Cot[c + d*x])^(5/2))/(5*
d) - (2*a^2*(e*Cot[c + d*x])^(7/2))/(7*d*e) + (a^2*e^(5/2)*Log[Sqrt[e] + Sq
rt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d) - (a^2*e^(5
/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sq
rt[2]*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx &= -\frac{2a^2(e\cot(c+dx))^{7/2}}{7de} + \int 2a^2\cot(c+dx)(e\cot(c+dx))^{5/2} \\
&= -\frac{2a^2(e\cot(c+dx))^{7/2}}{7de} + \left(2a^2\right)\int \cot(c+dx)(e\cot(c+dx)) \\
&= -\frac{2a^2(e\cot(c+dx))^{7/2}}{7de} + \frac{\left(2a^2\right)\int (e\cot(c+dx))^{7/2}\,dx}{e} \\
&= -\frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c+dx))^{7/2}}{7de} - \left(2a^2e\right)\int (e \\
&= \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c}{7de} \\
&= \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c}{7de} \\
&= \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c}{7de} \\
&= \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c}{7de} \\
&= \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c}{7de} \\
&= \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c}{7de} \\
&= \frac{\sqrt{2}\,a^2e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{\sqrt{2}\,a^2e^{5/2}\,\mathrm{ta}}{}
\end{aligned}$$

Mathematica [A]
time = 1.25, size = 187, normalized size = 0.70

$$-\frac{a^2(e\cot(c+dx))^{5/2}\left(-70\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)+70\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)-280\sqrt{\cot(c+dx)}+56\cot^{\frac{5}{2}}(c+dx)+20\cot^{\frac{7}{2}}(c+dx)-35\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)+35\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{70d\cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^2,x]
```

```
[Out] -1/70*(a^2*(e*Cot[c + d*x])^(5/2)*(-70*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[
c + d*x]]] + 70*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] - 280*Sqrt[C
ot[c + d*x]] + 56*Cot[c + d*x]^(5/2) + 20*Cot[c + d*x]^(7/2) - 35*Sqrt[2]*L
```

```
og[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] + 35*Sqrt[2]*Log[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(d*Cot[c + d*x]^(5/2))
```

Maple [A]
time = 0.45, size = 187, normalized size = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{2e(e\cot(dx+c))^{\frac{5}{2}}}{5}-2e^3\sqrt{e\cot(dx+c)}+\frac{e^3(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(}\right.\right.}{}\right.}{}$
default	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{2e(e\cot(dx+c))^{\frac{5}{2}}}{5}-2e^3\sqrt{e\cot(dx+c)}+\frac{e^3(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^2/e*(1/7*(e*cot(d*x+c))^(7/2)+2/5*e*(e*cot(d*x+c))^(5/2)-2*e^3*(e*co
t(d*x+c))^(1/2)+1/4*e^3*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Maxima [A]
time = 0.53, size = 166, normalized size = 0.62

$$-\frac{\left(70\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+70\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+35\sqrt{2}a^2\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-35\sqrt{2}a^2\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{280a^2}{\sqrt{\tan(dx+c)}}+\frac{56a^2}{\tan(dx+c)^{\frac{5}{2}}}+\frac{20a^2}{\tan(dx+c)^{\frac{7}{2}}}\right)e^{\frac{5}{2}}}{70d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/70*(70*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))))
+ 70*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 35
*sqrt(2)*a^2*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 35*sqrt
(2)*a^2*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 280*a^2/sqr
t(tan(d*x + c)) + 56*a^2/tan(d*x + c)^(5/2) + 20*a^2/tan(d*x + c)^(7/2))*e^
(5/2)/d
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int (e\cot(c+dx))^{\frac{5}{2}}\,dx+\int 2(e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)\,dx+\int (e\cot(c+dx))^{\frac{5}{2}}\cot^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*cot(d*x+c))**2,x)
```

```
[Out] a**2*(Integral((e*cot(c + d*x))**(5/2), x) + Integral(2*(e*cot(c + d*x))**(
5/2)*cot(c + d*x), x) + Integral((e*cot(c + d*x))**(5/2)*cot(c + d*x)**2, x
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(5/2), x)
```

Mupad [B]
time = 1.73, size = 125, normalized size = 0.46

$$\frac{4\,a^2\,e^2\,\sqrt{e\cot(c+d\,x)}}{d}-\frac{4\,a^2\,(e\cot(c+d\,x))^{5/2}}{5\,d}-\frac{2\,a^2\,(e\cot(c+d\,x))^{7/2}}{7\,d\,e}+\frac{(-1)^{1/4}\,a^2\,e^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,2\mathrm{i}}{d}+\frac{2\,(-1)^{1/4}\,a^2\,e^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}\,1\mathrm{i}}{\sqrt{e}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))^2,x)
```

```
[Out] (4*a^2*e^2*(e*cot(c + d*x))^(1/2))/d - (4*a^2*(e*cot(c + d*x))^(5/2))/(5*d)
 - (2*a^2*(e*cot(c + d*x))^(7/2))/(7*d*e) + ((-1)^(1/4)*a^2*e^(5/2)*atan(((
-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*2i)/d + (2*(-1)^(1/4)*a^2*e^(5/2
)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2)*1i)/e^(1/2)))/d
```

3.9 $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx$

Optimal. Leaf size=246

$$-\frac{\sqrt{2}\,a^2e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{\sqrt{2}\,a^2e^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{4a^2(e\cot($$

```
[Out] -4/3*a^2*(e*cot(d*x+c))^(3/2)/d-2/5*a^2*(e*cot(d*x+c))^(5/2)/d/e+1/2*a^2*e^
(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)
-1/2*a^2*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2)
)/d*2^(1/2)-a^2*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1
/2)/d+a^2*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d
```

Rubi [A]

time = 0.16, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3624, 12, 16, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt{2}\,a^2e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{\sqrt{2}\,a^2e^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{d}+\frac{a^2e^{3/2}\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,d}-\frac{a^2e^{3/2}\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,d}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}-\frac{4a^2(e\cot(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2,x]
```

```
[Out] -((Sqrt[2]*a^2*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
d) + (Sqrt[2]*a^2*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]
])/d - (4*a^2*(e*Cot[c + d*x])^(3/2))/(3*d) - (2*a^2*(e*Cot[c + d*x])^(5/2)
)/(5*d*e) + (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[
e*Cot[c + d*x]]])/(Sqrt[2]*d) - (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c +
d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = -\frac{2a^2(e\cot(c+dx))^{5/2}}{5de} + \int 2a^2\cot(c+dx)(e\cot(c+dx))^{3/2}$$

$$= -\frac{2a^2(e\cot(c+dx))^{5/2}}{5de} + (2a^2)\int \cot(c+dx)(e\cot(c+dx))$$

$$= -\frac{2a^2(e\cot(c+dx))^{5/2}}{5de} + \frac{(2a^2)\int (e\cot(c+dx))^{5/2}\,dx}{e}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}}{3d} - \frac{2a^2(e\cot(c+dx))^{5/2}}{5de} - (2a^2e)\int$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}}{3d} - \frac{2a^2(e\cot(c+dx))^{5/2}}{5de} + \frac{(2a^2e^2)\,\mathrm{Sul}}{}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}}{3d} - \frac{2a^2(e\cot(c+dx))^{5/2}}{5de} + \frac{(4a^2e^2)\,\mathrm{Sul}}{}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}}{3d} - \frac{2a^2(e\cot(c+dx))^{5/2}}{5de} - \frac{(2a^2e^2)\,\mathrm{Sul}}{}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}}{3d} - \frac{2a^2(e\cot(c+dx))^{5/2}}{5de} + \frac{(a^2e^{3/2})\,\mathrm{Su}}{}$$

$$= -\frac{4a^2(e\cot(c+dx))^{3/2}}{3d} - \frac{2a^2(e\cot(c+dx))^{5/2}}{5de} + \frac{a^2e^{3/2}\log\big(}{}$$

$$= -\frac{\sqrt{2}\,a^2e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{\sqrt{2}\,a^2e^{3/2}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.40, size = 52, normalized size = 0.21

$$-\frac{2a^2(e\cot(c+dx))^{3/2}\left(10+3\cot(c+dx)-10\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (-2*a^2*(e*Cot[c + d*x])^(3/2)*(10 + 3*Cot[c + d*x] - 10*Hypergeometric2F1[
3/4, 1, 7/4, -Cot[c + d*x]^2]))/(15*d)
```

Maple [A]
time = 0.44, size = 172, normalized size = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^3\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\,\mathrm{arcta}\cdots\right.}{\cdots}\right.}{de}$
default	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^3\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\,\mathrm{arcta}\cdots\right.}{\cdots}\right.}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^2/e*(1/5*(e*cot(d*x+c))^(5/2)+2/3*e*(e*cot(d*x+c))^(3/2)-1/4*e^3/(e^
2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)
+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^
(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1
/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Maxima [A]

time = 0.52, size = 146, normalized size = 0.59

$$\frac{\left(15\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^2-\frac{40a^2}{\tan(dx+c)^{\frac{3}{2}}}-\frac{12a^2}{\tan(dx+c)^{\frac{5}{2}}}\right)e^{\frac{3}{2}}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*l
og(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)/
sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^2 - 40*a^2/tan(d*x + c)^(3/2) -
 12*a^2/tan(d*x + c)^(5/2))*e^(3/2)/d
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int(e\cot(c+dx))^{\frac{3}{2}}\,dx+\int 2(e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)\,dx+\int(e\cot(c+dx))^{\frac{3}{2}}\cot^{2}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*cot(d*x+c))**2,x)
```

```
[Out] a**2*(Integral((e*cot(c + d*x))**(3/2), x) + Integral(2*(e*cot(c + d*x))**(
3/2)*cot(c + d*x), x) + Integral((e*cot(c + d*x))**(3/2)*cot(c + d*x)**2, x
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(3/2), x)
```

Mupad [B]
time = 0.95, size = 104, normalized size = 0.42

$$\frac{2\,{\left(-1\right)}^{1/4}\,a^2\,e^{3/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}{\sqrt{e}}\right)}{d}-\frac{2\,a^2\,{\left(e\,\mathrm{cot}(c+d\,x)\right)}^{5/2}}{5\,d\,e}-\frac{4\,a^2\,{\left(e\,\mathrm{cot}(c+d\,x)\right)}^{3/2}}{3\,d}+\frac{{\left(-1\right)}^{1/4}\,a^2\,e^{3/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}\,1{}\mathrm{i}}{\sqrt{e}}\right)\,2{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^2,x)
```

```
[Out] (2*(-1)^(1/4)*a^2*e^(3/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))
)/d - (2*a^2*(e*cot(c + d*x))^(5/2))/(5*d*e) - (4*a^2*(e*cot(c + d*x))^(3/2
))/(3*d) + ((-1)^(1/4)*a^2*e^(3/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2)*
1i)/e^(1/2))*2i)/d
```

3.10 $\int \sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))^2\,dx$

Optimal. Leaf size=244

$$-\frac{\sqrt{2}\,a^2\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{\sqrt{2}\,a^2\sqrt{e}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{4a^2\sqrt{e\,\text{c}}}{}$$

```
[Out] -2/3*a^2*(e*cot(d*x+c))^(3/2)/d/e-1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(
1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)+1/2*a^2*ln(e^(1/2)+cot(d*x+c)*
e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-a^2*arctan(1-2^(1/2
)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)*e^(1/2)/d+a^2*arctan(1+2^(1/2)*(e*c
ot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)*e^(1/2)/d-4*a^2*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3624, 12, 16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt{2}\,a^2\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{\sqrt{2}\,a^2\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{d}-\frac{2a^2(e\cot(c+dx))^{3/2}}{3de}-\frac{4a^2\sqrt{e\cot(c+dx)}}{d}-\frac{a^2\sqrt{e}\,\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,d}+\frac{a^2\sqrt{e}\,\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^2,x]
```

```
[Out] -((Sqrt[2]*a^2*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
d) + (Sqrt[2]*a^2*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]
])/d - (4*a^2*Sqrt[e*Cot[c + d*x]])/d - (2*a^2*(e*Cot[c + d*x])^(3/2))/(3*d
*e) - (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[
c + d*x]]])/(Sqrt[2]*d) + (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] +
 Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))^2\,dx = -\frac{2a^2(e\cot(c+dx))^{3/2}}{3de} + \int 2a^2\cot(c+dx)\sqrt{e\cot(c+dx)}\;d$$

$$= -\frac{2a^2(e\cot(c+dx))^{3/2}}{3de} + \left(2a^2\right)\int \cot(c+dx)\sqrt{e\cot(c+dx)}$$

$$= -\frac{2a^2(e\cot(c+dx))^{3/2}}{3de} + \frac{\left(2a^2\right)\int (e\cot(c+dx))^{3/2}\,dx}{e}$$

$$= -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} - \left(2a^2e\right)\int \frac{}{\sqrt{}}$$

$$= -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} + \frac{\left(2a^2e^2\right)\text{Subs}}{}$$

$$= -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} + \frac{\left(4a^2e^2\right)\text{Subs}}{}$$

$$= -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} + \frac{\left(2a^2e\right)\text{Subst}}{}$$

$$= -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} - \frac{\left(a^2\sqrt{e}\right)\text{Sub}}{}$$

$$= -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} - \frac{a^2\sqrt{e}\,\log\left(\right.}{}$$

$$= -\frac{\sqrt{2}\,a^2\sqrt{e}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{\sqrt{2}\,a^2\sqrt{e}}{}$$

Mathematica [A]

time = 0.46, size = 175, normalized size = 0.72

$$-\frac{a^2\sqrt{e\cot(c+dx)}\left(6\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-6\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)+24\sqrt{\cot(c+dx)}+4\cot^{\frac{3}{2}}(c+dx)+3\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-3\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{6d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^2,x]
```

```
[Out] -1/6*(a^2*Sqrt[e*Cot[c + d*x]]*(6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d
*x]]] - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24*Sqrt[Cot[c +
d*x]] + 4*Cot[c + d*x]^(3/2) + 3*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
 + Cot[c + d*x]] - 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d
*x]]]))/(d*Sqrt[Cot[c + d*x]])
```

Maple [A]
time = 0.44, size = 170, normalized size = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}+2e\sqrt{e\cot(dx+c)}-\frac{e(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{}}\right.\right.}{}\right.}{de}$
default	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}+2e\sqrt{e\cot(dx+c)}-\frac{e(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{}}\right.\right.}{}\right.}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^2/e*(1/3*(e*cot(d*x+c))^(3/2)+2*e*(e*cot(d*x+c))^(1/2)-1/4*e*(e^2)^(
1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^
2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2
)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Maxima [A]
time = 0.55, size = 153, normalized size = 0.63

$$\frac{\left(6\sqrt{2}\,a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}\,a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+3\sqrt{2}\,a^2\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-3\sqrt{2}\,a^2\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{24a^2}{\sqrt{\tan(dx+c)}}-\frac{4a^2}{\tan(dx+c)^{\frac{3}{2}}}\right)e^{\frac{1}{2}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 6
*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 3*sqrt
(2)*a^2*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 3*sqrt(2)*a^
2*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 24*a^2/sqrt(tan(d
*x + c)) - 4*a^2/tan(d*x + c)^(3/2))*e^(1/2)/d
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sqrt{e\cot\left(c+dx\right)}\,dx+\int 2\sqrt{e\cot\left(c+dx\right)}\cot\left(c+dx\right)dx+\int\sqrt{e\cot\left(c+dx\right)}\cot^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)*(a+a*cot(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(2*sqrt(e*cot(c + d*x))*c
ot(c + d*x), x) + Integral(sqrt(e*cot(c + d*x))*cot(c + d*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^2*sqrt(e*cot(d*x + c)), x)
```

Mupad [B]
time = 0.70, size = 104, normalized size = 0.43

$$-\frac{4\,a^2\,\sqrt{e\cot\left(c+d\,x\right)}}{d}-\frac{2\,a^2\,{\left(e\cot\left(c+d\,x\right)\right)}^{3/2}}{3\,d\,e}-\frac{{\left(-1\right)}^{1/4}\,a^2\,\sqrt{e}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,2{}\mathrm{i}}{d}-\frac{2\,{\left(-1\right)}^{1/4}\,a^2\,\sqrt{e}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}\,1{}\mathrm{i}}{\sqrt{e}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^2,x)
```

```
[Out] - (4*a^2*(e*cot(c + d*x))^(1/2))/d - (2*a^2*(e*cot(c + d*x))^(3/2))/(3*d*e)
 - ((-1)^(1/4)*a^2*e^(1/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)
)*2i)/d - (2*(-1)^(1/4)*a^2*e^(1/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2)
*1i)/e^(1/2)))/d
```

3.11 $\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal. Leaf size=222

$$\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

```
[Out] -1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1
/2)/e^(1/2)+1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1
/2))/d*2^(1/2)/e^(1/2)+a^2*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2
^(1/2)/d/e^(1/2)-a^2*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)
/d/e^(1/2)-2*a^2*(e*cot(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.14, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3624, 12, 16, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{d\sqrt{e}}-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}-\frac{a^2\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{a^2\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cot[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*Sqrt[e]
) - (Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*Sqr
t[e]) - (2*a^2*Sqrt[e*Cot[c + d*x]])/(d*e) - (a^2*Log[Sqrt[e] + Sqrt[e]*Cot
[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d*Sqrt[e]) + (a^2*Log[S
qrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d*S
qrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx &= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}+\int \frac{2a^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx \\
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}+\left(2a^2\right)\int \frac{\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx \\
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}+\frac{\left(2a^2\right)\int \sqrt{e\cot(c+dx)}\,dx}{e} \\
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}-\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx,x,e\cot(c+dx)\right)}{d} \\
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}-\frac{\left(4a^2\right)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d} \\
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}-\frac{\left(2a^2\right)\text{S}}{} \\
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}-\frac{a^2\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}\,x+x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}- \\
&= -\frac{2a^2\sqrt{e\cot(c+dx)}}{de}-\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right.}{\sqrt{2}\,d\sqrt{e}} \\
&= \frac{\sqrt{2}\,a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}-\frac{\sqrt{2}\,a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+}}{\sqrt{e}}\right.}{d\sqrt{e}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.27, size = 53, normalized size = 0.24

$$-\frac{2a^2\sqrt{e\cot(c+dx)}\left(3+2\cot(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cot[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (-2*a^2*Sqrt[e*Cot[c + d*x]]*(3 + 2*Cot[c + d*x]*Hypergeometric2F1[3/4, 1,
7/4, -Cot[c + d*x]^2]))/(3*d*e)
```

Maple [A]
time = 0.41, size = 155, normalized size = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(\sqrt{e\cot(dx+c)}+\frac{e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\sqrt{2}\right.\right.}{4(e^2)^{\frac{1}{4}}}\right.}{de}$
default	$-\frac{2a^2\left(\sqrt{e\cot(dx+c)}+\frac{e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\sqrt{2}\right.\right.}{4(e^2)^{\frac{1}{4}}}\right.}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^2/e*((e*cot(d*x+c))^(1/2)+1/4*e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c
)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)+1)))
```

Maxima [A]
time = 0.52, size = 132, normalized size = 0.59

$$-\frac{\left(\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^2+\frac{4a^2}{\sqrt{\tan(dx+c)}}\right)e^{\left(-\frac{1}{2}\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*log(
```

```
sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)/sqr
t(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^2 + 4*a^2/sqrt(tan(d*x + c)))*e^(-
1/2)/d
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{1}{\sqrt{e\cot{\left(c+dx \right)}}}\, dx+\int \frac{2\cot{\left(c+dx \right)}}{\sqrt{e\cot{\left(c+dx \right)}}}\, dx+\int \frac{\cot^{2}{\left(c+dx \right)}}{\sqrt{e\cot{\left(c+dx \right)}}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
[Out] a**2*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(2*cot(c + d*x)/sqrt(e*
cot(c + d*x)), x) + Integral(cot(c + d*x)**2/sqrt(e*cot(c + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*cot(d*x + c) + a)^2/sqrt(e*cot(d*x + c)), x)
```

Mupad [**B**]
time = 0.44, size = 86, normalized size = 0.39

$$\frac{2\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}{\sqrt{e}}\right)}{d\,\sqrt{e}}-\frac{2\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}{\sqrt{e}}\right)}{d\,\sqrt{e}}-\frac{2\,a^2\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}{d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(c + d*x))^2/(e*cot(c + d*x))^(1/2),x)
[Out] (2*(-1)^(1/4)*a^2*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/(d*e^
(1/2)) - (2*(-1)^(1/4)*a^2*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)
))/(d*e^(1/2)) - (2*a^2*(e*cot(c + d*x))^(1/2))/(d*e)
```

3.12 $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=222

$$\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{2a^2}{de\sqrt{e\cot(c+dx)}}$$

```
[Out] 1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(3/
2)*2^(1/2)-1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/
2))/d/e^(3/2)*2^(1/2)+a^2*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^
(1/2)/d/e^(3/2)-a^2*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/
d/e^(3/2)+2*a^2/d/e/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3623, 12, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{de^{3/2}}+\frac{a^2\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,de^{3/2}}-\frac{a^2\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,de^{3/2}}+\frac{2a^2}{de\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*e^(3/2)
) - (Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*e^(
3/2)) + (2*a^2)/(d*e*Sqrt[e*Cot[c + d*x]]) + (a^2*Log[Sqrt[e] + Sqrt[e]*Cot
[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d*e^(3/2)) - (a^2*Log[S
qrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d*e
^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx &= \frac{2a^2}{de\sqrt{e\cot(c+dx)}}+\frac{\int \frac{2a^2e}{\sqrt{e\cot(c+dx)}}\,dx}{e^2}\\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}}+\frac{(2a^2)\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx}{e}\\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx,x,e\cot(c+dx)\right)}{d}\\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{(4a^2)\,\mathrm{Subst}\left(\int \frac{1}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{de}-\frac{(2a^2)\,\mathrm{Su}}{}\\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}}+\frac{a^2\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}\,de^{3/2}}+\\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}}+\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}\,de^{3/2}}\\
&= \frac{\sqrt{2}\,a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}-\frac{\sqrt{2}\,a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c}}{\sqrt{e}}\right.}{de^{3/2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.91, size = 236, normalized size = 1.06

$$\frac{a^2(1+\cot(c+dx))^2\left(-4\cos^2(c+dx)\cot(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)+3\sin(c+dx)\left(4\cos(c+dx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+\sqrt{2}\cot^{\frac{3}{2}}(c+dx)\left(2\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\sin(c+dx)\right)\right)}{6d(e\cot(c+dx))^{3/2}(\cos(c+dx)+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]

[Out] (a^2*(1 + Cot[c + d*x])^2*(-4*Cos[c + d*x]^2*Cot[c + d*x]*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + 3*Sin[c + d*x]*(4*Cos[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Sqrt[2]*Cot[c + d*x]^(3/2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sin[c + d*x])))/(6*d*(e*Cot[c + d*x])^(3/2)*(Cos[c + d*x] + Sin[c + d*x])^2)

Maple [A]
time = 0.42, size = 159, normalized size = 0.72

method	result
derivativedivides	$-\frac{2a^2\left(-\frac{1}{\sqrt{e\cot(dx+c)}}+\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\right.\right.}{4e}}{de}$
default	$-\frac{2a^2\left(-\frac{1}{\sqrt{e\cot(dx+c)}}+\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\right.\right.}{4e}}{de}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/d*a^2/e*(-1/(e*cot(d*x+c))^(1/2)+1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))

Maxima [A]
time = 0.52, size = 139, normalized size = 0.63

$$-\frac{\left(2\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}a^2\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}a^2\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-4a^2\sqrt{\tan(dx+c)}\right)e^{\left(-\frac{3}{2}\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")

[Out] -1/2*(2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*a^2*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*a^2*l

```
og(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 4*a^2*sqrt(tan(d*x +
 c)))*e^(-3/2)/d
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{1}{\left(e\cot \left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{2\cot \left(c+dx\right)}{\left(e\cot \left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{\cot^{2}\left(c+dx\right)}{\left(e\cot \left(c+dx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(3/2),x)
[Out] a**2*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(2*cot(c + d*x)/(e*co
t(c + d*x))**(3/2), x) + Integral(cot(c + d*x)**2/(e*cot(c + d*x))**(3/2),
x))
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(3/2), x)
```

Mupad [B]

time = 0.59, size = 86, normalized size = 0.39

$$\frac{2\,a^2}{d\,e\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}+\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}{\sqrt{e}}\right)\,2{}\mathrm{i}}{d\,e^{3/2}}+\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}{\sqrt{e}}\right)\,2{}\mathrm{i}}{d\,e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(c + d*x))^2/(e*cot(c + d*x))^(3/2),x)
[Out] (2*a^2)/(d*e*(e*cot(c + d*x))^(1/2)) + ((-1)^(1/4)*a^2*atan(((-1)^(1/4)*(e*
cot(c + d*x))^(1/2))/e^(1/2))*2i)/(d*e^(3/2)) + ((-1)^(1/4)*a^2*atanh(((-1)
^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*2i)/(d*e^(3/2))
```

3.13 $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=247

$$-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{2a^2}{3de(e\cot(c+dx)}$$

```
[Out] 2/3*a^2/d/e/(e*cot(d*x+c))^(3/2)+1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1
/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)-1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e
^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)-a^2*arctan(1-2^(1/2)
*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d/e^(5/2)+a^2*arctan(1+2^(1/2)*(e*co
t(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d/e^(5/2)+4*a^2/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$,
Rules used = {3623, 12, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{\sqrt{2}\,a^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{de^{5/2}}+\frac{a^2\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,de^{5/2}}-\frac{a^2\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,de^{5/2}}+\frac{4a^2}{de^2\sqrt{e\cot(c+dx)}}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] -((Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*e^(5/
2))) + (Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*
e^(5/2)) + (2*a^2)/(3*d*e*(e*Cot[c + d*x])^(3/2)) + (4*a^2)/(d*e^2*Sqrt[e*C
ot[c + d*x]]) + (a^2*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Co
t[c + d*x]]])/(Sqrt[2]*d*e^(5/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x]
 + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d*e^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{\int \frac{2a^2 e}{(e\cot(c+dx))^{3/2}}\,dx}{e^2}$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{(2a^2)\int \frac{1}{(e\cot(c+dx))^{3/2}}\,dx}{e}$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} - \frac{(2a^2)\int \sqrt{e\cot(c+dx)}\,dx}{e^3}$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\cot\right.}{de^2}$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} + \frac{(4a^2)\,\mathrm{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\,c}\right.}{de^2}$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} - \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\,c}\right.}{de^2}$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} + \frac{a^2\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,\right.}{\sqrt{2}\,de^{5/2}}$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\right.}{\sqrt{2}\,de^{5/2}}$$

$$= -\frac{\sqrt{2}\,a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{\sqrt{2}\,a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c}{\sqrt{e}}\right.}{de^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.35, size = 233, normalized size = 0.94

$$\frac{a^2\left(48\cos^2(c+dx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+\sin(c+dx)\left(8\cos(c+dx)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)+3\sqrt{2}\cot^{\frac{5}{2}}(c+dx)\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\sin(c+dx)\right)\right)(1+\tan(c+dx))^2}{12de^2\sqrt{e\cot(c+dx)}\,(\cos(c+dx)+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(48*Cos[c + d*x]^2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] +
Sin[c + d*x]*(8*Cos[c + d*x]*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^
2] + 3*Sqrt[2]*Cot[c + d*x]^(5/2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]
 - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c +
d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])
*Sin[c + d*x]))*(1 + Tan[c + d*x])^2)/(12*d*e^2*Sqrt[e*Cot[c + d*x]]*(Cos[c
 + d*x] + Sin[c + d*x])^2)
```

Maple [A]
time = 0.39, size = 174, normalized size = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\cdots\right)\cdots\right)}{4e\left(e^2\right)^{\frac{1}{4}}}\cdots\right)}{de}$
default	$-\frac{2a^2\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\cdots\right)\cdots\right)}{4e\left(e^2\right)^{\frac{1}{4}}}\cdots\right)}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^2/e*(-1/4/e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/3/(e*cot(d
*x+c))^(3/2)-2/e/(e*cot(d*x+c))^(1/2))
```

Maxima [A]
time = 0.50, size = 147, normalized size = 0.60

$$\frac{\left(3\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^2+4\left(a^2+\frac{6a^2}{\tan(dx+c)}\right)\tan(dx+c)^{\frac{3}{2}}\right)e^{\left(-\frac{5}{2}\right)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*log
(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)/sq
rt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^2 + 4*(a^2 + 6*a^2/tan(d*x + c))*
tan(d*x + c)^(3/2))*e^(-5/2)/d
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int\frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx+\int\frac{2\cot\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx+\int\frac{\cot^{2}\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(5/2),x)
```

```
[Out] a**2*(Integral((e*cot(c + d*x))**(-5/2), x) + Integral(2*cot(c + d*x)/(e*co
t(c + d*x))**(5/2), x) + Integral(cot(c + d*x)**2/(e*cot(c + d*x))**(5/2),
x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(5/2), x)
```

Mupad [**B**]
time = 0.70, size = 99, normalized size = 0.40

$$\frac{4\,a^2\cot(c+d\,x)+\frac{2\,a^2}{3}}{d\,e\,{\left(e\cot\left(c+d\,x\right)\right)}^{3/2}}+\frac{2\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{d\,e^{5/2}}-\frac{2\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{d\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(c + d*x))^2/(e*cot(c + d*x))^(5/2),x)
```

```
[Out] (4*a^2*cot(c + d*x) + (2*a^2)/3)/(d*e*(e*cot(c + d*x))^(3/2)) + (2*(-1)^(1/
4)*a^2*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/(d*e^(5/2)) - (2*
(-1)^(1/4)*a^2*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/(d*e^(5/
2))
```

3.14 $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=249

$$-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{7/2}}+\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{7/2}}+\frac{2a^2}{5de(e\cot(c+dx)}$$

```
[Out] 2/5*a^2/d/e/(e*cot(d*x+c))^(5/2)+4/3*a^2/d/e^2/(e*cot(d*x+c))^(3/2)-1/2*a^2
*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(7/2)*2^(1
/2)+1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e
^(7/2)*2^(1/2)-a^2*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d
/e^(7/2)+a^2*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d/e^(7/
2)
```

Rubi [A]
time = 0.18, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3623, 12, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt{2}\,a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{7/2}}+\frac{\sqrt{2}\,a^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{de^{7/2}}-\frac{a^2\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,de^{7/2}}+\frac{a^2\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{\sqrt{2}\,de^{7/2}}+\frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] -((Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*e^(7/
2))) + (Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(d*
e^(7/2)) + (2*a^2)/(5*d*e*(e*Cot[c + d*x])^(5/2)) + (4*a^2)/(3*d*e^2*(e*Cot
[c + d*x])^(3/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[
e*Cot[c + d*x]]])/(Sqrt[2]*d*e^(7/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Cot[c +
d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(Sqrt[2]*d*e^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{\int \frac{2a^2 e}{(e\cot(c+dx))^{5/2}}\,dx}{e^2}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{(2a^2)\int \frac{1}{(e\cot(c+dx))^{5/2}}\,dx}{e}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} - \frac{(2a^2)\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx}{e^3}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\,(e^2+x^2)}\,dx,\right.}{de^2}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} + \frac{(4a^2)\,\mathrm{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e}\right.}{de^2}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e}\right.}{de^3}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} - \frac{a^2\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}\,x-x^2}\,d\right.}{\sqrt{2}\,de^{7/}}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx\right.}{\sqrt{2}\,de}$$

$$= -\frac{\sqrt{2}\,a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{7/2}} + \frac{\sqrt{2}\,a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c}}{\sqrt{e}}\right.}{de^{7/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.43, size = 141, normalized size = 0.57

$$\frac{2a^2\sin(c+dx)\left(10\cos(c+dx)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)+15\cos(c+dx)\cot(c+dx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+3\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(c+dx)\right)\sin(c+dx)\right)(1+\tan(c+dx))^2}{15de^3\sqrt{e\cot(c+dx)}\,(\cos(c+dx)+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] (2*a^2*Sin[c + d*x]*(10*Cos[c + d*x]*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c
 + d*x]^2] + 15*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4, -
Cot[c + d*x]^2] + 3*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2]*Sin[c
 + d*x])*(1 + Tan[c + d*x])^2)/(15*d*e^3*Sqrt[e*Cot[c + d*x]]*(Cos[c + d*x]
 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.39, size = 174, normalized size = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx-}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e^3}\right.}{de}$
default	$-\frac{2a^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx-}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e^3}\right.}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^2/e*(-1/4/e^3*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/5/(e*cot
(d*x+c))^(5/2)-2/3/e/(e*cot(d*x+c))^(3/2))
```

Maxima [A]
time = 0.52, size = 156, normalized size = 0.63

$$\frac{\left(30\sqrt{2}\,a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+30\sqrt{2}\,a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+15\sqrt{2}\,a^2\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-15\sqrt{2}\,a^2\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+4\left(3a^2+\frac{10a^2}{\tan(dx+c)}\right)\tan(dx+c)^{\frac{5}{2}}\right)e^{\left(-\frac{7}{2}\right)}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/30*(30*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) +
 30*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 15*
sqrt(2)*a^2*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 15*sqrt(
2)*a^2*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 4*(3*a^2 + 1
0*a^2/tan(d*x + c))*tan(d*x + c)^(5/2))*e^(-7/2)/d
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{7}{2}}}\, dx + \int \frac{2\cot\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{7}{2}}}\, dx + \int \frac{\cot^{2}\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{7}{2}}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(7/2),x)
```

```
[Out] a**2*(Integral((e*cot(c + d*x))**(-7/2), x) + Integral(2*cot(c + d*x)/(e*co
t(c + d*x))**(7/2), x) + Integral(cot(c + d*x)**2/(e*cot(c + d*x))**(7/2),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(7/2), x)
```

Mupad [B]
time = 1.29, size = 99, normalized size = 0.40

$$\frac{\frac{4\,a^2\,\mathrm{cot}\left(c+d\,x\right)}{3}+\frac{2\,a^2}{5}}{d\,e\,{\left(e\,\mathrm{cot}\left(c+d\,x\right)\right)}^{5/2}}-\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}{\sqrt{e}}\right)\,2{}\mathrm{i}}{d\,e^{7/2}}-\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}{\sqrt{e}}\right)\,2{}\mathrm{i}}{d\,e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(c + d*x))^2/(e*cot(c + d*x))^(7/2),x)
```

```
[Out] ((4*a^2*cot(c + d*x))/3 + (2*a^2)/5)/(d*e*(e*cot(c + d*x))^(5/2)) - ((-1)^(
1/4)*a^2*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*2i)/(d*e^(7/2))
- ((-1)^(1/4)*a^2*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*2i)/(d
*e^(7/2))
```

3.15 $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx$

Optimal. Leaf size=186

$$\frac{2\sqrt{2}\,a^3e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d}+\frac{4a^3e^2\sqrt{e\cot(c+dx)}}{d}+\frac{4a^3e(e\cot(c+dx))^{3/2}}{3d}-\frac{4a^3(e\cot(c+
$$

```
[Out] 4/3*a^3*e*(e*cot(d*x+c))^(3/2)/d-4/5*a^3*(e*cot(d*x+c))^(5/2)/d-40/63*a^3*(
e*cot(d*x+c))^(7/2)/d/e-2/9*(e*cot(d*x+c))^(7/2)*(a^3+a^3*cot(d*x+c))/d/e+2
*a^3*e^(5/2)*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))
^(1/2))*2^(1/2)/d+4*a^3*e^2*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.22, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3647, 3711, 3609, 3613, 211}

$$\frac{2\sqrt{2}\,a^3e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d}+\frac{4a^3e^2\sqrt{e\cot(c+dx)}}{d}-\frac{2(a^3\cot(c+dx)+a^3)\,(e\cot(c+dx))^{7/2}}{9de}-\frac{40a^3(e\cot(c+dx))^{7/2}}{63de}-\frac{4a^3(e\cot(c+dx))^{5/2}}{5d}+\frac{4a^3e(e\cot(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3,x]
```

```
[Out] (2*Sqrt[2]*a^3*e^(5/2)*ArcTan[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqr
t[e*Cot[c + d*x]])])/d + (4*a^3*e^2*Sqrt[e*Cot[c + d*x]])/d + (4*a^3*e*(e*C
ot[c + d*x])^(3/2))/(3*d) - (4*a^3*(e*Cot[c + d*x])^(5/2))/(5*d) - (40*a^3*
(e*Cot[c + d*x])^(7/2))/(63*d*e) - (2*(e*Cot[c + d*x])^(7/2)*(a^3 + a^3*Cot
[c + d*x]))/(9*d*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx = -\frac{2(e\cot(c+dx))^{7/2}\left(a^3+a^3\cot(c+dx)\right)}{9de} - \frac{2\int(e\cot(c+dx)}{}$$

$$= -\frac{40a^3(e\cot(c+dx))^{7/2}}{63de} - \frac{2(e\cot(c+dx))^{7/2}\left(a^3+a^3\cot(c\right.}{9de}$$

$$= -\frac{4a^3(e\cot(c+dx))^{5/2}}{5d} - \frac{40a^3(e\cot(c+dx))^{7/2}}{63de} - \frac{2(e\cot(c}{}$$

$$= \frac{4a^3e(e\cot(c+dx))^{3/2}}{3d} - \frac{4a^3(e\cot(c+dx))^{5/2}}{5d} - \frac{40a^3(e\cot(}{63}$$

$$= \frac{4a^3e^2\sqrt{e\cot(c+dx)}}{d} + \frac{4a^3e(e\cot(c+dx))^{3/2}}{3d} - \frac{4a^3(e\cot(}{5}$$

$$= \frac{4a^3e^2\sqrt{e\cot(c+dx)}}{d} + \frac{4a^3e(e\cot(c+dx))^{3/2}}{3d} - \frac{4a^3(e\cot(}{5}$$

$$= \frac{2\sqrt{2}\,a^3e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} + \frac{4a^3e^2\sqrt{e\cot(c}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.11, size = 729, normalized size = 3.92

$$-\frac{2\cos^2(c+dx)(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin(c+dx)}{9d(\cos(c+dx)+\sin(c+dx))^3}-\frac{6\cos(c+dx)(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^2(c+dx)}{7d(\cos(c+dx)+\sin(c+dx))^3}-\frac{4(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)}{5d(\cos(c+dx)+\sin(c+dx))^3}+\frac{\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)}{d\cot^{\frac{5}{2}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}-\frac{\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)}{d\cot^{\frac{5}{2}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}+\frac{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^3(c+dx)}{\sqrt{2}\,d\cot^{\frac{5}{2}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}-\frac{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^3(c+dx)}{\sqrt{2}\,d\cot^{\frac{5}{2}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}+\frac{4(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)\tan(c+dx)}{3d(\cos(c+dx)+\sin(c+dx))^3}-\frac{4(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\sin^3(c+dx)\tan(c+dx)}{3d(\cos(c+dx)+\sin(c+dx))^3}+\frac{4(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)\tan^2(c+dx)}{d(\cos(c+dx)+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3,x]
```

```
[Out] (-2*Cos[c + d*x]^2*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*
x])/(9*d*(Cos[c + d*x] + Sin[c + d*x])^3) - (6*Cos[c + d*x]*(e*Cot[c + d*x]
)^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^2)/(7*d*(Cos[c + d*x] + Sin[c +
 d*x])^3) - (4*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3
)/(5*d*(Cos[c + d*x] + Sin[c + d*x])^3) + (Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[
Cot[c + d*x]]]*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3
)/(d*Cot[c + d*x]^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) - (Sqrt[2]*ArcTan[
1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])
^3*Sin[c + d*x]^3)/(d*Cot[c + d*x]^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) +
 ((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*Log[1 - Sqrt[2]*Sqrt[Cot[c
+ d*x]] + Cot[c + d*x]]*Sin[c + d*x]^3)/(Sqrt[2]*d*Cot[c + d*x]^(5/2)*(Cos[
c + d*x] + Sin[c + d*x])^3) - ((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^
3*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^3)/(Sqrt[
2]*d*Cot[c + d*x]^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (4*(e*Cot[c + d*
x])^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3*Tan[c + d*x])/(3*d*(Cos[c +
 d*x] + Sin[c + d*x])^3) - (4*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3
*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]*Sin[c + d*x]^3*Tan[c + d*x
])/(3*d*(Cos[c + d*x] + Sin[c + d*x])^3) + (4*(e*Cot[c + d*x])^(5/2)*(a + a
*Cot[c + d*x])^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/(d*(Cos[c + d*x] + Sin[c +
d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(157) = 314$.
time = 0.48, size = 354, normalized size = 1.90

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{9}{2}}}{9}+\frac{3e(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{2e^2(e\cot(dx+c))^{\frac{5}{2}}}{5}-\frac{2e^3(e\cot(dx+c))^{\frac{3}{2}}}{3}-2e^4\sqrt{e\cot(dx+c)}+2e^5\left(\frac{(e^2)^{\frac{1}{4}}}{\cdots}\right.\right.}{\cdots}$

default	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{9}{2}}}{9}+\frac{3e(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{2e^2(e\cot(dx+c))^{\frac{5}{2}}}{5}-\frac{2e^3(e\cot(dx+c))^{\frac{3}{2}}}{3}-2e^4\sqrt{e\cot(dx+c)}+2e^5\left(\frac{(e^2}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*(1/9*(e*cot(d*x+c))^(9/2)+3/7*e*(e*cot(d*x+c))^(7/2)+2/5*e^2*(
e*cot(d*x+c))^(5/2)-2/3*e^3*(e*cot(d*x+c))^(3/2)-2*e^4*(e*cot(d*x+c))^(1/2)
+2*e^5*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8/(e^2)^(1/4)*2
^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*
arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]

time = 0.51, size = 127, normalized size = 0.68

$$-\frac{2\left(315\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)\right)a^3-\frac{630\,a^3}{\sqrt{\tan(dx+c)}}-\frac{210\,a^3}{\tan(dx+c)^{\frac{3}{2}}}+\frac{126\,a^3}{\tan(dx+c)^{\frac{5}{2}}}+\frac{135\,a^3}{\tan(dx+c)^{\frac{7}{2}}}+\frac{35\,a^3}{\tan(dx+c)^{\frac{9}{2}}}\right)e^{\frac{5}{2}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/315*(315*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) +
 sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))))*a^3 - 630*a
^3/sqrt(tan(d*x + c)) - 210*a^3/tan(d*x + c)^(3/2) + 126*a^3/tan(d*x + c)^(
5/2) + 135*a^3/tan(d*x + c)^(7/2) + 35*a^3/tan(d*x + c)^(9/2))*e^(5/2)/d
```

Fricas [A]

time = 4.53, size = 258, normalized size = 1.39

$$\frac{2\left(315\left(\sqrt{2}\,a^3\cos(2\,dx+2\,c)^2e^{\frac{5}{2}}-2\,\sqrt{2}\,a^3\cos(2\,dx+2\,c)\,e^{\frac{5}{2}}+\sqrt{2}\,a^3e^{\frac{5}{2}}\right)\arctan\left(-\frac{\left(\sqrt{2}\cos(2\,dx+2\,c)-\sqrt{2}\sin(2\,dx+2\,c)+\sqrt{2}\right)\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}}{2\,(\cos(2\,dx+2\,c)+1)}\right)+\left(721\,a^3\cos(2\,dx+2\,c)^2e^{\frac{5}{2}}-1330\,a^3\cos(2\,dx+2\,c)\,e^{\frac{5}{2}}+469\,a^3e^{\frac{5}{2}}-15\left(23\,a^3\cos(2\,dx+2\,c)\,e^{\frac{5}{2}}-5\,a^3e^{\frac{5}{2}}\right)\sin(2\,dx+2\,c)\right)\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}\right)}{315\left(d\cos(2\,dx+2\,c)^2-2\,d\cos(2\,dx+2\,c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 2/315*(315*(sqrt(2)*a^3*cos(2*d*x + 2*c)^2*e^(5/2) - 2*sqrt(2)*a^3*cos(2*d*
x + 2*c)*e^(5/2) + sqrt(2)*a^3*e^(5/2))*arctan(-1/2*(sqrt(2)*cos(2*d*x + 2*
```

```
c) - sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*
d*x + 2*c))/(cos(2*d*x + 2*c) + 1)) + (721*a^3*cos(2*d*x + 2*c)^2*e^(5/2) -
 1330*a^3*cos(2*d*x + 2*c)*e^(5/2) + 469*a^3*e^(5/2) - 15*(23*a^3*cos(2*d*x
 + 2*c)*e^(5/2) - 5*a^3*e^(5/2))*sin(2*d*x + 2*c))*sqrt((cos(2*d*x + 2*c) +
 1)/sin(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c)^2 - 2*d*cos(2*d*x + 2*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int (e\cot(c+dx))^{\frac{5}{2}}\,dx+\int 3(e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)\,dx+\int 3(e\cot(c+dx))^{\frac{5}{2}}\cot^2(c+dx)\,dx+\int (e\cot(c+dx))^{\frac{5}{2}}\cot^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*cot(d*x+c))**3,x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(5/2), x) + Integral(3*(e*cot(c + d*x))**(
5/2)*cot(c + d*x), x) + Integral(3*(e*cot(c + d*x))**(5/2)*cot(c + d*x)**2,
 x) + Integral((e*cot(c + d*x))**(5/2)*cot(c + d*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(5/2), x)
```

Mupad [B]
time = 2.44, size = 177, normalized size = 0.95

$$\frac{4\,a^3\,e^2\,\sqrt{e\cot(c+d\,x)}}{d}-\frac{4\,a^3\,(e\cot(c+d\,x))^{5/2}}{5\,d}-\frac{6\,a^3\,(e\cot(c+d\,x))^{7/2}}{7\,d\,e}-\frac{2\,a^3\,(e\cot(c+d\,x))^{9/2}}{9\,d\,e^2}+\frac{4\,a^3\,e\,(e\cot(c+d\,x))^{3/2}}{3\,d}-\frac{\sqrt{2}\,a^3\,e^{5/2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))^3,x)
```

```
[Out] (4*a^3*e^2*(e*cot(c + d*x))^(1/2))/d - (4*a^3*(e*cot(c + d*x))^(5/2))/(5*d)
 - (6*a^3*(e*cot(c + d*x))^(7/2))/(7*d*e) - (2*a^3*(e*cot(c + d*x))^(9/2))/
(9*d*e^2) + (4*a^3*e*(e*cot(c + d*x))^(3/2))/(3*d) - (2^(1/2)*a^3*e^(5/2)*(
2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*c
ot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/
2)))))/d
```

3.16 $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx$

Optimal. Leaf size=160

$$-\frac{2\sqrt{2}\,a^3e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d}+\frac{4a^3e\sqrt{e\cot(c+dx)}}{d}-\frac{4a^3(e\cot(c+dx))^{3/2}}{3d}-\frac{32a^3(e\cot(c+$$

```
[Out] -4/3*a^3*(e*cot(d*x+c))^(3/2)/d-32/35*a^3*(e*cot(d*x+c))^(5/2)/d/e-2/7*(e*c
ot(d*x+c))^(5/2)*(a^3+a^3*cot(d*x+c))/d/e-2*a^3*e^(3/2)*arctanh(1/2*(e^(1/2
)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d+4*a^3*e*(e*co
t(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.18, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3647, 3711, 3609, 3613, 214}

$$-\frac{2\sqrt{2}\,a^3e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,\cot(c+dx)+\sqrt{e}}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{32a^3(e\cot(c+dx))^{5/2}}{35de}-\frac{4a^3(e\cot(c+dx))^{3/2}}{3d}+\frac{4a^3e\sqrt{e\cot(c+dx)}}{d}-\frac{2(a^3\cot(c+dx)+a^3)\,(e\cot(c+dx))^{5/2}}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^3,x]
```

```
[Out] (-2*Sqrt[2]*a^3*e^(3/2)*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*S
qrt[e*Cot[c + d*x]])])/d + (4*a^3*e*Sqrt[e*Cot[c + d*x]])/d - (4*a^3*(e*Cot
[c + d*x])^(3/2))/(3*d) - (32*a^3*(e*Cot[c + d*x])^(5/2))/(35*d*e) - (2*(e*
Cot[c + d*x])^(5/2)*(a^3 + a^3*Cot[c + d*x]))/(7*d*e)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
```

```
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx &= -\frac{2(e\cot(c+dx))^{5/2}\left(a^3+a^3\cot(c+dx)\right)}{7de}-\frac{2\int(e\cot(c+dx)\\
&= -\frac{32a^3(e\cot(c+dx))^{5/2}}{35de}-\frac{2(e\cot(c+dx))^{5/2}\left(a^3+a^3\cot(c+\right.}{7de}\\
&= -\frac{4a^3(e\cot(c+dx))^{3/2}}{3d}-\frac{32a^3(e\cot(c+dx))^{5/2}}{35de}-\frac{2(e\cot(c+}{}\\
&= \frac{4a^3e\sqrt{e\cot(c+dx)}}{d}-\frac{4a^3(e\cot(c+dx))^{3/2}}{3d}-\frac{32a^3(e\cot(c+}{35de}\\
&= \frac{4a^3e\sqrt{e\cot(c+dx)}}{d}-\frac{4a^3(e\cot(c+dx))^{3/2}}{3d}-\frac{32a^3(e\cot(c+}{35de}\\
&= -\frac{2\sqrt{2}\,a^3e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d}+\frac{4a^3e\sqrt{e\cot(c}{d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 2.91, size = 332, normalized size = 2.08

$$\frac{a^3(e\cot(c+dx))^{3/2}(1+\cot(c+dx))^3\sin(c+dx)\left(-60\cos^2(c+dx)\cot^{\frac{3}{2}}(c+dx)+210\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\sin^2(c+dx)-210\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\sin^2(c+dx)+840\sqrt{\cot(c+dx)}\sin^2(c+dx)-280\cot^{\frac{3}{2}}(c+dx)\sin^2(c+dx)+280\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\sin^2(c+dx)+105\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^2(c+dx)-105\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^2(c+dx)-126\cot^{\frac{3}{2}}(c+dx)\sin(2(c+dx))\right)}{210d\cot^{\frac{3}{2}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^3,x]
```

```
[Out] (a^3*(e*Cot[c + d*x])^(3/2)*(1 + Cot[c + d*x])^3*Sin[c + d*x]*(-60*Cos[c +
d*x]^2*Cot[c + d*x]^(3/2) + 210*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x
]]]*Sin[c + d*x]^2 - 210*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sin
[c + d*x]^2 + 840*Sqrt[Cot[c + d*x]]*Sin[c + d*x]^2 - 280*Cot[c + d*x]^(3/2
)*Sin[c + d*x]^2 + 280*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -C
ot[c + d*x]^2]*Sin[c + d*x]^2 + 105*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*
x]] + Cot[c + d*x]]*Sin[c + d*x]^2 - 105*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c
 + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 - 126*Cot[c + d*x]^(3/2)*Sin[2*(c +
 d*x)]))/(210*d*Cot[c + d*x]^(3/2)*(Cos[c + d*x] + Sin[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(135) = 270$.

time = 0.50, size = 339, normalized size = 2.12

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{3e(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e^2(e\cot(dx+c))^{\frac{3}{2}}}{3}-2e^3\sqrt{e\cot(dx+c)}+2e^4\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(\cdots}{e\cot(\cdots}\right.\right.}{\cdots}\right.\right.}{\cdots}$
default	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{3e(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e^2(e\cot(dx+c))^{\frac{3}{2}}}{3}-2e^3\sqrt{e\cot(dx+c)}+2e^4\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(\cdots}{e\cot(\cdots}\right.\right.}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*(1/7*(e*cot(d*x+c))^(7/2)+3/5*e*(e*cot(d*x+c))^(5/2)+2/3*e^2*(
e*cot(d*x+c))^(3/2)-2*e^3*(e*cot(d*x+c))^(1/2)+2*e^4*(1/8/e*(e^2)^(1/4)*2^(
1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)
)/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*ar
ctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e
```

```
*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)
)))
```

Maxima [A]
time = 0.51, size = 122, normalized size = 0.76

$$-\frac{\left(105\left(\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^3-\frac{420\,a^3}{\sqrt{\tan\left(dx+c\right)}}+\frac{140\,a^3}{\tan(dx+c)^{\frac{3}{2}}}+\frac{126\,a^3}{\tan(dx+c)^{\frac{5}{2}}}+\frac{30\,a^3}{\tan(dx+c)^{\frac{7}{2}}}\right)e^{\frac{3}{2}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/105*(105*(sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
 sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^3 - 420*a
^3/sqrt(tan(d*x + c)) + 140*a^3/tan(d*x + c)^(3/2) + 126*a^3/tan(d*x + c)^(
5/2) + 30*a^3/tan(d*x + c)^(7/2))*e^(3/2)/d
```

Fricas [A]
time = 2.95, size = 248, normalized size = 1.55

$$\frac{105\left(\sqrt{2}\,a^3\cos\left(2\,dx+2\,c\right)e^{\frac{3}{2}}-\sqrt{2}\,a^3e^{\frac{3}{2}}\right)\log\left(\left(\sqrt{2}\,\cos\left(2\,dx+2\,c\right)-\sqrt{2}\,\sin\left(2\,dx+2\,c\right)-\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}+2\,\sin\left(2\,dx+2\,c\right)+1\right)\sin\left(2\,dx+2\,c\right)-2\left(55\,a^3\cos\left(2\,dx+2\,c\right)^2e^{\frac{3}{2}}-30\,a^3\cos\left(2\,dx+2\,c\right)e^{\frac{3}{2}}-85\,a^3e^{\frac{3}{2}}-21\left(13\,a^3\cos\left(2\,dx+2\,c\right)e^{\frac{3}{2}}-7\,a^3e^{\frac{3}{2}}\right)\sin\left(2\,dx+2\,c\right)\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{105\left(d\cos\left(2\,dx+2\,c\right)-d\right)\sin\left(2\,dx+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/105*(105*(sqrt(2)*a^3*cos(2*d*x + 2*c)*e^(3/2) - sqrt(2)*a^3*e^(3/2))*log
((sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) - sqrt(2))*sqrt((cos(
2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)) + 2*sin(2*d*x + 2*c) + 1)*sin(2*d*x + 2
*c) - 2*(55*a^3*cos(2*d*x + 2*c)^2*e^(3/2) - 30*a^3*cos(2*d*x + 2*c)*e^(3/2
) - 85*a^3*e^(3/2) - 21*(13*a^3*cos(2*d*x + 2*c)*e^(3/2) - 7*a^3*e^(3/2))*s
in(2*d*x + 2*c))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)))/((d*cos(2*d
*x + 2*c) - d)*sin(2*d*x + 2*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}\,dx+\int 3\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}\cot\left(c+dx\right)dx+\int 3\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}\cot^2\left(c+dx\right)dx+\int\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}\cot^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*cot(d*x+c))**3,x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(3/2), x) + Integral(3*(e*cot(c + d*x))**(
3/2)*cot(c + d*x), x) + Integral(3*(e*cot(c + d*x))**(3/2)*cot(c + d*x)**2,
 x) + Integral((e*cot(c + d*x))**(3/2)*cot(c + d*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(3/2), x)
```

Mupad [B]
time = 1.62, size = 143, normalized size = 0.89

$$\frac{4\,a^3\,e\,\sqrt{e\cot\left(c+d\,x\right)}}{d}-\frac{6\,a^3\,{\left(e\cot(c+d\,x)\right)}^{5/2}}{5\,d\,e}-\frac{2\,a^3\,{\left(e\cot(c+d\,x)\right)}^{7/2}}{7\,d\,e^2}-\frac{4\,a^3\,{\left(e\cot(c+d\,x)\right)}^{3/2}}{3\,d}+\frac{\sqrt{2}\,a^3\,e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,a^6\,e^{9/2}\,\sqrt{e\cot\left(c+d\,x\right)}\,32\mathrm{i}}{32\,a^6\,e^5+32\,a^6\,e^5\cot(c+d\,x)}\right)\,2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^3,x)
```

```
[Out] (4*a^3*e*(e*cot(c + d*x))^(1/2))/d - (6*a^3*(e*cot(c + d*x))^(5/2))/(5*d*e)
 - (2*a^3*(e*cot(c + d*x))^(7/2))/(7*d*e^2) - (4*a^3*(e*cot(c + d*x))^(3/2)
)/(3*d) + (2^(1/2)*a^3*e^(3/2)*atan((2^(1/2)*a^6*e^(9/2)*(e*cot(c + d*x))^(
1/2)*32i)/(32*a^6*e^5 + 32*a^6*e^5*cot(c + d*x)))*2i)/d
```

3.17 $\int \sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))^3\,dx$

Optimal. Leaf size=138

$$-\frac{2\sqrt{2}\,a^3\sqrt{e}\,\mathrm{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{4a^3\sqrt{e\cot(c+dx)}}{d}-\frac{8a^3(e\cot(c+dx))^{3/2}}{5de}-\frac{2(e\cot(c+dx)}{}$$

```
[Out] -8/5*a^3*(e*cot(d*x+c))^(3/2)/d/e-2/5*(e*cot(d*x+c))^(3/2)*(a^3+a^3*cot(d*x
+c))/d/e-2*a^3*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c
))^(1/2))*2^(1/2)*e^(1/2)/d-4*a^3*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3647, 3711, 3609, 3613, 211}

$$-\frac{2\sqrt{2}\,a^3\sqrt{e}\,\mathrm{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{8a^3(e\cot(c+dx))^{3/2}}{5de}-\frac{4a^3\sqrt{e\cot(c+dx)}}{d}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{3/2}}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^3,x]
```

```
[Out] (-2*Sqrt[2]*a^3*Sqrt[e]*ArcTan[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sq
rt[e*Cot[c + d*x]])])/d - (4*a^3*Sqrt[e*Cot[c + d*x]])/d - (8*a^3*(e*Cot[c
+ d*x])^(3/2))/(5*d*e) - (2*(e*Cot[c + d*x])^(3/2)*(a^3 + a^3*Cot[c + d*x])
)/(5*d*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
```

```
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))^3\,dx = -\frac{2(e\cot(c+dx))^{3/2}\left(a^3+a^3\cot(c+dx)\right)}{5de} - \frac{2\int\sqrt{e\cot(c+a}}{}$$

$$= -\frac{8a^3(e\cot(c+dx))^{3/2}}{5de} - \frac{2(e\cot(c+dx))^{3/2}\left(a^3+a^3\cot(c+\right.}{5de}$$

$$= -\frac{4a^3\sqrt{e\cot(c+dx)}}{d} - \frac{8a^3(e\cot(c+dx))^{3/2}}{5de} - \frac{2(e\cot(c+d}{}$$

$$= -\frac{4a^3\sqrt{e\cot(c+dx)}}{d} - \frac{8a^3(e\cot(c+dx))^{3/2}}{5de} - \frac{2(e\cot(c+d}{}$$

$$= -\frac{2\sqrt{2}\,a^3\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} - \frac{4a^3\sqrt{e\cot(c+}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.68, size = 315, normalized size = 2.28

$$-\frac{a^3\sqrt{e\cot(c+dx)}\,(1+\cot(c+dx))^3\sin(c+dx)\left(-20\sqrt{\cot(c+dx)}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\sin(2(c+dx))+3\left(4\cos^2(c+dx)\sqrt{\cot(c+dx)}+10\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\sin^2(c+dx)-10\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\sin^2(c+dx)+40\sqrt{\cot(c+dx)}\sin^2(c+dx)+5\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^2(c+dx)-5\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^2(c+dx)+10\sqrt{\cot(c+dx)}\sin(2(c+dx))\right)\right)}{30d\sqrt{\cot(c+dx)}\,(\cos(c+dx)+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^3,x]
```

```
[Out] -1/30*(a^3*Sqrt[e*Cot[c + d*x]]*(1 + Cot[c + d*x])^3*Sin[c + d*x]*(-20*Sqrt
[Cot[c + d*x]]*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]*Sin[2*(c + d
*x)] + 3*(4*Cos[c + d*x]^2*Sqrt[Cot[c + d*x]] + 10*Sqrt[2]*ArcTan[1 - Sqrt[
2]*Sqrt[Cot[c + d*x]]]*Sin[c + d*x]^2 - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[
Cot[c + d*x]]]*Sin[c + d*x]^2 + 40*Sqrt[Cot[c + d*x]]*Sin[c + d*x]^2 + 5*Sq
rt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 - 5
*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2
+ 10*Sqrt[Cot[c + d*x]]*Sin[2*(c + d*x)])))/(d*Sqrt[Cot[c + d*x]]*(Cos[c +
d*x] + Sin[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(117) = 234$.
time = 0.48, size = 323, normalized size = 2.34

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{5}{2}}}{5}+e(e\cot(dx+c))^{\frac{3}{2}}+2e^2\sqrt{e\cot(dx+c)}-2e^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\,\mathrm{co}}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\,\mathrm{co}}}\right.\right.}{}\right.\right.}{}$
default	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{5}{2}}}{5}+e(e\cot(dx+c))^{\frac{3}{2}}+2e^2\sqrt{e\cot(dx+c)}-2e^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\,\mathrm{co}}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\,\mathrm{co}}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*(1/5*(e*cot(d*x+c))^(5/2)+e*(e*cot(d*x+c))^(3/2)+2*e^2*(e*cot(
d*x+c))^(1/2)-2*e^3*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8/
(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^
2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2
^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]

time = 0.51, size = 101, normalized size = 0.73

$$\frac{2\left(5\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)\right)a^3-\frac{10a^3}{\sqrt{\tan\left(dx+c\right)}}-\frac{5a^3}{\tan\left(dx+c\right)^{\frac{3}{2}}}-\frac{a^3}{\tan\left(dx+c\right)^{\frac{5}{2}}}\right)e^{\frac{1}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2/5*(5*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))))*a^3 - 10*a^3/sqr
t(tan(d*x + c)) - 5*a^3/tan(d*x + c)^(3/2) - a^3/tan(d*x + c)^(5/2))*e^(1/2
)/d
```

Fricas [A]
time = 2.52, size = 189, normalized size = 1.37

$$-\frac{2\left(5\left(\sqrt{2}a^3\cos\left(2dx+2c\right)e^{\frac{1}{2}}-\sqrt{2}a^3e^{\frac{1}{2}}\right)\arctan\left(-\frac{\left(\sqrt{2}\cos\left(2dx+2c\right)-\sqrt{2}\sin\left(2dx+2c\right)+\sqrt{2}\right)\sqrt{\frac{\cos\left(2dx+2c\right)+1}{\sin\left(2dx+2c\right)}}}{2\left(\cos\left(2dx+2c\right)+1\right)}\right)+\left(9a^3\cos\left(2dx+2c\right)e^{\frac{1}{2}}-5a^3e^{\frac{1}{2}}\sin\left(2dx+2c\right)-11a^3e^{\frac{1}{2}}\right)\sqrt{\frac{\cos\left(2dx+2c\right)+1}{\sin\left(2dx+2c\right)}}\right)}{5\left(d\cos\left(2dx+2c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/5*(5*(sqrt(2)*a^3*cos(2*d*x + 2*c)*e^(1/2) - sqrt(2)*a^3*e^(1/2))*arctan
(-1/2*(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt(
(cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))/(cos(2*d*x + 2*c) + 1)) + (9*a^3*c
os(2*d*x + 2*c)*e^(1/2) - 5*a^3*e^(1/2)*sin(2*d*x + 2*c) - 11*a^3*e^(1/2))*
sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\sqrt{e\cot\left(c+dx\right)}\,dx+\int3\sqrt{e\cot\left(c+dx\right)}\cot\left(c+dx\right)dx+\int3\sqrt{e\cot\left(c+dx\right)}\cot^2\left(c+dx\right)dx+\int\sqrt{e\cot\left(c+dx\right)}\cot^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)*(a+a*cot(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(3*sqrt(e*cot(c + d*x))*c
ot(c + d*x), x) + Integral(3*sqrt(e*cot(c + d*x))*cot(c + d*x)**2, x) + Int
egral(sqrt(e*cot(c + d*x))*cot(c + d*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^3,x, algorithm="giac")

[Out] integrate((a*cot(d*x + c) + a)^3*sqrt(e*cot(d*x + c)), x)

Mupad [B]
time = 0.99, size = 136, normalized size = 0.99

$$\frac{\sqrt{2}\,a^3\,\sqrt{e}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\,\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\,\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,{\left(e\,\cot\left(c+d\,x\right)\right)}^{3/2}}{2\,e^{3/2}}\right)\right)}{d}-\frac{2\,a^3\,{\left(e\,\cot\left(c+d\,x\right)\right)}^{3/2}}{d\,e}-\frac{2\,a^3\,{\left(e\,\cot\left(c+d\,x\right)\right)}^{5/2}}{5\,d\,e^2}-\frac{4\,a^3\,\sqrt{e\,\cot\left(c+d\,x\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^3,x)

[Out] (2^(1/2)*a^3*e^(1/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)))))/d - (2*a^3*(e*cot(c + d*x))^(3/2))/(d*e) - (2*a^3*(e*cot(c + d*x))^(5/2))/(5*d*e^2) - (4*a^3*(e*cot(c + d*x))^(1/2))/d

3.18 $\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal. Leaf size=117

$$\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}}-\frac{16a^3\sqrt{e\cot(c+dx)}}{3de}-\frac{2\sqrt{e\cot(c+dx)}\,(a^3+a^3\cot(c+dx))}{3de}$$

```
[Out] 2*a^3*arctanh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2)
)*2^(1/2)/d/e^(1/2)-16/3*a^3*(e*cot(d*x+c))^(1/2)/d/e-2/3*(a^3+a^3*cot(d*x+
c))*(e*cot(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3647, 3711, 3613, 214}

$$-\frac{16a^3\sqrt{e\cot(c+dx)}}{3de}-\frac{2(a^3\cot(c+dx)+a^3)\sqrt{e\cot(c+dx)}}{3de}+\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{e}\,\cot(c+dx)+\sqrt{e}}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (2*Sqrt[2]*a^3*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot
[c + d*x]])])/(d*Sqrt[e]) - (16*a^3*Sqrt[e*Cot[c + d*x]])/(3*d*e) - (2*Sqrt
[e*Cot[c + d*x]]*(a^3 + a^3*Cot[c + d*x]))/(3*d*e)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
```

```
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = -\frac{2\sqrt{e\cot(c+dx)}\,(a^3+a^3\cot(c+dx))}{3de} - \frac{2\int \frac{-a^3e-3a^3e\cot(c+dx)-4a^3e\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}}\,d}{3e}$$

$$= -\frac{16a^3\sqrt{e\cot(c+dx)}}{3de} - \frac{2\sqrt{e\cot(c+dx)}\,(a^3+a^3\cot(c+dx))}{3de} - \frac{2\int \frac{3a^3e-3a^3}{\sqrt{e\cot}}}{3}$$

$$= -\frac{16a^3\sqrt{e\cot(c+dx)}}{3de} - \frac{2\sqrt{e\cot(c+dx)}\,(a^3+a^3\cot(c+dx))}{3de} + \frac{(12a^6e)\,\mathrm{Sub}}{}$$

$$= \frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}} - \frac{16a^3\sqrt{e\cot(c+dx)}}{3de} - \frac{2\sqrt{e\cot(c-}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 5.46, size = 292, normalized size = 2.50

$$-\frac{a^3(1+\cot(c+dx))^3\sin(c+dx)\left(4\cos^2(c+dx)+8\cos^2(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)+6\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\sqrt{\cot(c+dx)}\sin^2(c+dx)-6\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\sqrt{\cot(c+dx)}\sin^2(c+dx)+3\sqrt{2}\sqrt{\cot(c+dx)}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^2(c+dx)-3\sqrt{2}\sqrt{\cot(c+dx)}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^2(c+dx)+18\sin(2(c+dx))\right)}{6d\sqrt{e\cot(c+dx)}\,(\cos(c+dx)+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] -1/6*(a^3*(1 + Cot[c + d*x])^3*Sin[c + d*x]*(4*Cos[c + d*x]^2 + 8*Cos[c + d
*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + 6*Sqrt[2]*ArcTan[1
```

```
- Sqrt[2]*Sqrt[Cot[c + d*x]]*Sqrt[Cot[c + d*x]]*Sin[c + d*x]^2 - 6*Sqrt[2]
*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sin[c + d*x]^2 +
 3*Sqrt[2]*Sqrt[Cot[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c +
d*x]]*Sin[c + d*x]^2 - 3*Sqrt[2]*Sqrt[Cot[c + d*x]]*Log[1 + Sqrt[2]*Sqrt[Co
t[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 + 18*Sin[2*(c + d*x)]))/(d*Sqrt[
e*Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(98) = 196$.
time = 0.45, size = 309, normalized size = 2.64

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}+3e\sqrt{e\cot(dx+c)}-2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{\ }}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{\ }}\right.\right.}{}\right.\right.}{}$
default	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}+3e\sqrt{e\cot(dx+c)}-2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{\ }}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{\ }}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*(1/3*(e*cot(d*x+c))^(3/2)+3*e*(e*cot(d*x+c))^(1/2)-2*e^2*(1/8/
e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(
-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(e^2)^(1/4)*2^(1/2)*(ln((
e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d
*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/
2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.51, size = 96, normalized size = 0.82

$$\frac{\left(3\left(\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^3-\frac{18a^3}{\sqrt{\tan(dx+c)}}-\frac{2a^3}{\tan(dx+c)^{\frac{3}{2}}}\right)e^{\left(-\frac{1}{2}\right)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt
(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^3 - 18*a^3/sqr
t(tan(d*x + c)) - 2*a^3/tan(d*x + c)^(3/2))*e^(-1/2)/d
```

Fricas [A]
time = 2.58, size = 168, normalized size = 1.44

$$\frac{\left(3\sqrt{2}\,a^3\log\left(-\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)-\sqrt{2}\sin\left(2\,dx+2\,c\right)-\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}+2\sin\left(2\,dx+2\,c\right)+1\right)\sin\left(2\,dx+2\,c\right)-2\left(a^3\cos\left(2\,dx+2\,c\right)+9\,a^3\sin\left(2\,dx+2\,c\right)+a^3\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\right)e^{\left(-\frac{1}{2}\right)}}{3\,d\sin\left(2\,dx+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*sqrt(2)*a^3*log(-(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c
) - sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)) + 2*sin(2*d*x +
2*c) + 1)*sin(2*d*x + 2*c) - 2*(a^3*cos(2*d*x + 2*c) + 9*a^3*sin(2*d*x + 2*
c) + a^3)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)))*e^(-1/2)/(d*sin(2*
d*x + 2*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\frac{1}{\sqrt{e\cot\left(c+dx\right)}}\,dx+\int\frac{3\cot\left(c+dx\right)}{\sqrt{e\cot\left(c+dx\right)}}\,dx+\int\frac{3\cot^2\left(c+dx\right)}{\sqrt{e\cot\left(c+dx\right)}}\,dx+\int\frac{\cot^3\left(c+dx\right)}{\sqrt{e\cot\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a**3*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(3*cot(c + d*x)/sqrt(e*
cot(c + d*x)), x) + Integral(3*cot(c + d*x)**2/sqrt(e*cot(c + d*x)), x) + I
ntegral(cot(c + d*x)**3/sqrt(e*cot(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^3/sqrt(e*cot(d*x + c)), x)
```

Mupad [B]
time = 0.62, size = 100, normalized size = 0.85

$$\frac{2\sqrt{2}\,a^3\,\mathrm{atanh}\left(\frac{32\sqrt{2}\,a^6\,\sqrt{e}\,\sqrt{e\,\cot\left(c+d\,x\right)}}{32\,a^6\,e+32\,a^6\,e\,\cot\left(c+d\,x\right)}\right)}{d\,\sqrt{e}}-\frac{2\,a^3\,{\left(e\,\cot\left(c+d\,x\right)\right)}^{3/2}}{3\,d\,e^2}-\frac{6\,a^3\,\sqrt{e\,\cot\left(c+d\,x\right)}}{d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(1/2),x)
```

```
[Out] (2*2^(1/2)*a^3*atanh((32*2^(1/2)*a^6*e^(1/2)*(e*cot(c + d*x))^(1/2))/(32*a^
6*e + 32*a^6*e*cot(c + d*x))))/(d*e^(1/2)) - (2*a^3*(e*cot(c + d*x))^(3/2))
/(3*d*e^2) - (6*a^3*(e*cot(c + d*x))^(1/2))/(d*e)
```

3.19 $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2\sqrt{2}\,a^3\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}}-\frac{4a^3\sqrt{e\cot(c+dx)}}{de^2}+\frac{2(a^3+a^3\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

```
[Out] 2*a^3*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))
*2^(1/2)/d/e^(3/2)+2*(a^3+a^3*cot(d*x+c))/d/e/(e*cot(d*x+c))^(1/2)-4*a^3*(e
*cot(d*x+c))^(1/2)/d/e^2
```

Rubi [A]

time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3646, 3711, 3613, 211}

$$\frac{2\sqrt{2}\,a^3\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}}-\frac{4a^3\sqrt{e\cot(c+dx)}}{de^2}+\frac{2(a^3\cot(c+dx)+a^3)}{de\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[2]*a^3*ArcTan[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[
c + d*x]])])/(d*e^(3/2)) - (4*a^3*Sqrt[e*Cot[c + d*x]])/(d*e^2) + (2*(a^3 +
 a^3*Cot[c + d*x]))/(d*e*Sqrt[e*Cot[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
```

```
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2(a^3+a^3\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} - \frac{2\int \frac{-2a^3e^2-a^3e^2\cot(c+dx)-a^3e^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^3}$$

$$= -\frac{4a^3\sqrt{e\cot(c+dx)}}{de^2} + \frac{2(a^3+a^3\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} - \frac{2\int \frac{-a^3e^2-a^3e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^3}$$

$$= -\frac{4a^3\sqrt{e\cot(c+dx)}}{de^2} + \frac{2(a^3+a^3\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} + \frac{(4a^6e)\,\mathrm{Subst}\left(\int \frac{1}{-2a^6e^4-ex^2} \ldots\right.}{d \ldots}$$

$$= \frac{2\sqrt{2}\,a^3\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}} - \frac{4a^3\sqrt{e\cot(c+dx)}}{de^2} + \frac{2(a^3+a^3\cot \ldots}{de\sqrt{e\cot(\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 3.10, size = 311, normalized size = 2.73

$$\frac{a^3(1+\cot(c+dx))^3\left(-4\cos^3(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)+\sin(c+dx)\left(-4\cos^2(c+dx)+2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot^{\frac{3}{2}}(c+dx)\sin^2(c+dx)-2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot^{\frac{3}{2}}(c+dx)\sin^2(c+dx)+\sqrt{2}\cot^{\frac{3}{2}}(c+dx)\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^2(c+dx)-\sqrt{2}\cot^{\frac{3}{2}}(c+dx)\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^2(c+dx)+2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)\sin(2(c+dx))\right)\right)}{2d(e\cot(c+dx))^{3/2}(\cos(c+dx)+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (a^3*(1 + Cot[c + d*x])^3*(-4*Cos[c + d*x]^3*Hypergeometric2F1[3/4, 1, 7/4,
 -Cot[c + d*x]^2] + Sin[c + d*x]*(-4*Cos[c + d*x]^2 + 2*Sqrt[2]*ArcTan[1 -
Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cot[c + d*x]^(3/2)*Sin[c + d*x]^2 - 2*Sqrt[2]*A
rcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cot[c + d*x]^(3/2)*Sin[c + d*x]^2 + S
qrt[2]*Cot[c + d*x]^(3/2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]
]*Sin[c + d*x]^2 - Sqrt[2]*Cot[c + d*x]^(3/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c +
d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 + 2*Hypergeometric2F1[-1/4, 1, 3/4, -C
ot[c + d*x]^2]*Sin[2*(c + d*x)])))/(2*d*(e*Cot[c + d*x])^(3/2)*(Cos[c + d*x
] + Sin[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(99) = 198$.
time = 0.42, size = 305, normalized size = 2.68

method	result
derivativedivides	$-\frac{2a^3\left(\sqrt{e\cot(dx+c)}+2e\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan \ldots}{8 \ldots}\right.\right.}{\ldots}$
default	$-\frac{2a^3\left(\sqrt{e\cot(dx+c)}+2e\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan \ldots}{8 \ldots}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*((e*cot(d*x+c))^(1/2)+2*e*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*co
t(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c
)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1))+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-e/(e*cot(d*
x+c))^(1/2))
```

Maxima [A]
time = 0.53, size = 86, normalized size = 0.75

$$-\frac{2\left(\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)\right)a^3-a^3\sqrt{\tan(dx+c)}+\frac{a^3}{\sqrt{\tan(dx+c)}}\right)e^{\left(-\frac{3}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*((sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))))*a^3 - a^3*sqrt(tan(
d*x + c)) + a^3/sqrt(tan(d*x + c)))*e^(-3/2)/d
```

Fricas [A]
time = 2.73, size = 178, normalized size = 1.56

$$\frac{2\left(\left(\sqrt{2}\,a^3\cos\left(2\,dx+2\,c\right)+\sqrt{2}\,a^3\right)\arctan\left(-\frac{\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)-\sqrt{2}\sin\left(2\,dx+2\,c\right)+\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{2\left(\cos\left(2\,dx+2\,c\right)+1\right)}\right)-\left(a^3\cos\left(2\,dx+2\,c\right)-a^3\sin\left(2\,dx+2\,c\right)+a^3\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\right)}{d\cos\left(2\,dx+2\,c\right)e^{\frac{3}{2}}+de^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((sqrt(2)*a^3*cos(2*d*x + 2*c) + sqrt(2)*a^3)*arctan(-1/2*(sqrt(2)*cos(2*
d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1
)/sin(2*d*x + 2*c))/(cos(2*d*x + 2*c) + 1)) - (a^3*cos(2*d*x + 2*c) - a^3*s
in(2*d*x + 2*c) + a^3)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)))/(d*co
s(2*d*x + 2*c)*e^(3/2) + d*e^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int\frac{3\cot\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int\frac{3\cot^2\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int\frac{\cot^3\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(3/2),x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(3*cot(c + d*x)/(e*co
t(c + d*x))**(3/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(3/2)
, x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^3/(e*cot(d*x + c))^(3/2), x)
```

Mupad [B]
time = 0.58, size = 119, normalized size = 1.04

$$\frac{2\,a^3}{d\,e\,\sqrt{e\cot\left(c+d\,x\right)}}-\frac{2\,a^3\,\sqrt{e\cot\left(c+d\,x\right)}}{d\,e^2}-\frac{\sqrt{2}\,a^3\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,{\left(e\cot\left(c+d\,x\right)\right)}^{3/2}}{2\,e^{3/2}}\right)\right)}{d\,e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(3/2),x)
```

```
[Out] (2*a^3)/(d*e*(e*cot(c + d*x))^(1/2)) - (2*a^3*(e*cot(c + d*x))^(1/2))/(d*e^
2) - (2^(1/2)*a^3*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2
*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*
x))^(3/2))/(2*e^(3/2)))))/(d*e^(3/2))
```

3.20 $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}}+\frac{16a^3}{3de^2\sqrt{e\cot(c+dx)}}+\frac{2(a^3+a^3\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

```
[Out] 2/3*(a^3+a^3*cot(d*x+c))/d/e/(e*cot(d*x+c))^(3/2)-2*a^3*arctanh(1/2*(e^(1/2
)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d/e^(5/2)+16/3*
a^3/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3646, 3709, 3613, 214}

$$-\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{e}\,\cot(c+dx)+\sqrt{e}}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}}+\frac{16a^3}{3de^2\sqrt{e\cot(c+dx)}}+\frac{2(a^3\cot(c+dx)+a^3)}{3de(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[2]*a^3*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Co
t[c + d*x]])])/(d*e^(5/2)) + (16*a^3)/(3*d*e^2*Sqrt[e*Cot[c + d*x]]) + (2*(
a^3 + a^3*Cot[c + d*x]))/(3*d*e*(e*Cot[c + d*x])^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
```

```
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \frac{2(a^3+a^3\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} - \frac{2\int \frac{-4a^3e^2-3a^3e^2\cot(c+dx)-a^3e^2\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{3e^3}$$

$$= \frac{16a^3}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} - \frac{2\int \frac{-3a^3e^3+3a^3e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{3e^5}$$

$$= \frac{16a^3}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} + \frac{(12a^6e)\,\mathrm{Subst}\left(\int \frac{1}{18a^6e^6-ex^2}\,dx \right.}{d}$$

$$= -\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}} + \frac{16a^3}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3}{3de(e\cot}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.11, size = 417, normalized size = 3.56

$$-\frac{2\cos^3(c+dx)\cot(c+dx)(a+a\cot(c+dx))^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)}{3d(e\cot(c+dx))^{5/2}(\cos(c+dx)+\sin(c+dx))^3} + \frac{6\cos^2(c+dx)(a+a\cot(c+dx))^3\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)\sin(c+dx)}{d(e\cot(c+dx))^{5/2}(\cos(c+dx)+\sin(c+dx))^3} + \frac{2\cos(c+dx)(a+a\cot(c+dx))^3\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)\sin^2(c+dx)}{3d(e\cot(c+dx))^{5/2}(\cos(c+dx)+\sin(c+dx))^3} + \frac{3\cot^{\frac{5}{2}}(c+dx)(a+a\cot(c+dx))^3\left(2\left(\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)\right)+\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\sin^3(c+dx)}{4d(e\cot(c+dx))^{5/2}(\cos(c+dx)+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Cot[c + d*x])^3*Hypergeometric2F1[3/
4, 1, 7/4, -Cot[c + d*x]^2])/(3*d*(e*Cot[c + d*x])^(5/2)*(Cos[c + d*x] + Si
n[c + d*x])^3) + (6*Cos[c + d*x]^2*(a + a*Cot[c + d*x])^3*Hypergeometric2F1
[-1/4, 1, 3/4, -Cot[c + d*x]^2]*Sin[c + d*x])/(d*(e*Cot[c + d*x])^(5/2)*(Co
s[c + d*x] + Sin[c + d*x])^3) + (2*Cos[c + d*x]*(a + a*Cot[c + d*x])^3*Hype
rgeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2]*Sin[c + d*x]^2)/(3*d*(e*Cot[c
+ d*x])^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (3*Cot[c + d*x]^(5/2)*(a +
 a*Cot[c + d*x])^3*(2*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - Sqr
t[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
 + Cot[c + d*x]])*Sin[c + d*x]^3)/(4*d*(e*Cot[c + d*x])^(5/2)*(Cos[c + d*x]
 + Sin[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(98) = 196$.

time = 0.44, size = 303, normalized size = 2.59

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{4e}\cdots\right.}{\cdots}$
default	$-\frac{2a^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{4e}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*(1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4/(e^2)^(
1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^
2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2
)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/3*e/(e*cot(d*x+c))^(3/2)-3/(e*cot(d*
x+c))^(1/2))
```

Maxima [A]

time = 0.53, size = 97, normalized size = 0.83

$$-\frac{\left(3\left(\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)\right)a^3-2\left(a^3+\frac{9a^3}{\tan\left(dx+c\right)}\right)\tan\left(dx+c\right)^{\frac{3}{2}}\right)e^{\left(-\frac{5}{2}\right)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqr
t(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^3 - 2*(a^3 +
9*a^3/tan(d*x + c))*tan(d*x + c)^(3/2))*e^(-5/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(89) = 178$.
time = 4.42, size = 182, normalized size = 1.56

$$\frac{3\left(\sqrt{2}a^3\cos\left(2dx+2c\right)+\sqrt{2}a^3\right)\log\left(\left(\sqrt{2}\cos\left(2dx+2c\right)-\sqrt{2}\sin\left(2dx+2c\right)-\sqrt{2}\right)\sqrt{\frac{\cos\left(2dx+2c\right)+1}{\sin\left(2dx+2c\right)}}+2\sin\left(2dx+2c\right)+1\right)-2\left(a^3\cos\left(2dx+2c\right)-9a^3\sin\left(2dx+2c\right)-a^3\right)\sqrt{\frac{\cos\left(2dx+2c\right)+1}{\sin\left(2dx+2c\right)}}}{3\left(d\cos\left(2dx+2c\right)e^{\frac{5}{2}}+de^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(sqrt(2)*a^3*cos(2*d*x + 2*c) + sqrt(2)*a^3)*log((sqrt(2)*cos(2*d*x
+ 2*c) - sqrt(2)*sin(2*d*x + 2*c) - sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/si
n(2*d*x + 2*c)) + 2*sin(2*d*x + 2*c) + 1) - 2*(a^3*cos(2*d*x + 2*c) - 9*a^3
*sin(2*d*x + 2*c) - a^3)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)))/(d*
cos(2*d*x + 2*c)*e^(5/2) + d*e^(5/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx+\int\frac{3\cot\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx+\int\frac{3\cot^2\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx+\int\frac{\cot^3\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(5/2),x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(-5/2), x) + Integral(3*cot(c + d*x)/(e*co
t(c + d*x))**(5/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(5/2)
, x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(5/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^3/(e*cot(d*x + c))^(5/2), x)
```

Mupad [B]
time = 0.71, size = 101, normalized size = 0.86

$$\frac{\frac{2\,a^3\,e}{3}+6\,a^3\,e\cot(c+d\,x)}{d\,e^2\,{\left(e\cot\left(c+d\,x\right)\right)}^{3/2}}-\frac{2\,\sqrt{2}\,a^3\,\mathrm{atanh}\left(\frac{32\,\sqrt{2}\,a^6\,d\,e^{5/2}\,\sqrt{e\cot\left(c+d\,x\right)}}{32\,a^6\,d\,e^3+32\,a^6\,d\,e^3\cot(c+d\,x)}\right)}{d\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(5/2),x)
```

```
[Out] ((2*a^3*e)/3 + 6*a^3*e*cot(c + d*x))/(d*e^2*(e*cot(c + d*x))^(3/2)) - (2*2^
(1/2)*a^3*atanh((32*2^(1/2)*a^6*d*e^(5/2)*(e*cot(c + d*x))^(1/2))/(32*a^6*d
*e^3 + 32*a^6*d*e^3*cot(c + d*x))))/(d*e^(5/2))
```

3.21 $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{2\sqrt{2}\,a^3\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{de^{7/2}}+\frac{8a^3}{5de^2(e\cot(c+dx))^{3/2}}+\frac{4a^3}{de^3\sqrt{e\cot(c+dx)}}+\frac{2(a^3+a^3\cot(c+a}{5de(e\cot(c+dx))}$$

```
[Out] 8/5*a^3/d/e^2/(e*cot(d*x+c))^(3/2)+2/5*(a^3+a^3*cot(d*x+c))/d/e/(e*cot(d*x+
c))^(5/2)-2*a^3*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+
c))^(1/2))*2^(1/2)/d/e^(7/2)+4*a^3/d/e^3/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3646, 3709, 3610, 3613, 211}

$$-\frac{2\sqrt{2}\,a^3\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{de^{7/2}}+\frac{4a^3}{de^3\sqrt{e\cot(c+dx)}}+\frac{8a^3}{5de^2(e\cot(c+dx))^{3/2}}+\frac{2(a^3\cot(c+dx)+a^3)}{5de(e\cot(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] (-2*Sqrt[2]*a^3*ArcTan[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot
[c + d*x]])])/(d*e^(7/2)) + (8*a^3)/(5*d*e^2*(e*Cot[c + d*x])^(3/2)) + (4*a
^3)/(d*e^3*Sqrt[e*Cot[c + d*x]]) + (2*(a^3 + a^3*Cot[c + d*x]))/(5*d*e*(e*C
ot[c + d*x])^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \frac{2(a^3+a^3\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{2\int \frac{-6a^3e^2-5a^3e^2\cot(c+dx)-a^3e^2\cot^2(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx}{5e^3}$$

$$= \frac{8a^3}{5de^2(e\cot(c+dx))^{3/2}} + \frac{2(a^3+a^3\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{2\int \frac{-5a^3e^3+5a^3e^3\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{5e^5}$$

$$= \frac{8a^3}{5de^2(e\cot(c+dx))^{3/2}} + \frac{4a^3}{de^3\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{2\int$$

$$= \frac{8a^3}{5de^2(e\cot(c+dx))^{3/2}} + \frac{4a^3}{de^3\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} + \frac{(20a$$

$$= -\frac{2\sqrt{2}\,a^3\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{7/2}} + \frac{8a^3}{5de^2(e\cot(c+dx))^{3/2}} + \frac{4}{de^3\sqrt{e\cot}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 3.44, size = 269, normalized size = 1.91

$$\frac{a^3\left(120\cos^3(c+dx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+\sin(c+dx)\left(40\cos^2(c+dx)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)+\sin(c+dx)\left(8\cos(c+dx)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(c+dx)\right)+5\sqrt{2}\cot^{\frac{7}{2}}(c+dx)\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\sin(c+dx)\right)\right)\right)(1+\tan(c+dx))^3}{20de^3\sqrt{e\cot(c+dx)}\,(\cos(c+dx)+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] (a^3*(120*Cos[c + d*x]^3*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] +
 Sin[c + d*x]*(40*Cos[c + d*x]^2*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d
*x]^2] + Sin[c + d*x]*(8*Cos[c + d*x]*Hypergeometric2F1[-5/4, 1, -1/4, -Cot
[c + d*x]^2] + 5*Sqrt[2]*Cot[c + d*x]^(7/2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[
c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqr
t[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[
c + d*x]])*Sin[c + d*x])))*(1 + Tan[c + d*x])^3)/(20*d*e^3*Sqrt[e*Cot[c + d
*x]]*(Cos[c + d*x] + Sin[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(120) = 240$.
time = 0.44, size = 323, normalized size = 2.29

method	result
derivativedivides	$-\frac{2a^3\left(-\frac{e}{5(e\cot(dx+c))^{\frac{5}{2}}}-\frac{1}{(e\cot(dx+c))^{\frac{3}{2}}}-\frac{2}{e\sqrt{e\cot(dx+c)}}+\frac{-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(a}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(a}}\right.\right.}{}}{}\right.}{}$
default	$-\frac{2a^3\left(-\frac{e}{5(e\cot(dx+c))^{\frac{5}{2}}}-\frac{1}{(e\cot(dx+c))^{\frac{3}{2}}}-\frac{2}{e\sqrt{e\cot(dx+c)}}+\frac{-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(a}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(a}}\right.\right.}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*(-1/5*e/(e*cot(d*x+c))^(5/2)-1/(e*cot(d*x+c))^(3/2)-2/e/(e*cot
(d*x+c))^(1/2)+1/e*(-1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*co
```

```
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4/
(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^
2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2
^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.53, size = 101, normalized size = 0.72

$$\frac{2\left(5\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)\right)a^3+\left(a^3+\frac{5a^3}{\tan(dx+c)}+\frac{10a^3}{\tan(dx+c)^2}\right)\tan\left(dx+c\right)^{\frac{5}{2}}\right)e^{\left(-\frac{7}{2}\right)}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*(5*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))))*a^3 + (a^3 + 5*a
^3/tan(d*x + c) + 10*a^3/tan(d*x + c)^2)*tan(d*x + c)^(5/2))*e^(-7/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(109) = 218.
time = 3.80, size = 235, normalized size = 1.67

$$-\frac{2\left(5\left(\sqrt{2}a^3\cos\left(2dx+2c\right)^2+2\sqrt{2}a^3\cos\left(2dx+2c\right)+\sqrt{2}a^3\right)\arctan\left(-\frac{\left(\sqrt{2}\cos(2dx+2c)-\sqrt{2}\sin(2dx+2c)+\sqrt{2}\right)\sqrt{\frac{\cos\left(2dx+2c\right)+1}{\sin\left(2dx+2c\right)}}}{2\left(\cos(2dx+2c)+1\right)}\right)+\left(5a^3\cos\left(2dx+2c\right)^2-5a^3-\left(9a^3\cos\left(2dx+2c\right)+11a^3\right)\sin\left(2dx+2c\right)\right)\sqrt{\frac{\cos\left(2dx+2c\right)+1}{\sin\left(2dx+2c\right)}}\right)}{5\left(d\cos\left(2dx+2c\right)^2e^{\frac{7}{2}}+2d\cos\left(2dx+2c\right)e^{\frac{7}{2}}+de^{\frac{7}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(5*(sqrt(2)*a^3*cos(2*d*x + 2*c)^2 + 2*sqrt(2)*a^3*cos(2*d*x + 2*c) +
sqrt(2)*a^3)*arctan(-1/2*(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*
c) + sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))/(cos(2*d*x + 2*
c) + 1)) + (5*a^3*cos(2*d*x + 2*c)^2 - 5*a^3 - (9*a^3*cos(2*d*x + 2*c) + 11
*a^3)*sin(2*d*x + 2*c))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)))/(d*c
os(2*d*x + 2*c)^2*e^(7/2) + 2*d*cos(2*d*x + 2*c)*e^(7/2) + d*e^(7/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx+\int\frac{3\cot\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx+\int\frac{3\cot^2\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx+\int\frac{\cot^3\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(7/2),x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(-7/2), x) + Integral(3*cot(c + d*x)/(e*co
t(c + d*x))**(7/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(7/2)
, x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(7/2), x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 1.26, size = 126, normalized size = 0.89

$$\frac{4\,e\,a^3\,{\mathrm{cot}\left(c+d\,x\right)}^2+2\,e\,a^3\,\mathrm{cot}\left(c+d\,x\right)+\frac{2\,e\,a^3}{5}}{d\,e^2\,{\left(e\,\mathrm{cot}\left(c+d\,x\right)\right)}^{5/2}}+\frac{\sqrt{2}\,a^3\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\,\mathrm{cot}\left(c+d\,x\right)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,{\left(e\,\mathrm{cot}\left(c+d\,x\right)\right)}^{3/2}}{2\,e^{3/2}}\right)\right)}{d\,e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(7/2),x)
```

```
[Out] ((2*a^3*e)/5 + 4*a^3*e*cot(c + d*x)^2 + 2*a^3*e*cot(c + d*x))/(d*e^2*(e*cot
(c + d*x))^(5/2)) + (2^(1/2)*a^3*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(
2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2
)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)))))/(d*e^(7/2))
```

3.22 $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=165

$$\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{de^{9/2}}+\frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}}+\frac{4a^3}{3de^3(e\cot(c+dx))^{3/2}}-\frac{4a^3}{de^4\sqrt{e\cot(c+}}$$

```
[Out] 32/35*a^3/d/e^2/(e*cot(d*x+c))^(5/2)+4/3*a^3/d/e^3/(e*cot(d*x+c))^(3/2)+2/7
*(a^3+a^3*cot(d*x+c))/d/e/(e*cot(d*x+c))^(7/2)+2*a^3*arctanh(1/2*(e^(1/2)+c
ot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d/e^(9/2)-4*a^3/d/
e^4/(e*cot(d*x+c))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3646, 3709, 3610, 3613, 214}

$$\frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{e}\,\cot(c+dx)+\sqrt{e}}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{de^{9/2}}-\frac{4a^3}{de^4\sqrt{e\cot(c+dx)}}+\frac{4a^3}{3de^3(e\cot(c+dx))^{3/2}}+\frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}}+\frac{2(a^3\cot(c+dx)+a^3)}{7de(e\cot(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(9/2),x]
```

```
[Out] (2*Sqrt[2]*a^3*ArcTanh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot
[c + d*x]])])/(d*e^(9/2)) + (32*a^3)/(35*d*e^2*(e*Cot[c + d*x])^(5/2)) + (4
*a^3)/(3*d*e^3*(e*Cot[c + d*x])^(3/2)) - (4*a^3)/(d*e^4*Sqrt[e*Cot[c + d*x]
]) + (2*(a^3 + a^3*Cot[c + d*x]))/(7*d*e*(e*Cot[c + d*x])^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \frac{2(a^3+a^3\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{-8a^3e^2-7a^3e^2\cot(c+dx)-a^3e^2\cot^2(c+dx)}{(e\cot(c+dx))^{7/2}}dx}{7e^3}$$

$$= \frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}} + \frac{2(a^3+a^3\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{-7a^3e^3+7a^3e^3\cot(c+dx)}{(e\cot(c+dx))^{5/2}}dx}{7e^5}$$

$$= \frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}} + \frac{4a^3}{3de^3(e\cot(c+dx))^{3/2}} + \frac{2(a^3+a^3\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} -$$

$$= \frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}} + \frac{4a^3}{3de^3(e\cot(c+dx))^{3/2}} - \frac{4a^3}{de^4\sqrt{e\cot(c+dx)}} +$$

$$= \frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}} + \frac{4a^3}{3de^3(e\cot(c+dx))^{3/2}} - \frac{4a^3}{de^4\sqrt{e\cot(c+dx)}} +$$

$$= \frac{2\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{9/2}} + \frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.12, size = 174, normalized size = 1.05

$$\frac{2a^3\cos(c+dx)(1+\cot(c+dx))^3\left(35\cos^2(c+dx)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)+35\cos^2(c+dx)\cot(c+dx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+5\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\cot^2(c+dx)\right)\sin^2(c+dx)+\frac{21}{2}\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(c+dx)\right)\sin(2(c+dx))\right)}{35d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(9/2),x]
```

```
[Out] (2*a^3*Cos[c + d*x]*(1 + Cot[c + d*x])^3*(35*Cos[c + d*x]^2*Hypergeometric2
F1[-3/4, 1, 1/4, -Cot[c + d*x]^2] + 35*Cos[c + d*x]^2*Cot[c + d*x]*Hypergeo
metric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + 5*Hypergeometric2F1[-7/4, 1, -3/
4, -Cot[c + d*x]^2]*Sin[c + d*x]^2 + (21*Hypergeometric2F1[-5/4, 1, -1/4, -
Cot[c + d*x]^2]*Sin[2*(c + d*x)])/2))/(35*d*(e*Cot[c + d*x])^(9/2)*(Cos[c +
 d*x] + Sin[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(140) = 280.
time = 0.43, size = 338, normalized size = 2.05

method	result

derivativedivides	$-\frac{2a^3\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
default	$-\frac{2a^3\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^3/e^2*(1/e^2*(-1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/4
/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-
2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/7*e/(e*cot(d*x+c))^(7/2)-3/
5/(e*cot(d*x+c))^(5/2)+2/e^2/(e*cot(d*x+c))^(1/2)-2/3/e/(e*cot(d*x+c))^(3/2
))
```

Maxima [A]
time = 0.51, size = 125, normalized size = 0.76

$$\frac{\left(2\left(15\,a^3+\frac{63\,a^3}{\tan(dx+c)}+\frac{70\,a^3}{\tan(dx+c)^2}-\frac{210\,a^3}{\tan(dx+c)^3}\right)\tan\left(dx+c\right)^{\frac{7}{2}}+105\left(\sqrt{2}\,\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\,\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)\right)a^3\right)e^{\left(-\frac{9}{2}\right)}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] 1/105*(2*(15*a^3 + 63*a^3/tan(d*x + c) + 70*a^3/tan(d*x + c)^2 - 210*a^3/ta
n(d*x + c)^3)*tan(d*x + c)^(7/2) + 105*(sqrt(2)*log(sqrt(2)/sqrt(tan(d*x +
c)) + 1/tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan
(d*x + c) + 1))*a^3)*e^(-9/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(125) = 250$.

time = 3.91, size = 252, normalized size = 1.53

$$\frac{105 \left(\sqrt{2}\, a^3 \cos\left(2\, dx + 2\, c\right)^2 + 2 \sqrt{2}\, a^3 \cos\left(2\, dx + 2\, c\right) + \sqrt{2}\, a^3\right) \log\left(-\left(\sqrt{2}\, \cos\left(2\, dx + 2\, c\right) - \sqrt{2}\, \sin\left(2\, dx + 2\, c\right) - \sqrt{2}\right) \sqrt{\frac{\cos\left(2\, dx + 2\, c\right) + 1}{\sin\left(2\, dx + 2\, c\right)}} + 2\, \sin\left(2\, dx + 2\, c\right) + 1\right) - 2 \left(55\, a^3 \cos\left(2\, dx + 2\, c\right)^2 + 30\, a^3 \cos\left(2\, dx + 2\, c\right) - 85\, a^3 + 21 \left(13\, a^3 \cos\left(2\, dx + 2\, c\right) + 7\, a^3\right) \sin\left(2\, dx + 2\, c\right)\right) \sqrt{\frac{\cos\left(2\, dx + 2\, c\right) + 1}{\sin\left(2\, dx + 2\, c\right)}}}{105 \left(d \cos\left(2\, dx + 2\, c\right)^2 e^{\frac{9}{2}} + 2\, d \cos\left(2\, dx + 2\, c\right) e^{\frac{9}{2}} + de^{\frac{9}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(105*(sqrt(2)*a^3*cos(2*d*x + 2*c)^2 + 2*sqrt(2)*a^3*cos(2*d*x + 2*c)
 + sqrt(2)*a^3)*log(-(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) -
 sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)) + 2*sin(2*d*x + 2*c
) + 1) - 2*(55*a^3*cos(2*d*x + 2*c)^2 + 30*a^3*cos(2*d*x + 2*c) - 85*a^3 +
21*(13*a^3*cos(2*d*x + 2*c) + 7*a^3)*sin(2*d*x + 2*c))*sqrt((cos(2*d*x + 2*
c) + 1)/sin(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c)^2*e^(9/2) + 2*d*cos(2*d*x +
2*c)*e^(9/2) + d*e^(9/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int \frac{1}{\left(e \cot\left(c + dx\right)\right)^{\frac{9}{2}}}\, dx + \int \frac{3 \cot\left(c + dx\right)}{\left(e \cot\left(c + dx\right)\right)^{\frac{9}{2}}}\, dx + \int \frac{3 \cot^2\left(c + dx\right)}{\left(e \cot\left(c + dx\right)\right)^{\frac{9}{2}}}\, dx + \int \frac{\cot^3\left(c + dx\right)}{\left(e \cot\left(c + dx\right)\right)^{\frac{9}{2}}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(9/2),x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(-9/2), x) + Integral(3*cot(c + d*x)/(e*co
t(c + d*x))**(9/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(9/2)
, x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(9/2), x))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 1.92, size = 129, normalized size = 0.78

$$\frac{-4\, e\, a^3 \cot(c + d\, x)^3 + \frac{4\, e\, a^3 \cot(c+d\, x)^2}{3} + \frac{6\, e\, a^3 \cot(c+d\, x)}{5} + \frac{2\, e\, a^3}{7}}{d\, e^2 \left(e \cot\left(c + d\, x\right)\right)^{7/2}} + \frac{2 \sqrt{2}\, a^3 \operatorname{atanh}\left(\frac{32 \sqrt{2}\, a^6\, d\, e^{9/2} \sqrt{e \cot\left(c + d\, x\right)}}{32\, a^6\, d\, e^5 + 32\, a^6\, d\, e^5 \cot(c+d\, x)}\right)}{d\, e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(9/2),x)
```

```
[Out] ((2*a^3*e)/7 + (4*a^3*e*cot(c + d*x)^2)/3 - 4*a^3*e*cot(c + d*x)^3 + (6*a^3
*e*cot(c + d*x))/5)/(d*e^2*(e*cot(c + d*x))^(7/2)) + (2*2^(1/2)*a^3*atanh((
32*2^(1/2)*a^6*d*e^(9/2)*(e*cot(c + d*x))^(1/2))/(32*a^6*d*e^5 + 32*a^6*d*e
^5*cot(c + d*x))))/(d*e^(9/2))
```

3.23 $\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx$

Optimal. Leaf size=111

$$\frac{e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad}-\frac{e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

```
[Out] e^(5/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d-1/2*e^(5/2)*arctan(1/2*(e^
(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a/d*2^(1/2)-2*e^2*(
e*cot(d*x+c))^(1/2)/a/d
```

Rubi [A]

time = 0.30, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3647, 3734, 3613, 211, 3715, 65}

$$\frac{e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad}-\frac{e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x]),x]
```

```
[Out] (e^(5/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(a*d) - (e^(5/2)*ArcTan[(Sqr
t[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])])/(Sqrt[2]*a*d)
 - (2*e^2*Sqrt[e*Cot[c + d*x]])/(a*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
```

```
EqQ[c^2 - d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = -\frac{2e^2\sqrt{e\cot(c+dx)}}{ad} - \frac{2\int \frac{\frac{ae^3}{2}+\frac{1}{2}ae^3\cot(c+dx)+\frac{1}{2}ae^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))}\,dx}{a}$$

$$= -\frac{2e^2\sqrt{e\cot(c+dx)}}{ad} - \frac{\int \frac{\frac{a^2e^3}{2}+\frac{1}{2}a^2e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a^3} - \frac{1}{2}e^3\int \frac{1+\cot^2(c+}{\sqrt{e\cot(c+dx)}\,(a+a}$$

$$= -\frac{2e^2\sqrt{e\cot(c+dx)}}{ad} - \frac{e^3\text{Subst}\left(\int \frac{1}{\sqrt{-ex}\,(a-ax)}\,dx, x, -\cot(c+dx)\right)}{2d} + \frac{(ae^6)}{}$$

$$= -\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad} - \frac{2e^2\sqrt{e\cot(c+dx)}}{ad} + \frac{e^2\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\right.}{}$$

$$= \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad} - \frac{2e^2\sqrt{e\,c}}{}$$

Mathematica [A]
time = 0.93, size = 110, normalized size = 0.99

$$-\frac{\left(\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\text{ArcTan}\left(\sqrt{\cot(c+dx)}\right)+4\sqrt{\cot(c+dx)}\right)(e\cot(c+dx))^{5/2}}{2ad\cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x]),x]
```

```
[Out] -1/2*((Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - Sqrt[2]*ArcTan[1 +
Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[Sqrt[Cot[c + d*x]]] + 4*Sqrt[Cot[c +
 d*x]])*(e*Cot[c + d*x])^(5/2))/(a*d*Cot[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(92) = 184$.
time = 0.52, size = 312, normalized size = 2.81

method	result

derivativedivides	$-\frac{2e^2\left(\sqrt{e\cot(dx+c)}-\frac{e\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\cdots\right.\right.}{8e}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{2e^2\left(\sqrt{e\cot(dx+c)}-\frac{e\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\cdots\right.\right.}{8e}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a*e^2*((e*cot(d*x+c))^(1/2)-1/2*e*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*co
t(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c
)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1))+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/2*e^(1/2)
*arctan((e*cot(d*x+c))^(1/2)/e^(1/2)))
```

Maxima [A]

time = 0.50, size = 88, normalized size = 0.79

$$\frac{\left(\frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)}{a}+\frac{2\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a}-\frac{4}{a\sqrt{\tan(dx+c)}}\right)e^{\frac{5}{2}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))))/a + 2*arctan(1/sqr
t(tan(d*x + c)))/a - 4/(a*sqrt(tan(d*x + c))))*e^(5/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(80) = 160$.
time = 4.07, size = 169, normalized size = 1.52

$$-\frac{\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)-\sqrt{2}\sin\left(2\,dx+2\,c\right)+\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{2\left(\cos\left(2\,dx+2\,c\right)+1\right)}\right)e^{\frac{5}{2}}+2\arctan\left(\frac{\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\sin\left(2\,dx+2\,c\right)}{\cos\left(2\,dx+2\,c\right)+1}\right)e^{\frac{5}{2}}+4\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\,e^{\frac{5}{2}}}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*arctan(-1/2*(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2
*c) + sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))/(cos(2*d*x + 2
*c) + 1))*e^(5/2) + 2*arctan(sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*
sin(2*d*x + 2*c)/(cos(2*d*x + 2*c) + 1))*e^(5/2) + 4*sqrt((cos(2*d*x + 2*c)
 + 1)/sin(2*d*x + 2*c))*e^(5/2))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}{\cot\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c)),x)
```

```
[Out] Integral((e*cot(c + d*x))**(5/2)/(cot(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)/(a*cot(d*x + c) + a), x)
```

Mupad [B]
time = 0.68, size = 123, normalized size = 1.11

$$\frac{e^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{a\,d}-\frac{2\,e^2\,\sqrt{e\cot\left(c+d\,x\right)}}{a\,d}+\frac{\sqrt{2}\,e^{5/2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,{\left(e\cot\left(c+d\,x\right)\right)}^{3/2}}{2\,e^{3/2}}\right)\right)}{4\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(5/2)/(a + a*cot(c + d*x)),x)
```

```
[Out] (e^(5/2)*atan((e*cot(c + d*x))^(1/2)/e^(1/2)))/(a*d) - (2*e^2*(e*cot(c + d*
x))^(1/2))/(a*d) + (2^(1/2)*e^(5/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2)
)/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(
1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)))))/(4*a*d)
```

3.24 $\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx$

Optimal. Leaf size=87

$$-\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad}+\frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad}$$

```
[Out] -e^(3/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d+1/2*e^(3/2)*arctanh(1/2*(
e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a/d*2^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3654, 3613, 214, 3715, 65, 211}

$$\frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad}-\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x]),x]
```

```
[Out] -((e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(a*d)) + (e^(3/2)*ArcTanh[
(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])])/(Sqrt[2]*
a*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx = \frac{\int \frac{-ae^2+ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2} + \frac{1}{2}e^2\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))}\,dx$$

$$= \frac{e^2\text{Subst}\left(\int \frac{1}{\sqrt{-ex}\,(a-ax)}\,dx, x, -\cot(c+dx)\right)}{2d} - \frac{e^4\text{Subst}\left(\int \frac{1}{2a^2e^4-ex^2}\,dx, x, \frac{-ae^2 \cdots}{\sqrt{e\,c \cdots}}\right.}{d}$$

$$= \frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad} - \frac{e\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}$$

$$= -\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad} + \frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad}$$

Mathematica [A]
time = 3.78, size = 107, normalized size = 1.23

$$-\frac{(e\cot(c+dx))^{3/2}\left(4\text{ArcTan}\left(\sqrt{\cot(c+dx)}\right)+\sqrt{2}\left(\log\left(-1+\sqrt{2}\,\sqrt{\cot(c+dx)}-\cot(c+dx)\right)-\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{4ad\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x]),x]
```

```
[Out] -1/4*((e*Cot[c + d*x])^(3/2)*(4*ArcTan[Sqrt[Cot[c + d*x]]] + Sqrt[2]*(Log[-
1 + Sqrt[2]*Sqrt[Cot[c + d*x]] - Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c
 + d*x]] + Cot[c + d*x]])))/(a*d*Cot[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(71) = 142$.
time = 0.52, size = 298, normalized size = 3.43

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{16e}\cdots\right.}{\cdots}$
default	$-\frac{2e^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{16e}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a*e^2*(-1/16/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/16/(e^2)^
(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)
/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/2/e^(1/2)*arctan((e*cot(d*x+c))^(1/
2)/e^(1/2)))
```

Maxima [A]
time = 0.56, size = 83, normalized size = 0.95

$$\frac{\left(\frac{\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a}-\frac{4\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a}\right)e^{\frac{3}{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2
)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/a - 4*arctan(1/sqr
t(tan(d*x + c)))/a)*e^(3/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(61) = 122$.
time = 2.25, size = 142, normalized size = 1.63

$$\frac{\sqrt{2}\,e^{\frac{3}{2}}\log\left(-\left(\sqrt{2}\,\cos\left(2\,dx+2\,c\right)-\sqrt{2}\,\sin\left(2\,dx+2\,c\right)-\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}+2\,\sin\left(2\,dx+2\,c\right)+1\right)+4\,\arctan\left(\frac{\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)+1}\right)e^{\frac{3}{2}}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*e^(3/2)*log(-(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2
*c) - sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)) + 2*sin(2*d*x
+ 2*c) + 1) + 4*arctan(sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*
d*x + 2*c)/(cos(2*d*x + 2*c) + 1))*e^(3/2))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}{\cot\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c)),x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)/(cot(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)/(a*cot(d*x + c) + a), x)
```

Mupad [B]
time = 0.49, size = 79, normalized size = 0.91

$$\frac{\sqrt{2}\,e^{3/2}\,\mathrm{atanh}\left(\frac{12\,\sqrt{2}\,e^{25/2}\,\sqrt{e\cot\left(c+d\,x\right)}}{12\,e^{13}\,\cot(c+d\,x)+12\,e^{13}}\right)}{2\,a\,d}-\frac{e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)/(a + a*cot(c + d*x)),x)
```

```
[Out] (2^(1/2)*e^(3/2)*atanh((12*2^(1/2)*e^(25/2)*(e*cot(c + d*x))^(1/2))/(12*e^1
3*cot(c + d*x) + 12*e^13)))/(2*a*d) - (e^(3/2)*atan((e*cot(c + d*x))^(1/2)/
e^(1/2)))/(a*d)
```

3.25 $\int \frac{\sqrt{e \cot(c+dx)}}{a+a \cot(c+dx)} \, dx$

Optimal. Leaf size=87

$$\frac{\sqrt{e} \operatorname{ArcTan}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad}+\frac{\sqrt{e} \operatorname{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2}\, ad}$$

```
[Out] arctan((e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d+1/2*arctan(1/2*(e^(1/2)-co
t(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*e^(1/2)/a/d*2^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3653, 3613, 211, 3715, 65}

$$\frac{\sqrt{e} \operatorname{ArcTan}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad}+\frac{\sqrt{e} \operatorname{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2}\, ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x]),x]
```

```
[Out] (Sqrt[e]*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(a*d) + (Sqrt[e]*ArcTan[(Sqr
t[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])])/(Sqrt[2]*a*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3653

```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a*c + b*d + (b*c -
a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[d*((b*c - a*d
)/(c^2 + d^2)), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx &= \frac{\int \frac{ae+ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2}-\frac{1}{2}e\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))}\,dx\\ &= -\frac{e\text{Subst}\left(\int \frac{1}{\sqrt{-ex}\,(a-ax)}\,dx,x,-\cot(c+dx)\right)}{2d}-\frac{e^2\text{Subst}\left(\int \frac{1}{-2a^2e^2-ex^2}\,dx,x,-\ldots\right)}{d}\\ &= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad}+\frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx,x,\sqrt{e\cot(c+dx)}\right)}{d}\\ &= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad}+\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 98, normalized size = 1.13

$$\frac{\left(\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+2\text{ArcTan}\left(\sqrt{\cot(c+dx)}\right)\right)\sqrt{e\cot(c+dx)}}{2ad\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x]),x]
```

```
[Out] ((Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[
2]*Sqrt[Cot[c + d*x]]] + 2*ArcTan[Sqrt[Cot[c + d*x]]])*Sqrt[e*Cot[c + d*x]]
)/(2*a*d*Sqrt[Cot[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(71) = 142$.
time = 0.53, size = 304, normalized size = 3.49

method	result
derivativedivides	$-\frac{2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a*e^2*(1/2/e*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8/(e
^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2
)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)
^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(
1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/2/e^(3/2)*arctan((e*cot(d*x+c)
)^(1/2)/e^(1/2)))
```

Maxima [A]
time = 0.54, size = 75, normalized size = 0.86

$$-\frac{\left(\frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)}{a}-\frac{2\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a}\right)e^{\frac{1}{2}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))))/a - 2*arctan(1/sq
rt(tan(d*x + c)))/a)*e^(1/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(62) = 124$.
time = 4.79, size = 140, normalized size = 1.61

$$\frac{\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)-\sqrt{2}\sin\left(2\,dx+2\,c\right)+\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{2\left(\cos\left(2\,dx+2\,c\right)+1\right)}\right)e^{\frac{1}{2}}-2\arctan\left(\frac{\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\sin\left(2\,dx+2\,c\right)}{\cos\left(2\,dx+2\,c\right)+1}\right)e^{\frac{1}{2}}}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*arctan(-1/2*(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*
c) + sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))/(cos(2*d*x + 2*
c) + 1))*e^(1/2) - 2*arctan(sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*s
in(2*d*x + 2*c)/(cos(2*d*x + 2*c) + 1))*e^(1/2))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\cot\left(c+dx\right)}}{\cot\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))/(cot(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))/(a*cot(d*x + c) + a), x)
```

Mupad [B]
time = 0.37, size = 102, normalized size = 1.17

$$\frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{a\,d}-\frac{\sqrt{2}\,\sqrt{e}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,{\left(e\cot\left(c+d\,x\right)\right)}^{3/2}}{2\,e^{3/2}}\right)\right)}{4\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)/(a + a*cot(c + d*x)),x)
```

```
[Out] (e^(1/2)*atan((e*cot(c + d*x))^(1/2)/e^(1/2)))/(a*d) - (2^(1/2)*e^(1/2)*(2*
atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot
(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)
))))/(4*a*d)
```

3.26 $\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))}\,dx$

Optimal. Leaf size=83

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad\sqrt{e}}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,(1+\cot(c+dx))}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad\sqrt{e}}$$

```
[Out] -arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d/e^(1/2)-1/2*arctanh(1/2*(1+cot(d*
x+c))*e^(1/2)*2^(1/2)/(e*cot(d*x+c))^(1/2))/a/d*2^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3655, 3613, 214, 3715, 65, 211}

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad\sqrt{e}}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,(\cot(c+dx)+1)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])),x]
```

```
[Out] -(ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]]/(a*d*Sqrt[e])) - ArcTanh[(Sqrt[e]*(1
 + Cot[c + d*x]))/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])]/(Sqrt[2]*a*d*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))}\,dx = \frac{1}{2}\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))}\,dx + \frac{\int \frac{a-a\cot(c+dx)}{\sqrt{e\cot(c+dx}}}{2a^2}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-ex}\,(a-ax)}\,dx, x, -\cot(c+dx)\right)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{2a^2-ex}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,(1+\cot(c+dx))}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\,c}\right.}{de}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad\sqrt{e}} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,(1+\cot(c+dx))}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ad\sqrt{e}}$$

Mathematica [A]
time = 0.55, size = 107, normalized size = 1.29

$$-\frac{\sqrt{\cot(c+dx)}\left(4\text{ArcTan}\left(\sqrt{\cot(c+dx)}\right)+\sqrt{2}\left(-\log\left(-1+\sqrt{2}\,\sqrt{\cot(c+dx)}-\cot(c+dx)\right)+\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{4ad\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])),x]
```

```
[Out] -1/4*(Sqrt[Cot[c + d*x]]*(4*ArcTan[Sqrt[Cot[c + d*x]]] + Sqrt[2]*(-Log[-1 +
 Sqrt[2]*Sqrt[Cot[c + d*x]] - Cot[c + d*x]] + Log[1 + Sqrt[2]*Sqrt[Cot[c +
d*x]] + Cot[c + d*x]])))/(a*d*Sqrt[e*Cot[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(68) = 136$.
time = 0.53, size = 304, normalized size = 3.66

method	result
derivativedivides	$-\frac{2e^2\left(\frac{\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{2e^{\frac{5}{2}}}+\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2e^2\left(\frac{\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{2e^{\frac{5}{2}}}+\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)\cdots\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a*e^2*(1/2/e^(5/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))+1/2/e^2*(1/8/e
*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-
2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(e^2)^(1/4)*2^(1/2)*(ln((e
*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*
x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2
)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)+1))))
```

Maxima [A]

time = 0.51, size = 83, normalized size = 1.00

$$-\frac{\left(\frac{\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)}{a}+\frac{4\arctan\left(\frac{1}{\sqrt{\tan\left(dx+c\right)}}\right)}{a}\right)e^{\left(-\frac{1}{2}\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*((sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(
2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/a + 4*arctan(1/sq
rt(tan(d*x + c)))/a)*e^(-1/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(55) = 110$.
time = 2.14, size = 139, normalized size = 1.67

$$\frac{\left(\sqrt{2}\log\left(\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)-\sqrt{2}\sin\left(2\,dx+2\,c\right)-\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}+2\sin\left(2\,dx+2\,c\right)+1\right)+4\arctan\left(\frac{\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\sin\left(2\,dx+2\,c\right)}{\cos\left(2\,dx+2\,c\right)+1}\right)\right)e^{\left(-\frac{1}{2}\right)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*log((sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) - sqr
t(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)) + 2*sin(2*d*x + 2*c) +
1) + 4*arctan(sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c
)/(cos(2*d*x + 2*c) + 1)))*e^(-1/2)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{1}{\sqrt{e\cot\left(c+dx\right)}\cot\left(c+dx\right)+\sqrt{e\cot\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c)),x)
```

```
[Out] Integral(1/(sqrt(e*cot(c + d*x))*cot(c + d*x) + sqrt(e*cot(c + d*x))), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)*sqrt(e*cot(d*x + c))), x)
```

Mupad [B]
time = 0.52, size = 79, normalized size = 0.95

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{a\,d\,\sqrt{e}}-\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{12\,\sqrt{2}\,e^{9/2}\,\sqrt{e\cot\left(c+d\,x\right)}}{12\,e^5\,\cot(c+d\,x)+12\,e^5}\right)}{2\,a\,d\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))),x)
```

```
[Out] - atan((e*cot(c + d*x))^(1/2)/e^(1/2))/(a*d*e^(1/2)) - (2^(1/2)*atanh((12*2
^(1/2)*e^(9/2)*(e*cot(c + d*x))^(1/2))/(12*e^5*cot(c + d*x) + 12*e^5)))/(2*
a*d*e^(1/2))
```

3.27 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx$

Optimal. Leaf size=111

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{3/2}}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{3/2}}+\frac{2}{ade\sqrt{e\cot(c+dx)}}$$

```
[Out] arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d/e^(3/2)-1/2*arctan(1/2*(e^(1/2)-co
t(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a/d/e^(3/2)*2^(1/2)+2/a/d/e
/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3650, 3734, 3613, 211, 3715, 65}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{3/2}}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{3/2}}+\frac{2}{ade\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])),x]
```

```
[Out] ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]]/(a*d*e^(3/2)) - ArcTan[(Sqrt[e] - Sqrt
[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])]/(Sqrt[2]*a*d*e^(3/2)) + 2
/(a*d*e*Sqrt[e*Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
```

```
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx = \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{2\int \frac{-\frac{ae^2}{2}-\frac{1}{2}ae^2\cot(c+dx)-\frac{1}{2}ae^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))}dx}{ae^3}$$

$$= \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{\int \frac{-\frac{1}{2}a^2e^2-\frac{1}{2}a^2e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^3e^3} - \frac{\int \frac{}{\sqrt{e\cot(c}}}{}$$

$$= \frac{2}{ade\sqrt{e\cot(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-ex}\,(a-ax)}dx, x, -\cot(c+\right.}{2de}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ade^{3/2}} + \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{\text{Sub}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{3/2}} - \frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ade^{3/2}} +$$

Mathematica [**A**]
time = 2.17, size = 176, normalized size = 1.59

$$\frac{2\left(1+2\cot^2(c+dx)+\cot^4(c+dx)-\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot^{\frac{5}{2}}(c+dx)\csc^2(2(c+dx))+\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot^{\frac{5}{2}}(c+dx)\csc^2(2(c+dx))+2\text{ArcTan}\left(\sqrt{\cot(c+dx)}\right)\cot^{\frac{5}{2}}(c+dx)\csc^2(2(c+dx))\right)\sin^4(c+dx)}{ade\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])),x]
```

```
[Out] (2*(1 + 2*Cot[c + d*x]^2 + Cot[c + d*x]^4 - Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]]]*Cot[c + d*x]^(5/2)*Csc[2*(c + d*x)]^2 + Sqrt[2]*ArcTan[1 +
Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cot[c + d*x]^(5/2)*Csc[2*(c + d*x)]^2 + 2*ArcTa
n[Sqrt[Cot[c + d*x]]]*Cot[c + d*x]^(5/2)*Csc[2*(c + d*x)]^2)*Sin[c + d*x]^4
)/(a*d*e*Sqrt[e*Cot[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(92) = 184.
time = 0.50, size = 319, normalized size = 2.87

method	result

derivativedivides	$-\frac{2e^2\left(-\frac{1}{e^3\sqrt{e\cot(dx+c)}}+\frac{-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\,\mathrm{a}\right.}{}}{}\right)}{}$
default	$-\frac{2e^2\left(-\frac{1}{e^3\sqrt{e\cot(dx+c)}}+\frac{-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\,\mathrm{a}\right.}{}}{}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a*e^2*(-1/e^3/(e*cot(d*x+c))^(1/2)+1/2/e^3*(-1/8/e*(e^2)^(1/4)*2^(1/2)
*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e
*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan
(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)+1))-1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1
/2/e^(7/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2)))
```

Maxima [A]
time = 0.52, size = 88, normalized size = 0.79

$$\frac{\left(\frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)}{a}+\frac{2\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a}+\frac{4\sqrt{\tan(dx+c)}}{a}\right)e^{\left(-\frac{3}{2}\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))))/a + 2*arctan(1/sqr
t(tan(d*x + c)))/a + 4*sqrt(tan(d*x + c))/a)*e^(-3/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(80) = 160$.

time = 2.57, size = 213, normalized size = 1.92

$$-\frac{\left(\sqrt{2}\,\cos\left(2\,dx+2\,c\right)+\sqrt{2}\right)\arctan\left(-\frac{\left(\sqrt{2}\,\cos(2\,dx+2\,c)-\sqrt{2}\,\sin(2\,dx+2\,c)+\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{2\,(\cos(2\,dx+2\,c)+1)}\right)+2\left(\cos\left(2\,dx+2\,c\right)+1\right)\arctan\left(\frac{\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\,\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)+1}\right)-4\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\,\sin\left(2\,dx+2\,c\right)}{2\left(ad\cos\left(2\,dx+2\,c\right)e^{\frac{3}{2}}+ade^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*((sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan(-1/2*(sqrt(2)*cos(2*d*x +
 2*c) - sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin
(2*d*x + 2*c))/(cos(2*d*x + 2*c) + 1)) + 2*(cos(2*d*x + 2*c) + 1)*arctan(sq
rt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c)/(cos(2*d*x + 2
*c) + 1)) - 4*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c
))/(a*d*cos(2*d*x + 2*c)*e^(3/2) + a*d*e^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}\cot\left(c+dx\right)+\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c)),x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*cot(c + d*x) + (e*cot(c + d*x))**(3/2))
, x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)*(e*cot(d*x + c))^(3/2)), x)
```

Mupad [B]
time = 0.64, size = 123, normalized size = 1.11

$$\frac{2}{a\,d\,e\,\sqrt{e\cot\left(c+d\,x\right)}}+\frac{\mathrm{atan}\left(\frac{\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{a\,d\,e^{3/2}}+\frac{\sqrt{2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,{\left(e\cot(c+d\,x)\right)}^{3/2}}{2\,e^{3/2}}\right)\right)}{4\,a\,d\,e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))),x)
```

```
[Out] 2/(a*d*e*(e*cot(c + d*x))^(1/2)) + atan((e*cot(c + d*x))^(1/2)/e^(1/2))/(a*
d*e^(3/2)) + (2^(1/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)))
 + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c
+ d*x))^(3/2))/(2*e^(3/2)))))/(4*a*d*e^(3/2))
```

3.28 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx$

Optimal. Leaf size=135

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{5/2}}+\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{2}{ade^2\sqrt{e\cot(c+}}$$

```
[Out] -arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d/e^(5/2)+2/3/a/d/e/(e*cot(d*x+c))^
(3/2)+1/2*arctanh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(
1/2))/a/d/e^(5/2)*2^(1/2)-2/a/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3650, 3730, 12, 16, 3654, 3613, 214, 3715, 65, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{5/2}}-\frac{2}{ade^2\sqrt{e\cot(c+dx)}}+\frac{2}{3ade(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])),x]
```

```
[Out] -(ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]]/(a*d*e^(5/2))) + ArcTanh[(Sqrt[e] +
Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])]/(Sqrt[2]*a*d*e^(5/2))
 + 2/(3*a*d*e*(e*Cot[c + d*x])^(3/2)) - 2/(a*d*e^2*Sqrt[e*Cot[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx = \frac{2}{3ade(e\cot(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3ae^2}{2}-\frac{3}{2}ae^2\cot(c+dx)-\frac{3}{2}ae^2\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}}{3ae^3}$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}} + \frac{4\int \frac{\ }{4\sqrt{e\cot}}}{\ }$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}} + \frac{\int \frac{\ }{\sqrt{e\cot(c}}}{\ }$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}} + \frac{\int \frac{(e\cot(c+dx)}{a+a\cot(c+}}{e^4}$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}} + \frac{\int \frac{-ae^2+ae^2 c}{\sqrt{e\cot(c}}}{2a^2}$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\ }{2a}\right.}{\ }$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ade^{5/2}} + \frac{2}{3ade(e\cot(c+dx))^{3/2}} -$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}\,ade^{5/2}}$$

Mathematica [A]

time = 1.41, size = 131, normalized size = 0.97

$$\frac{-12\mathrm{ArcTan}\left(\sqrt{\cot(c+dx)}\right)\sqrt{\cot(c+dx)}-3\sqrt{2}\sqrt{\cot(c+dx)}\left(\log\left(-1+\sqrt{2}\sqrt{\cot(c+dx)}-\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)+8(-3+\tan(c+dx))}{12ade^2\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])),x]
```

```
[Out] (-12*ArcTan[Sqrt[Cot[c + d*x]]]*Sqrt[Cot[c + d*x]] - 3*Sqrt[2]*Sqrt[Cot[c +
 d*x]]*(Log[-1 + Sqrt[2]*Sqrt[Cot[c + d*x]] - Cot[c + d*x]] - Log[1 + Sqrt[
2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]) + 8*(-3 + Tan[c + d*x]))/(12*a*d*e^2
*Sqrt[e*Cot[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(113) = 226$.
time = 0.48, size = 333, normalized size = 2.47

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{1}{3e^3(e\cot(dx+c))^{\frac{3}{2}}}+\frac{1}{e^4\sqrt{e\cot(dx+c)}}+\frac{\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{2e^{\frac{9}{2}}}+\frac{-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c}{e\cot(dx+c}\right.\right.}{}}{}\right.}{}$
default	$-\frac{2e^2\left(-\frac{1}{3e^3(e\cot(dx+c))^{\frac{3}{2}}}+\frac{1}{e^4\sqrt{e\cot(dx+c)}}+\frac{\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{2e^{\frac{9}{2}}}+\frac{-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c}{e\cot(dx+c}\right.\right.}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a*e^2*(-1/3/e^3/(e*cot(d*x+c))^(3/2)+1/e^4/(e*cot(d*x+c))^(1/2)+1/2/e^
(9/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))+1/2/e^4*(-1/8/e*(e^2)^(1/4)*2^(1
/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))
/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arc
tan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)+1))+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*
```

```
cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))
))
```

Maxima [A]
time = 0.51, size = 109, normalized size = 0.81

$$-\frac{\left(\frac{8\left(\frac{3}{\tan(dx+c)}-1\right)\tan(dx+c)^{\frac{3}{2}}}{a}-\frac{3\left(\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)}{a}+\frac{12\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a}\right)e^{\left(-\frac{5}{2}\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(8*(3/tan(d*x + c) - 1)*tan(d*x + c)^(3/2)/a - 3*(sqrt(2)*log(sqrt(2)
/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)/sqrt(tan(d
*x + c)) + 1/tan(d*x + c) + 1))/a + 12*arctan(1/sqrt(tan(d*x + c)))/a)*e^(-
5/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(97) = 194.
time = 2.48, size = 229, normalized size = 1.70

$$\frac{12\left(\cos(2\,dx+2\,c)+1\right)\arctan\left(\frac{\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)+1}\right)+3\left(\sqrt{2}\cos(2\,dx+2\,c)+\sqrt{2}\right)\log\left(-\left(\sqrt{2}\cos(2\,dx+2\,c)-\sqrt{2}\sin(2\,dx+2\,c)-\sqrt{2}\right)\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}+2\sin(2\,dx+2\,c)+1\right)-8\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}\left(\cos(2\,dx+2\,c)+3\sin(2\,dx+2\,c)-1\right)}{12\left(ad\cos(2\,dx+2\,c)e^{\frac{5}{2}}+ade^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(12*(cos(2*d*x + 2*c) + 1)*arctan(sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*
x + 2*c))*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c) + 1)) + 3*(sqrt(2)*cos(2*d*x +
 2*c) + sqrt(2))*log(-(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c)
- sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)) + 2*sin(2*d*x + 2*
c) + 1) - 8*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c)
 + 3*sin(2*d*x + 2*c) - 1))/(a*d*cos(2*d*x + 2*c)*e^(5/2) + a*d*e^(5/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{5}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c)),x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(5/2)*cot(c + d*x) + (e*cot(c + d*x))**(5/2))
, x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)*(e*cot(d*x + c))^(5/2)), x)
```

Mupad [B]
time = 0.93, size = 132, normalized size = 0.98

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{12\,\sqrt{2}\,a^3\,d^3\,e^{21/2}\,\sqrt{e\cot\left(c+d\,x\right)}}{12\,a^3\,d^3\,e^{11}+12\,a^3\,d^3\,e^{11}\,\cot(c+d\,x)}\right)}{2\,a\,d\,e^{5/2}}-\frac{\mathrm{atan}\left(\frac{\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{a\,d\,e^{5/2}}-\frac{\frac{2\cot(c+d\,x)}{e}-\frac{2}{3\,e}}{a\,d\,\left(e\cot\left(c+d\,x\right)\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))),x)
```

```
[Out] (2^(1/2)*atanh((12*2^(1/2)*a^3*d^3*e^(21/2)*(e*cot(c + d*x))^(1/2))/(12*a^3
*d^3*e^11 + 12*a^3*d^3*e^11*cot(c + d*x))))/(2*a*d*e^(5/2)) - atan((e*cot(c
 + d*x))^(1/2)/e^(1/2))/(a*d*e^(5/2)) - ((2*cot(c + d*x))/e - 2/(3*e))/(a*d
*(e*cot(c + d*x))^(3/2))
```

3.29 $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx$

Optimal. Leaf size=281

$$-\frac{3e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d}-\frac{e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2d}+\frac{e^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2d}$$

```
[Out] -3/2*e^(5/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d-1/4*e^(5/2)*arctan(
1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1/4*e^(5/2)*arctan(1+
2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/8*e^(5/2)*ln(e^(1/2)+
cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)+1/8*e^(5/2)*
ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)+1
/2*e^2*(e*cot(d*x+c))^(1/2)/d/(a^2+a^2*cot(d*x+c))
```

Rubi [A]
time = 0.37, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3646, 3734, 12, 3557, 335, 217, 1179, 642, 1176, 631, 210, 3715, 65, 211}

$$-\frac{3e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d}-\frac{e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2d}+\frac{e^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{2\sqrt{2}\,a^2d}-\frac{e^{5/2}\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}\,a^2d}+\frac{e^{5/2}\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}\,a^2d}+\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (-3*e^(5/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(2*a^2*d) - (e^(5/2)*ArcT
an[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(2*Sqrt[2]*a^2*d) + (e^(5/2
)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(2*Sqrt[2]*a^2*d) + (
e^2*Sqrt[e*Cot[c + d*x]])/(2*d*(a^2 + a^2*Cot[c + d*x])) - (e^(5/2)*Log[Sqr
t[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(4*Sqrt[2]*a^2
*d) + (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c +
d*x]]])/(4*Sqrt[2]*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = \frac{e^2\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} - \frac{\int \frac{-\frac{1}{2}a^2e^3+a^2e^3\cot(c+dx)-\frac{3}{2}a^2e^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))}\,dx}{2a^3}$$

$$= \frac{e^2\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} - \frac{\int \frac{2a^3e^3}{\sqrt{e\cot(c+dx)}}\,dx}{4a^5} + \frac{(3e^3)\int \frac{1+\cot^2(c+dx}{\sqrt{e\cot(c+dx)}\,(a+}}{4a}$$

$$= \frac{e^2\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} - \frac{e^3\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2} + \frac{(3e^3)\,\text{Subst}\Big(\int \frac{1}{\sqrt{-ex}\,(a}}{4}$$

$$= \frac{e^2\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} - \frac{(3e^2)\,\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2ad} + \frac{e^4\text{S}}{}$$

$$= -\frac{3e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{e^2\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} + \frac{e^4\text{Subst}\Big(\int \frac{1}{e^2+}}{}$$

$$= -\frac{3e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{e^2\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} + \frac{e^3\text{Subst}\Big(\int \frac{e-x}{e^2+}}{}$$

$$= -\frac{3e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{e^2\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} - \frac{e^{5/2}\text{Subst}\Big(\int -}{}$$

$$= -\frac{3e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{e^2\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} - \frac{e^{5/2}\log\Big(\sqrt{e}+}{}$$

$$= -\frac{3e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2d} +$$

Mathematica [A]
time = 2.08, size = 224, normalized size = 0.80

$$\frac{(e\cot(c+dx))^{5/2}\left(-\frac{1}{2}(1+\cot(c+dx))\csc(c+dx)\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+12\text{ArcTan}\left(\sqrt{\cot(c+dx)}\right)+\sqrt{2}\log\left(-1+\sqrt{2}\sqrt{\cot(c+dx)}-\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)+2\cot^{\frac{3}{2}}(c+dx)\sec(c+dx)\right)(\cos(c+dx)+\sin(c+dx))}{4a^2d\cot^{\frac{5}{2}}(c+dx)(1+\cot(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x])^2,x]
```

```
[Out] ((e*Cot[c + d*x])^(5/2)*(-1/2*((1 + Cot[c + d*x])*Csc[c + d*x]*(2*Sqrt[2]*A
rcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[C
ot[c + d*x]]] + 12*ArcTan[Sqrt[Cot[c + d*x]]] + Sqrt[2]*Log[-1 + Sqrt[2]*Sq
```

```
rt[Cot[c + d*x]] - Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x
]] + Cot[c + d*x]]])) + 2*Cot[c + d*x]^(3/2)*Sec[c + d*x])*(Cos[c + d*x] + S
in[c + d*x]))/(4*a^2*d*Cot[c + d*x]^(5/2)*(1 + Cot[c + d*x])^2)
```

Maple [A]
time = 0.57, size = 191, normalized size = 0.68

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx-}}{(e^2)^{\frac{1}{4}}}\right.\right.}{16e}\right.}{}$
default	$-\frac{2e^3\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx-}}{(e^2)^{\frac{1}{4}}}\right.\right.}{16e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*e^3*(-1/16/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4*(e*co
t(d*x+c))^(1/2)/(e*cot(d*x+c)+e)+3/4/e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^
(1/2)))
```

Maxima [A]
time = 0.51, size = 161, normalized size = 0.57

$$\frac{\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^2}-\frac{12\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a^2}+\frac{4}{\left(a^2+\frac{a^2}{\tan(dx+c)}\right)\sqrt{\tan(dx+c)}}\right)e^{\frac{5}{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*log(s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)/sqrt
(tan(d*x + c)) + 1/tan(d*x + c) + 1))/a^2 - 12*arctan(1/sqrt(tan(d*x + c)))
/a^2 + 4/((a^2 + a^2/tan(d*x + c))*sqrt(tan(d*x + c))))*e^(5/2)/d
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\cot(c+dx))^{\frac{5}{2}}}{\cot^2(c+dx)+2\cot(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**2,x)
```

```
[Out] Integral((e*cot(c + d*x))**(5/2)/(cot(c + d*x)**2 + 2*cot(c + d*x) + 1), x)
/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)/(a*cot(d*x + c) + a)^2, x)
```

Mupad [B]
time = 0.90, size = 375, normalized size = 1.33

$$\frac{e^3\sqrt{e\cot(c+dx)}}{2\left(a^2de+a^2de\cot(c+dx)\right)}-\mathrm{atan}\left(\frac{e^{20}\sqrt{e\cot(c+dx)}\left(-\frac{e^{10}}{256a^8d^4}\right)^{1/4}16\mathrm{i}}{\frac{36e^{23}}{a^2d}+64a^2de^{18}\sqrt{-\frac{e^{10}}{256a^8d^4}}}-\frac{e^{15}\sqrt{e\cot(c+dx)}\left(-\frac{e^{10}}{256a^8d^4}\right)^{3/4}2304\mathrm{i}}{\frac{36e^{23}}{a^6d^3}+\frac{64e^{18}\sqrt{-\frac{e^{10}}{256a^8d^4}}}{a^2d}}\right)\left(-\frac{e^{10}}{256a^8d^4}\right)^{1/4}2\mathrm{i}-\frac{\mathrm{atan}\left(\frac{4e^{20}\sqrt{e\cot(c+dx)}\left(-\frac{e^{10}}{a^8d^4}\right)^{1/4}}{\frac{36e^{23}}{a^2d}-4a^2de^{18}\sqrt{-\frac{e^{10}}{a^8d^4}}}+\frac{36e^{15}\sqrt{e\cot(c+dx)}\left(-\frac{e^{10}}{a^8d^4}\right)^{3/4}}{\frac{36e^{23}}{a^6d^3}-\frac{4e^{18}\sqrt{-\frac{e^{10}}{a^8d^4}}}{a^2d}}\right)\left(-\frac{e^{10}}{a^8d^4}\right)^{1/4}}{2}-\frac{\mathrm{atan}\left(\frac{\sqrt{e\cot(c+dx)}\sqrt{-e^5}\,1\mathrm{i}}{e^3}\right)\sqrt{-e^5}\,3\mathrm{i}}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(5/2)/(a + a*cot(c + d*x))^2,x)
```

```
[Out] (e^3*(e*cot(c + d*x))^(1/2))/(2*(a^2*d*e + a^2*d*e*cot(c + d*x))) - atan((e
^20*(e*cot(c + d*x))^(1/2)*(-e^10/(256*a^8*d^4))^(1/4)*16i)/((36*e^23)/(a^2
*d) + 64*a^2*d*e^18*(-e^10/(256*a^8*d^4))^(1/2)) - (e^15*(e*cot(c + d*x))^(
```

```
1/2)*(-e^10/(256*a^8*d^4))^(3/4)*2304i)/((36*e^23)/(a^6*d^3) + (64*e^18*(-e
^10/(256*a^8*d^4))^(1/2))/(a^2*d))*(-e^10/(256*a^8*d^4))^(1/4)*2i - (atan(
(4*e^20*(e*cot(c + d*x))^(1/2)*(-e^10/(a^8*d^4))^(1/4))/((36*e^23)/(a^2*d)
- 4*a^2*d*e^18*(-e^10/(a^8*d^4))^(1/2)) + (36*e^15*(e*cot(c + d*x))^(1/2)*(
-e^10/(a^8*d^4))^(3/4))/((36*e^23)/(a^6*d^3) - (4*e^18*(-e^10/(a^8*d^4))^(1
/2))/(a^2*d)))*(-e^10/(a^8*d^4))^(1/4))/2 - (atan(((e*cot(c + d*x))^(1/2)*(
-e^5)^(1/2)*1i)/e^3)*(-e^5)^(1/2)*3i)/(2*a^2*d)
```

3.30 $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx$

Optimal. Leaf size=279

$$\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d}+\frac{e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2d}-\frac{e^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+}}{\sqrt{e}}\right.}{2\sqrt{2}\,a^2d}$$

```
[Out] 1/2*e^(3/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d+1/4*e^(3/2)*arctan(1
-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/4*e^(3/2)*arctan(1+2
^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/8*e^(3/2)*ln(e^(1/2)+c
ot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)+1/8*e^(3/2)*l
n(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)-1/
2*e*(e*cot(d*x+c))^(1/2)/d/(a^2+a^2*cot(d*x+c))
```

Rubi [A]
time = 0.39, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3648, 3734, 12, 16, 3557, 335, 303, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d}+\frac{e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2d}-\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{2\sqrt{2}\,a^2d}-\frac{e^{3/2}\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}\,a^2d}+\frac{e^{3/2}\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}\,a^2d}-\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(2*a^2*d) + (e^(3/2)*ArcTan[
1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(2*Sqrt[2]*a^2*d) - (e^(3/2)*A
rcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(2*Sqrt[2]*a^2*d) - (e*S
qrt[e*Cot[c + d*x]])/(2*d*(a^2 + a^2*Cot[c + d*x])) - (e^(3/2)*Log[Sqrt[e]
+ Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(4*Sqrt[2]*a^2*d) +
 (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]
])/(4*Sqrt[2]*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
```

```
!GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx = -\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int \frac{\frac{ae^2}{2}-ae^2\cot(c+dx)-\frac{1}{2}ae^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))}\,dx}{2a^2}$$

$$= -\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int -\frac{2a^2e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{4a^4} - \frac{e^2\int \frac{1+\cot^2(c+}{\sqrt{e\cot(c+dx)}\,(}}{4a}$$

$$= -\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{e^2\int \frac{\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2} - \frac{e^2\mathrm{Subst}\left(\int \frac{1}{\sqrt{-ex}\,(a-}\right.}{}$$

$$= -\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{e\int \sqrt{e\cot(c+dx)}\,dx}{2a^2} + \frac{e\mathrm{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x,\right.}{2ad}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{e^2\mathrm{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\right.}{}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{e^2\mathrm{Subst}\left(\int \frac{x^2}{e^2+x^4}\right.}{}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{e^2\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\right.}{}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{e^{3/2}\mathrm{Subst}\left(\int \frac{\sqrt{}}{-e-}\right.}{}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{e^{3/2}\log\left(\sqrt{e}+\right.}{}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2d} - \frac{e^3}{}$$

Mathematica [A]
time = 3.02, size = 312, normalized size = 1.12

$$-\frac{(e\cot(c+dx))^{3/2}\left(-4\sqrt{\cot(c+dx)}+4\cot^{\frac{3}{2}}(c+dx)-4\cot^{\frac{5}{2}}(c+dx)+4\cot^{\frac{7}{2}}(c+dx)-2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\cos(2(c+dx))\csc^4(c+dx)+2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\cos(2(c+dx))\csc^4(c+dx)-4\mathrm{ArcTan}\left(\sqrt{\cot(c+dx)}\right)\cos(2(c+dx))\csc^4(c+dx)+\sqrt{2}\cos(2(c+dx))\csc^4(c+dx)\log\left(-1+\sqrt{2}\sqrt{\cot(c+dx)}-\cot(c+dx)\right)-\sqrt{2}\cos(2(c+dx))\csc^4(c+dx)\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\sin^2(c+dx)}{8a^2d\cot^{\frac{3}{2}}(c+dx)\left(-1+\cot^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x])^2,x]
```

```
[Out] -1/8*((e*Cot[c + d*x])^(3/2)*(-4*Sqrt[Cot[c + d*x]] + 4*Cot[c + d*x]^(3/2)
- 4*Cot[c + d*x]^(5/2) + 4*Cot[c + d*x]^(7/2) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2
]*Sqrt[Cot[c + d*x]]]*Cos[2*(c + d*x)]*Csc[c + d*x]^4 + 2*Sqrt[2]*ArcTan[1
+ Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cos[2*(c + d*x)]*Csc[c + d*x]^4 - 4*ArcTan[Sq
rt[Cot[c + d*x]]]*Cos[2*(c + d*x)]*Csc[c + d*x]^4 + Sqrt[2]*Cos[2*(c + d*x)
]*Csc[c + d*x]^4*Log[-1 + Sqrt[2]*Sqrt[Cot[c + d*x]] - Cot[c + d*x]] - Sqrt
[2]*Cos[2*(c + d*x)]*Csc[c + d*x]^4*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Co
t[c + d*x]])*Sin[c + d*x]^2)/(a^2*d*Cot[c + d*x]^(3/2)*(-1 + Cot[c + d*x]^2
))
```

Maple [A]

time = 0.52, size = 197, normalized size = 0.71

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{-\frac{\sqrt{e\cot(dx+c)}}{2(e\cot(dx+c)+e)}+\frac{\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{2\sqrt{e}}}{2e}+\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+\cdots}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+\cdots}\right)\cdots}{\cdots}\right)}{d\,a^2\cdots}$
default	$-\frac{2e^3\left(-\frac{-\frac{\sqrt{e\cot(dx+c)}}{2(e\cot(dx+c)+e)}+\frac{\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{2\sqrt{e}}}{2e}+\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+\cdots}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+\cdots}\right)\cdots}{\cdots}\right)}{d\,a^2\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*e^3*(-1/2/e*(-1/2*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)+e)+1/2/e^(1/2
)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2)))+1/16/e/(e^2)^(1/4)*2^(1/2)*(ln((e*c
ot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+
c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)+1)))
```

Maxima [A]

time = 0.49, size = 161, normalized size = 0.58

$$-\frac{\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^2}-\frac{4\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a^2}+\frac{4}{\left(a^2+\frac{a^2}{\tan(dx+c)}\right)\sqrt{\tan(dx+c)}}\right)e^{\frac{3}{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*log(
sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)/sqr
t(tan(d*x + c)) + 1/tan(d*x + c) + 1))/a^2 - 4*arctan(1/sqrt(tan(d*x + c)))
/a^2 + 4/((a^2 + a^2/tan(d*x + c))*sqrt(tan(d*x + c))))*e^(3/2)/d
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\cot(c+dx))^{\frac{3}{2}}}{\cot^2(c+dx)+2\cot(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**2,x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)/(cot(c + d*x)**2 + 2*cot(c + d*x) + 1), x)
/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)/(a*cot(d*x + c) + a)^2, x)
```

Mupad [B]
time = 0.82, size = 376, normalized size = 1.35

$$-\frac{\mathrm{atan}\left(\frac{4\,e^{16}\,\sqrt{e\cot(c+d\,x)}\,{\left(-\frac{e^6}{a^8\,d^4}\right)}^{1/4}}{\frac{4\,e^{18}}{a^2\,d}+4\,a^2\,d\,e^{15}\,\sqrt{-\frac{e^6}{a^8\,d^4}}}+\frac{4\,e^{13}\,\sqrt{e\cot(c+d\,x)}\,{\left(-\frac{e^6}{a^8\,d^4}\right)}^{3/4}}{\frac{4\,e^{18}}{a^6\,d^3}+\frac{4\,e^{15}\,\sqrt{-\frac{e^6}{a^8\,d^4}}}{a^2\,d}}\right)\,{\left(-\frac{e^6}{a^8\,d^4}\right)}^{1/4}}{2}-\mathrm{atan}\left(\frac{e^{16}\,\sqrt{e\cot(c+d\,x)}\,{\left(-\frac{e^6}{256\,a^8\,d^4}\right)}^{1/4}\,16{}\mathrm{i}}{\frac{4\,e^{18}}{a^2\,d}-64\,a^2\,d\,e^{15}\,\sqrt{-\frac{e^6}{256\,a^8\,d^4}}}-\frac{e^{13}\,\sqrt{e\cot(c+d\,x)}\,{\left(-\frac{e^6}{256\,a^8\,d^4}\right)}^{3/4}\,256{}\mathrm{i}}{\frac{4\,e^{18}}{a^6\,d^3}-\frac{64\,e^{15}\,\sqrt{-\frac{e^6}{256\,a^8\,d^4}}}{a^2\,d}}\right)\,{\left(-\frac{e^6}{256\,a^8\,d^4}\right)}^{1/4}\,2{}\mathrm{i}-\frac{e^2\,\sqrt{e\cot(c+d\,x)}}{2\,\left(a^2\,d\,e+a^2\,d\,e\cot(c+d\,x)\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}\,\sqrt{-e^3}\,1{}\mathrm{i}}{e^2}\right)\,\sqrt{-e^3}\,1{}\mathrm{i}}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)/(a + a*cot(c + d*x))^2,x)
```

```
[Out] (atan(((e*cot(c + d*x))^(1/2)*(-e^3)^(1/2)*1i)/e^2)*(-e^3)^(1/2)*1i)/(2*a^2
*d) - atan((e^16*(e*cot(c + d*x))^(1/2)*(-e^6/(256*a^8*d^4))^(1/4)*16i)/((4
*e^18)/(a^2*d) - 64*a^2*d*e^15*(-e^6/(256*a^8*d^4))^(1/2)) - (e^13*(e*cot(c
 + d*x))^(1/2)*(-e^6/(256*a^8*d^4))^(3/4)*256i)/((4*e^18)/(a^6*d^3) - (64*e
^15*(-e^6/(256*a^8*d^4))^(1/2))/(a^2*d)))*(-e^6/(256*a^8*d^4))^(1/4)*2i - (
e^2*(e*cot(c + d*x))^(1/2))/(2*(a^2*d*e + a^2*d*e*cot(c + d*x))) - (atan((4
*e^16*(e*cot(c + d*x))^(1/2)*(-e^6/(a^8*d^4))^(1/4))/((4*e^18)/(a^2*d) + 4*
a^2*d*e^15*(-e^6/(a^8*d^4))^(1/2)) + (4*e^13*(e*cot(c + d*x))^(1/2)*(-e^6/(
a^8*d^4))^(3/4))/((4*e^18)/(a^6*d^3) + (4*e^15*(-e^6/(a^8*d^4))^(1/2))/(a^2
*d)))*(-e^6/(a^8*d^4))^(1/4))/2
```

3.31 $\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx$

Optimal. Leaf size=278

$$\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d}+\frac{\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2d}-\frac{\sqrt{e}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(}}{\sqrt{e}}\right.}{2\sqrt{2}\,a^2d}$$

```
[Out] 1/2*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d+1/4*arctan(1-2^(1/2)
*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)-1/4*arctan(1+2^(1/2)*(
e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)+1/8*ln(e^(1/2)+cot(d*x+c
)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/a^2/d*2^(1/2)-1/8*ln(e^(1/2
)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/a^2/d*2^(1/2)+1/
2*(e*cot(d*x+c))^(1/2)/d/(a^2+a^2*cot(d*x+c))
```

Rubi [A]
time = 0.34, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3649, 3734, 12, 3557, 335, 217, 1179, 642, 1176, 631, 210, 3715, 65, 211}

$$\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d}+\frac{\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2d}-\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{2\sqrt{2}\,a^2d}+\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}+\frac{\sqrt{e}\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}\,a^2d}-\frac{\sqrt{e}\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}\,a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (Sqrt[e]*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(2*a^2*d) + (Sqrt[e]*ArcTan[
1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(2*Sqrt[2]*a^2*d) - (Sqrt[e]*A
rcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(2*Sqrt[2]*a^2*d) + Sqrt
[e*Cot[c + d*x]]/(2*d*(a^2 + a^2*Cot[c + d*x])) + (Sqrt[e]*Log[Sqrt[e] + Sq
rt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(4*Sqrt[2]*a^2*d) - (Sq
rt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(
4*Sqrt[2]*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx &= \frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int \frac{-\frac{ae}{2}-ae\cot(c+dx)+\frac{1}{2}ae\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))}\,dx}{2a^2} \\
&= \frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int -\frac{2a^2e}{\sqrt{e\cot(c+dx)}}\,dx}{4a^4} - \frac{e\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+a\,c}}{4a} \\
&= \frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{e\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2} - \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{-ex}\,(a-ax)}\,d\right.}{4ad} \\
&= \frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2ad} - \frac{e^2\text{Subst}\left(\right.}{} \\
&= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{e^2\text{Subst}\left(\int \frac{1}{e^2+x^4}\,d\right.}{} \\
&= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{e\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,d\right.}{} \\
&= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{\sqrt{}}{-e-}\right.}{} \\
&= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{}\right.}{} \\
&= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2d} - \frac{\sqrt{}}{}
\end{aligned}$$

Mathematica [A]
time = 1.38, size = 207, normalized size = 0.74

$$\frac{\sqrt{e\cot(c+dx)}\csc(c+dx)(\cos(c+dx)+\sin(c+dx))\left(2+\frac{1}{2}\sqrt{\cot(c+dx)}\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+4\text{ArcTan}\left(\sqrt{\cot(c+dx)}\right)+\sqrt{2}\log\left(-1+\sqrt{2}\sqrt{\cot(c+dx)}-\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)(1+\tan(c+dx))}{4a^2d(1+\cot(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (Sqrt[e*Cot[c + d*x]]*Csc[c + d*x]*(Cos[c + d*x] + Sin[c + d*x])*(2 + (Sqrt
[Cot[c + d*x]]*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2
]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 4*ArcTan[Sqrt[Cot[c + d*x]]] + S
```

```
qrt[2]*Log[-1 + Sqrt[2]*Sqrt[Cot[c + d*x]] - Cot[c + d*x]] - Sqrt[2]*Log[1
+ Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*(1 + Tan[c + d*x]))/2))/(4*a^
2*d*(1 + Cot[c + d*x])^2)
```

Maple [A]
time = 0.54, size = 200, normalized size = 0.72

method	result
derivativedivides	$-\frac{2e^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{(e^2)^{\frac{1}{4}}}\right.\right.}{16e^3}\right.}{d}$
default	$-\frac{2e^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{(e^2)^{\frac{1}{4}}}\right.\right.}{16e^3}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*e^3*(1/16/e^3*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/2/e*(1
/2*(e*cot(d*x+c))^(1/2)/e/(e*cot(d*x+c)+e)+1/2/e^(3/2)*arctan((e*cot(d*x+c)
)^(1/2)/e^(1/2))))
```

Maxima [A]
time = 0.51, size = 161, normalized size = 0.58

$$-\frac{\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^2}-\frac{4\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a^2}-\frac{4}{\left(a^2+\frac{a^2}{\tan(dx+c)}\right)\sqrt{\tan(dx+c)}}\right)e^{\frac{1}{2}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*log(
sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)/sqr
t(tan(d*x + c)) + 1/tan(d*x + c) + 1))/a^2 - 4*arctan(1/sqrt(tan(d*x + c)))
/a^2 - 4/((a^2 + a^2/tan(d*x + c))*sqrt(tan(d*x + c))))*e^(1/2)/d
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\cot(c+dx)}}{\cot^2(c+dx)+2\cot(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))/(cot(c + d*x)**2 + 2*cot(c + d*x) + 1), x)/a*
*2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))/(a*cot(d*x + c) + a)^2, x)
```

Mupad [B]
time = 0.72, size = 366, normalized size = 1.32

$$\frac{\operatorname{atan}\left(\frac{4\,e^{12}\,\sqrt{e\cot(c+d\,x)}\,{\left(-\frac{e^2}{a^8\,d^4}\right)}^{1/4}}{\frac{4\,e^{13}}{a^2\,d}-4\,a^2\,d\,e^{12}\,\sqrt{-\frac{e^2}{a^8\,d^4}}}+\frac{4\,e^{11}\,\sqrt{e\cot(c+d\,x)}\,{\left(-\frac{e^2}{a^8\,d^4}\right)}^{3/4}}{\frac{4\,e^{13}}{a^6\,d^3}-\frac{4\,e^{12}\,\sqrt{-\frac{e^2}{a^8\,d^4}}}{a^2\,d}}\right)\,{\left(-\frac{e^2}{a^8\,d^4}\right)}^{1/4}}{2}+\operatorname{atan}\left(\frac{e^{12}\,\sqrt{e\cot(c+d\,x)}\,{\left(-\frac{e^2}{256\,a^8\,d^4}\right)}^{1/4}\,16{}\mathrm{i}}{\frac{4\,e^{13}}{a^2\,d}+64\,a^2\,d\,e^{12}\,\sqrt{-\frac{e^2}{256\,a^8\,d^4}}}-\frac{e^{11}\,\sqrt{e\cot(c+d\,x)}\,{\left(-\frac{e^2}{256\,a^8\,d^4}\right)}^{3/4}\,256{}\mathrm{i}}{\frac{4\,e^{13}}{a^6\,d^3}+\frac{64\,e^{12}\,\sqrt{-\frac{e^2}{256\,a^8\,d^4}}}{a^2\,d}}\right)\,{\left(-\frac{e^2}{256\,a^8\,d^4}\right)}^{1/4}\,2{}\mathrm{i}+\frac{e\,\sqrt{e\cot(c+d\,x)}}{2\,\left(a^2\,d\,e+a^2\,d\,e\cot(c+d\,x)\right)}-\frac{\sqrt{-e}\,\operatorname{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}\,1{}\mathrm{i}}{\sqrt{-e}}\right)\,1{}\mathrm{i}}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)/(a + a*cot(c + d*x))^2,x)
```

```
[Out] (atan((4*e^12*(e*cot(c + d*x))^(1/2)*(-e^2/(a^8*d^4))^(1/4))/((4*e^13)/(a^2
*d) - 4*a^2*d*e^12*(-e^2/(a^8*d^4))^(1/2)) + (4*e^11*(e*cot(c + d*x))^(1/2)
*(-e^2/(a^8*d^4))^(3/4))/((4*e^13)/(a^6*d^3) - (4*e^12*(-e^2/(a^8*d^4))^(1/
```

```
2))/(a^2*d)))*(-e^2/(a^8*d^4))^(1/4))/2 + atan((e^12*(e*cot(c + d*x))^(1/2)
*(-e^2/(256*a^8*d^4))^(1/4)*16i)/((4*e^13)/(a^2*d) + 64*a^2*d*e^12*(-e^2/(2
56*a^8*d^4))^(1/2)) - (e^11*(e*cot(c + d*x))^(1/2)*(-e^2/(256*a^8*d^4))^(3/
4)*256i)/((4*e^13)/(a^6*d^3) + (64*e^12*(-e^2/(256*a^8*d^4))^(1/2))/(a^2*d)
))*(-e^2/(256*a^8*d^4))^(1/4)*2i + (e*(e*cot(c + d*x))^(1/2))/(2*(a^2*d*e +
 a^2*d*e*cot(c + d*x))) - ((-e)^(1/2)*atan(((e*cot(c + d*x))^(1/2)*1i)/(-e)
^(1/2))*1i)/(2*a^2*d)
```

3.32 $\int \frac{1}{\sqrt{e \cot(c+dx)}\,(a+a\cot(c+dx))^2}\,dx$

Optimal. Leaf size=281

$$-\frac{3\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}}-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2d\sqrt{e}}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2d\sqrt{e}}$$

```
[Out] -3/2*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(1/2)-1/4*arctan(1-2^(1/2
)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)+1/4*arctan(1+2^(1/2)*
(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)+1/8*ln(e^(1/2)+cot(d*x+
c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)/e^(1/2)-1/8*ln(e^(1/
2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)/e^(1/2)-1
/2*(e*cot(d*x+c))^(1/2)/d/e/(a^2+a^2*cot(d*x+c))
```

Rubi [A]
time = 0.37, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3650, 3734, 12, 16, 3557, 335, 303, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{3\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}}-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2d\sqrt{e}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{2\sqrt{2}\,a^2d\sqrt{e}}-\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}+\frac{\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}\,a^2d\sqrt{e}}-\frac{\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}\,a^2d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^2),x]
```

```
[Out] (-3*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(2*a^2*d*Sqrt[e]) - ArcTan[1 - (S
qrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(2*Sqrt[2]*a^2*d*Sqrt[e]) + ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(2*Sqrt[2]*a^2*d*Sqrt[e]) - Sqrt[
e*Cot[c + d*x]]/(2*d*e*(a^2 + a^2*Cot[c + d*x])) + Log[Sqrt[e] + Sqrt[e]*Co
t[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]]/(4*Sqrt[2]*a^2*d*Sqrt[e]) - Log[
Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]]/(4*Sqrt[2]*a
^2*d*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
```

```
!GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))^2}\,dx = -\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int \frac{-\frac{3a^2e}{2}+a^2e\cot(c+dx)-\frac{1}{2}a^2e\cot^2(c+d\cdots}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))\cdots}}{2a^3e}\cdots$$

$$= -\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} + \frac{3\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx\cdots}}{4a}\cdots$$

$$= -\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int \frac{\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{2a^2} + \frac{3\text{Subs}\cdots}{\cdots}$$

$$= -\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int \sqrt{e\cot(c+dx)}\,dx}{2a^2e} - \frac{3\text{Subst}\cdots}{\cdots}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} + \cdots$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} + \cdots$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} - \cdots$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} + \cdots$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} + \cdots$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+\cdots}}{\sqrt{e}}\cdots\right.}{2\sqrt{2}\,a^2d\sqrt{e}}\cdots$$

Mathematica [A]
time = 0.88, size = 337, normalized size = 1.20

$$-\frac{\sqrt{\cot(c+dx)}\left(12\text{ArcTan}\left(\sqrt{\cot(c+dx)}\right)\cos(c+dx)-\sqrt{2}\cos(c+dx)\log\left(-1+\sqrt{2}\sqrt{\cot(c+dx)}-\cot(c+dx)\right)+\sqrt{2}\cos(c+dx)\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)+12\text{ArcTan}\left(\sqrt{\cot(c+dx)}\right)\sin(c+dx)+4\sqrt{\cot(c+dx)}\sin(c+dx)-\sqrt{2}\log\left(-1+\sqrt{2}\sqrt{\cot(c+dx)}-\cot(c+dx)\right)\sin(c+dx)+\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin(c+dx)+2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)(\cos(c+dx)+\sin(c+dx))-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)(\cos(c+dx)+\sin(c+dx))\right)}{8a^2d\sqrt{e\cot(c+dx)}\,(\cos(c+dx)+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^2),x]
```

```
[Out] -1/8*(Sqrt[Cot[c + d*x]]*(12*ArcTan[Sqrt[Cot[c + d*x]]]*Cos[c + d*x] - Sqrt
[2]*Cos[c + d*x]*Log[-1 + Sqrt[2]*Sqrt[Cot[c + d*x]] - Cot[c + d*x]] + Sqrt
[2]*Cos[c + d*x]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] + 12*Ar
cTan[Sqrt[Cot[c + d*x]]]*Sin[c + d*x] + 4*Sqrt[Cot[c + d*x]]*Sin[c + d*x] -
 Sqrt[2]*Log[-1 + Sqrt[2]*Sqrt[Cot[c + d*x]] - Cot[c + d*x]]*Sin[c + d*x] +
 Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x] +
2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]*(Cos[c + d*x] + Sin[c + d*
x]) - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*(Cos[c + d*x] + Sin[
c + d*x])))/(a^2*d*Sqrt[e*Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x]))
```

Maple [A]

time = 0.54, size = 197, normalized size = 0.70

method	result
derivativedivides	$-\frac{2e^3\left(\frac{\frac{\sqrt{e\cot(dx+c)}}{2e\cot(dx+c)+2e}+\frac{3\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{2\sqrt{e}}}{2e^3}-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c}}\right.\right.}{}\right.}{d\,a^2}$
default	$-\frac{2e^3\left(\frac{\frac{\sqrt{e\cot(dx+c)}}{2e\cot(dx+c)+2e}+\frac{3\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{2\sqrt{e}}}{2e^3}-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c}}\right.\right.}{}\right.}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*e^3*(1/2/e^3*(1/2*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)+e)+3/2/e^(1/2
)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2)))-1/16/e^3/(e^2)^(1/4)*2^(1/2)*(ln((e
*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*
x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2
)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)+1)))
```

Maxima [A]

time = 0.54, size = 161, normalized size = 0.57

$$\frac{\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^2}-\frac{12\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a^2}-\frac{4}{\left(a^2+\frac{a^2}{\tan(dx+c)}\right)\sqrt{\tan(dx+c)}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*log(s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)/sqrt
(tan(d*x + c)) + 1/tan(d*x + c) + 1))/a^2 - 12*arctan(1/sqrt(tan(d*x + c)))
/a^2 - 4/((a^2 + a^2/tan(d*x + c))*sqrt(tan(d*x + c))))*e^(-1/2)/d
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\cot{\left(c+dx \right)}}\cot^{2}{\left(c+dx \right)}+2\sqrt{e\cot{\left(c+dx \right)}}\cot{\left(c+dx \right)}+\sqrt{e\cot{\left(c+dx \right)}}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**2,x)
```

```
[Out] Integral(1/(sqrt(e*cot(c + d*x))*cot(c + d*x)**2 + 2*sqrt(e*cot(c + d*x))*c
ot(c + d*x) + sqrt(e*cot(c + d*x))), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)^2*sqrt(e*cot(d*x + c))), x)
```

Mupad [B]
time = 0.81, size = 366, normalized size = 1.30

$$\frac{\mathrm{atan}\left(\frac{4\,e^8\,\sqrt{e\,\cot\left(c+d\,x\right)}\,{\left(-\frac{1}{a^8\,d^4\,e^2}\right)}^{1/4}}{\frac{4\,e^8}{a^2\,d}+36\,a^2\,d\,e^9\,\sqrt{-\frac{1}{a^8\,d^4\,e^2}}}+\frac{36\,e^9\,\sqrt{e\,\cot\left(c+d\,x\right)}\,{\left(-\frac{1}{a^8\,d^4\,e^2}\right)}^{3/4}}{\frac{4\,e^8}{a^6\,d^3}+\frac{36\,e^9\,\sqrt{-\frac{1}{a^8\,d^4\,e^2}}}{a^2\,d}}\right)\,{\left(-\frac{1}{a^8\,d^4\,e^2}\right)}^{1/4}}{2}+\mathrm{atan}\left(\frac{e^8\,\sqrt{e\,\cot\left(c+d\,x\right)}\,{\left(-\frac{1}{256\,a^8\,d^4\,e^2}\right)}^{1/4}\,16{}\mathrm{i}}{\frac{4\,e^8}{a^2\,d}-576\,a^2\,d\,e^9\,\sqrt{-\frac{1}{256\,a^8\,d^4\,e^2}}}-\frac{e^9\,\sqrt{e\,\cot\left(c+d\,x\right)}\,{\left(-\frac{1}{256\,a^8\,d^4\,e^2}\right)}^{3/4}\,2304{}\mathrm{i}}{\frac{4\,e^8}{a^6\,d^3}-\frac{576\,e^9\,\sqrt{-\frac{1}{256\,a^8\,d^4\,e^2}}}{a^2\,d}}\right)\,{\left(-\frac{1}{256\,a^8\,d^4\,e^2}\right)}^{1/4}\,2{}\mathrm{i}-\frac{\sqrt{e\,\cot\left(c+d\,x\right)}}{2\,\left(a^2\,d\,e+a^2\,d\,e\,\cot\left(c+d\,x\right)\right)}-\frac{\mathrm{atan}\left(\frac{\sqrt{e\,\cot\left(c+d\,x\right)}\,1{}\mathrm{i}}{\sqrt{-e}}\right)\,3{}\mathrm{i}}{2\,a^2\,d\,\sqrt{-e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^2),x)
```

```
[Out] (atan((4*e^8*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d^4*e^2))^(1/4))/((4*e^8)/(a^2
*d) + 36*a^2*d*e^9*(-1/(a^8*d^4*e^2))^(1/2)) + (36*e^9*(e*cot(c + d*x))^(1/
2)*(-1/(a^8*d^4*e^2))^(3/4))/((4*e^8)/(a^6*d^3) + (36*e^9*(-1/(a^8*d^4*e^2)
)^(1/2))/(a^2*d)))*(-1/(a^8*d^4*e^2))^(1/4))/2 + atan((e^8*(e*cot(c + d*x))
^(1/2)*(-1/(256*a^8*d^4*e^2))^(1/4)*16i)/((4*e^8)/(a^2*d) - 576*a^2*d*e^9*(
-1/(256*a^8*d^4*e^2))^(1/2)) - (e^9*(e*cot(c + d*x))^(1/2)*(-1/(256*a^8*d^4
*e^2))^(3/4)*2304i)/((4*e^8)/(a^6*d^3) - (576*e^9*(-1/(256*a^8*d^4*e^2))^(1
/2))/(a^2*d)))*(-1/(256*a^8*d^4*e^2))^(1/4)*2i - (e*cot(c + d*x))^(1/2)/(2*
(a^2*d*e + a^2*d*e*cot(c + d*x))) - (atan(((e*cot(c + d*x))^(1/2)*1i)/(-e)^
(1/2))*3i)/(2*a^2*d*(-e)^(1/2))
```

3.33 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx$

Optimal. Leaf size=306

$$\frac{5\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}}-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2de^{3/2}}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2de^{3/2}}-$$

```
[Out] 5/2*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(3/2)-1/4*arctan(1-2^(1/2)
*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(3/2)*2^(1/2)+1/4*arctan(1+2^(1/2)*(
e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(3/2)*2^(1/2)-1/8*ln(e^(1/2)+cot(d*x+c
)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d/e^(3/2)*2^(1/2)+1/8*ln(e^(1/2
)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d/e^(3/2)*2^(1/2)+5/
2/a^2/d/e/(e*cot(d*x+c))^(1/2)-1/2/d/e/(a^2+a^2*cot(d*x+c))/(e*cot(d*x+c))^
(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3650, 3730, 3734, 12, 3557, 335, 217, 1179, 642, 1176, 631, 210, 3715, 65, 211}

$$\frac{5\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}}-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2de^{3/2}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{2\sqrt{2}\,a^2de^{3/2}}-\frac{\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}\,a^2de^{3/2}}+\frac{\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}\,a^2de^{3/2}}+\frac{5}{2a^2de\sqrt{e\cot(c+dx)}}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2),x]
```

```
[Out] (5*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(2*a^2*d*e^(3/2)) - ArcTan[1 - (Sq
rt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(2*Sqrt[2]*a^2*d*e^(3/2)) + ArcTan[1 +
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(2*Sqrt[2]*a^2*d*e^(3/2)) + 5/(2*a
^2*d*e*Sqrt[e*Cot[c + d*x]]) - 1/(2*d*e*Sqrt[e*Cot[c + d*x]]*(a^2 + a^2*Cot
[c + d*x])) - Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d
*x]]]/(4*Sqrt[2]*a^2*d*e^(3/2)) + Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt
[2]*Sqrt[e*Cot[c + d*x]]]/(4*Sqrt[2]*a^2*d*e^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx = -\frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c+dx))} - \frac{\int \frac{-\frac{5a^2e}{2}+a^2e\cot}{(e\cot(c+dx}}{}$$

$$= \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c}$$

$$= \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c}$$

$$= \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c}$$

$$= \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c}$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} + \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{}{2de}$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} + \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{}{2de}$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} + \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{}{2de}$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} + \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{}{2de}$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+}}{\sqrt{e}}\right.}{2\sqrt{2}\,a^2de^{3/2}}$$

Mathematica [A]
time = 1.40, size = 203, normalized size = 0.66

$$\frac{\cot^{\frac{3}{2}}(c+dx)\left(-\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)+\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+10\text{ArcTan}\left(\sqrt{\cot(c+dx)}\right)+\frac{-\log\left(-1+\sqrt{2}\sqrt{\cot(c+dx)}-\cot(c+dx)\right)+\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)}{\sqrt{2}}+\frac{2(5\cos(c+dx)+4\sin(c+dx))}{\sqrt{\cot(c+dx)}\,(\cos(c+dx)+\sin(c+dx))}\right)}{4a^2d(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2),x]
```

```
[Out] (Cot[c + d*x]^(3/2)*(-(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]) + Sq
rt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 10*ArcTan[Sqrt[Cot[c + d*x]]
] + (-Log[-1 + Sqrt[2]*Sqrt[Cot[c + d*x]] - Cot[c + d*x]] + Log[1 + Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/Sqrt[2] + (2*(5*Cos[c + d*x] + 4*Sin[c
 + d*x]))/(Sqrt[Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x]))))/(4*a^2*d*(e*
Cot[c + d*x])^(3/2))
```

Maple [A]
time = 0.53, size = 212, normalized size = 0.69

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{1}{e^4\sqrt{e\cot(dx+c)}}-\frac{\frac{\sqrt{e\cot(dx+c)}}{2e\cot(dx+c)+2e}+\frac{5\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{2\sqrt{e}}}{2e^4}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\,\mathrm{co}}{e\,\mathrm{co}}\right.\right.}{}\right.}{}$
default	$-\frac{2e^3\left(-\frac{1}{e^4\sqrt{e\cot(dx+c)}}-\frac{\frac{\sqrt{e\cot(dx+c)}}{2e\cot(dx+c)+2e}+\frac{5\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{2\sqrt{e}}}{2e^4}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\,\mathrm{co}}{e\,\mathrm{co}}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*e^3*(-1/e^4/(e*cot(d*x+c))^(1/2)-1/2/e^4*(1/2*(e*cot(d*x+c))^(1/2)
/(e*cot(d*x+c)+e)+5/2/e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2)))-1/16/e^
5*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(
-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Maxima [A]
time = 0.50, size = 174, normalized size = 0.57

$$\frac{\left(\frac{4\left(\frac{5}{\tan(dx+c)}+4\right)}{\frac{a^2}{\sqrt{\tan(dx+c)}}+\frac{a^2}{\tan(dx+c)^{\frac{3}{2}}}}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^2}+\frac{20\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a^2}\right)e^{\left(-\frac{3}{2}\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(4*(5/tan(d*x + c) + 4)/(a^2/sqrt(tan(d*x + c)) + a^2/tan(d*x + c)^(3/2
)) + (2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*log(s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)/sqrt
(tan(d*x + c)) + 1/tan(d*x + c) + 1))/a^2 + 20*arctan(1/sqrt(tan(d*x + c)))
/a^2)*e^(-3/2)/d
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\cot^2(c+dx)+2(e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**2,x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*cot(c + d*x)**2 + 2*(e*cot(c + d*x))**(
3/2)*cot(c + d*x) + (e*cot(c + d*x))**(3/2)), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(3/2)), x)
```

Mupad [B]
time = 0.92, size = 414, normalized size = 1.35

$$\frac{\frac{5\cot(c+dx)}{2}+2}{a^2\,d\,(e\cot(c+dx))^{3/2}+a^2\,d\,e\,\sqrt{e\cot(c+dx)}}-\frac{\mathrm{atan}\left(\frac{2048\,a^{10}\,d^5\,e^{13}\,\sqrt{e\cot(c+dx)}\,\left(-\frac{1}{a^8\,d^4\,e^6}\right)^{1/4}}{51200\,a^8\,d^4\,e^{12}-2048\,a^{12}\,d^6\,e^{15}\,\sqrt{-\frac{1}{a^8\,d^4\,e^6}}}+\frac{51200\,a^{14}\,d^7\,e^{16}\,\sqrt{e\cot(c+dx)}\,\left(-\frac{1}{a^8\,d^4\,e^6}\right)^{3/4}}{51200\,a^8\,d^4\,e^{12}-2048\,a^{12}\,d^6\,e^{15}\,\sqrt{-\frac{1}{a^8\,d^4\,e^6}}}\right)\,\left(-\frac{1}{a^8\,d^4\,e^6}\right)^{1/4}}{2}-\mathrm{atan}\left(\frac{a^{10}\,d^5\,e^{13}\,\sqrt{e\cot(c+dx)}\,\left(-\frac{1}{256\,a^8\,d^4\,e^6}\right)^{1/4}\,8192{}\mathrm{i}}{51200\,a^8\,d^4\,e^{12}+32768\,a^{12}\,d^6\,e^{15}\,\sqrt{-\frac{1}{256\,a^8\,d^4\,e^6}}}-\frac{a^{14}\,d^7\,e^{16}\,\sqrt{e\cot(c+dx)}\,\left(-\frac{1}{256\,a^8\,d^4\,e^6}\right)^{3/4}\,3276800{}\mathrm{i}}{51200\,a^8\,d^4\,e^{12}+32768\,a^{12}\,d^6\,e^{15}\,\sqrt{-\frac{1}{256\,a^8\,d^4\,e^6}}}\right)\,\left(-\frac{1}{256\,a^8\,d^4\,e^6}\right)^{1/4}\,2{}\mathrm{i}+\frac{\mathrm{atan}\left(\frac{\sqrt{e\cot(c+dx)}\,\sqrt{-e^3}\,1{}\mathrm{i}}{e^2}\right)\,\sqrt{-e^3}\,5{}\mathrm{i}}{2\,a^2\,d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^2),x)
```

```
[Out] ((5*cot(c + d*x))/2 + 2)/(a^2*d*(e*cot(c + d*x))^(3/2) + a^2*d*e*(e*cot(c +
 d*x))^(1/2)) - (atan((2048*a^10*d^5*e^13*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d
^4*e^6))^(1/4))/(51200*a^8*d^4*e^12 - 2048*a^12*d^6*e^15*(-1/(a^8*d^4*e^6))
^(1/2)) + (51200*a^14*d^7*e^16*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d^4*e^6))^(3
/4))/(51200*a^8*d^4*e^12 - 2048*a^12*d^6*e^15*(-1/(a^8*d^4*e^6))^(1/2)))*(-
1/(a^8*d^4*e^6))^(1/4))/2 - atan((a^10*d^5*e^13*(e*cot(c + d*x))^(1/2)*(-1/
(256*a^8*d^4*e^6))^(1/4)*8192i)/(51200*a^8*d^4*e^12 + 32768*a^12*d^6*e^15*(
-1/(256*a^8*d^4*e^6))^(1/2)) - (a^14*d^7*e^16*(e*cot(c + d*x))^(1/2)*(-1/(2
56*a^8*d^4*e^6))^(3/4)*3276800i)/(51200*a^8*d^4*e^12 + 32768*a^12*d^6*e^15*
(-1/(256*a^8*d^4*e^6))^(1/2)))*(-1/(256*a^8*d^4*e^6))^(1/4)*2i + (atan(((e*
cot(c + d*x))^(1/2)*(-e^3)^(1/2)*1i)/e^2)*(-e^3)^(1/2)*5i)/(2*a^2*d*e^3)
```

3.34 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx$

Optimal. Leaf size=331

$$-\frac{7\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}}+\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2de^{5/2}}-\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2de^{5/2}}$$

```
[Out] -7/2*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(5/2)+7/6/a^2/d/e/(e*cot(
d*x+c))^(3/2)-1/2/d/e/(e*cot(d*x+c))^(3/2)/(a^2+a^2*cot(d*x+c))+1/4*arctan(
1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(5/2)*2^(1/2)-1/4*arctan(1+
2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(5/2)*2^(1/2)-1/8*ln(e^(1/2)+
cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d/e^(5/2)*2^(1/2)+1/8*
ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d/e^(5/2)*2
^(1/2)-9/2/a^2/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3650, 3730, 3734, 12, 16, 3557, 335, 303, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{7\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}}+\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}\,a^2de^{5/2}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{2\sqrt{2}\,a^2de^{5/2}}-\frac{\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}\,a^2de^{5/2}}+\frac{\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{4\sqrt{2}\,a^2de^{5/2}}-\frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}+\frac{7}{6a^2de(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^2),x]
```

```
[Out] (-7*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(2*a^2*d*e^(5/2)) + ArcTan[1 - (S
qrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(2*Sqrt[2]*a^2*d*e^(5/2)) - ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(2*Sqrt[2]*a^2*d*e^(5/2)) + 7/(6*
a^2*d*e*(e*Cot[c + d*x])^(3/2)) - 9/(2*a^2*d*e^2*Sqrt[e*Cot[c + d*x]]) - 1/
(2*d*e*(e*Cot[c + d*x])^(3/2)*(a^2 + a^2*Cot[c + d*x])) - Log[Sqrt[e] + Sqr
t[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]]/(4*Sqrt[2]*a^2*d*e^(5/2))
 + Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]]/(4*Sq
rt[2]*a^2*d*e^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
```

```
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx = -\frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)} - \frac{\int \frac{-\frac{7a^2e}{2}+a^2e\,\mathrm{co}}{(e\cot(c+d}}{}$$

$$= \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(\right.}$$

$$= \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}} - \frac{}{2de(e\,\mathrm{c}}$$

$$= \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}} - \frac{}{2de(e\,\mathrm{c}}$$

$$= \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}} - \frac{}{2de(e\,\mathrm{c}}$$

$$= \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}} - \frac{}{2de(e\,\mathrm{c}}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{}{2}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{}{2}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{}{2}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{}{2}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e\cot(c+dx))^{3/2}} - \frac{}{2}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+}}{\sqrt{e}}\right.}{2\sqrt{2}\,a^2de^{5/2}}$$

Mathematica [A]

time = 6.36, size = 467, normalized size = 1.41

$$\frac{\cot^3(c+dx)\csc^2(c+dx)(\cos(c+dx)+\sin(c+dx))^2\left(-\frac{2}{3}+\frac{2}{3}\sec^2(c+dx)-\frac{\sin(c+dx)}{2(\cos(c+dx)+\sin(c+dx))}-4\tan(c+dx)\right)}{d(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}+\frac{\cot^{\frac{5}{2}}(c+dx)\csc^2(c+dx)(\cos(c+dx)+\sin(c+dx))^2\left(-\frac{16\mathrm{ArcTan}\left(\sqrt{\cot(c+dx)}\right)(1+\cot(c+dx))\csc^3(c+dx)\sec(c+dx)}{(1+\cot^2(c+dx))^2(1+\tan(c+dx))}+\frac{\cos(2(c+dx))\csc^3(c+dx)\left(-\log\left(-1+\sqrt{2}\sqrt{\cot(c+dx)}-\cot(c+dx)\right)+\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\sec(c+dx)}{\sqrt{2}(-1+\cot(c+dx))(1+\cot^2(c+dx))(1+\tan(c+dx))}+\frac{\left(-\sqrt{2}\left(-\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)+\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\right)+2\mathrm{ArcTan}\left(\sqrt{\cot(c+dx)}\right)\right)(1+\cot(c+dx))\csc^2(c+dx)\sec^2(c+dx)\sin(2(c+dx))}{2(1+\cot^2(c+dx))(1+\tan(c+dx))}\right)}{4d(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^2),x]
```

```
[Out] (Cot[c + d*x]^3*Csc[c + d*x]^2*(Cos[c + d*x] + Sin[c + d*x])^2*(-2/3 + (2*S
ec[c + d*x]^2)/3 - Sin[c + d*x]/(2*(Cos[c + d*x] + Sin[c + d*x])) - 4*Tan[c
 + d*x]))/(d*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^2) + (Cot[c + d*x]
^(5/2)*Csc[c + d*x]^2*(Cos[c + d*x] + Sin[c + d*x])^2*((-16*ArcTan[Sqrt[Cot
[c + d*x]]]*(1 + Cot[c + d*x])*Csc[c + d*x]^3*Sec[c + d*x])/((1 + Cot[c + d
*x]^2)^2*(1 + Tan[c + d*x])) + (Cos[2*(c + d*x)]*Csc[c + d*x]^3*(-Log[-1 +
Sqrt[2]*Sqrt[Cot[c + d*x]] - Cot[c + d*x]] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d
*x]] + Cot[c + d*x]])*Sec[c + d*x])/(Sqrt[2]*(-1 + Cot[c + d*x])*(1 + Cot[c
 + d*x]^2)*(1 + Tan[c + d*x])) + ((-(Sqrt[2]*(-ArcTan[1 - Sqrt[2]*Sqrt[Cot[
c + d*x]]] + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])) + 2*ArcTan[Sqrt[Cot[c
 + d*x]]])*(1 + Cot[c + d*x])*Csc[c + d*x]^2*Sec[c + d*x]^2*Sin[2*(c + d*x)
])/(2*(1 + Cot[c + d*x]^2)*(1 + Tan[c + d*x]))))/(4*d*(e*Cot[c + d*x])^(5/2
)*(a + a*Cot[c + d*x])^2)
```

Maple [A]
time = 0.52, size = 227, normalized size = 0.69

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{1}{3e^4(e\cot(dx+c))^{\frac{3}{2}}}+\frac{2}{e^5\sqrt{e\cot(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e}}\right.\right.}{}\right.}{}$
default	$-\frac{2e^3\left(-\frac{1}{3e^4(e\cot(dx+c))^{\frac{3}{2}}}+\frac{2}{e^5\sqrt{e\cot(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e}}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*e^3*(-1/3/e^4/(e*cot(d*x+c))^(3/2)+2/e^5/(e*cot(d*x+c))^(1/2)+1/16
/e^5/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
```

```
*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2
)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arct
an(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/2/e^5*(1/2*(e*cot(d*x+c)
)^(1/2)/(e*cot(d*x+c)+e)+7/2/e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))))
```

Maxima [A]
time = 0.51, size = 185, normalized size = 0.56

$$-\frac{\left(\frac{4\left(\frac{20}{\tan(dx+c)}+\frac{27}{\tan(dx+c)^2}-4\right)}{\frac{a^2}{\tan(dx+c)^{\frac{3}{2}}}+\frac{a^2}{\tan(dx+c)^{\frac{5}{2}}}}+\frac{3\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)}{a^2}+\frac{84\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a^2}\right)e^{\left(-\frac{5}{2}\right)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/24*(4*(20/tan(d*x + c) + 27/tan(d*x + c)^2 - 4)/(a^2/tan(d*x + c)^(3/2)
+ a^2/tan(d*x + c)^(5/2)) + 3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sq
rt(tan(d*x + c)))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*
x + c)))) - sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) +
sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/a^2 + 84*arc
tan(1/sqrt(tan(d*x + c)))/a^2)*e^(-5/2)/d
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\cot^2(c+dx)+2(e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{5}{2}}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**2,x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(5/2)*cot(c + d*x)**2 + 2*(e*cot(c + d*x))**(
5/2)*cot(c + d*x) + (e*cot(c + d*x))**(5/2)), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(5/2)), x)
```

Mupad [B]
time = 1.23, size = 425, normalized size = 1.28

$$-\frac{\mathrm{atan}\left(\frac{2048\,a^{10}\,d^5\,e^{18}\,\sqrt{e\,\cot\left(c+d\,x\right)}\,{\left(-\frac{1}{a^8\,d^4\,e^{10}}\right)}^{1/4}}{2048\,a^8\,d^4\,e^{16}+100352\,a^{12}\,d^6\,e^{21}\,\sqrt{-\frac{1}{a^8\,d^4\,e^{10}}}}+\frac{100352\,a^{14}\,d^7\,e^{23}\,\sqrt{e\,\cot\left(c+d\,x\right)}\,{\left(-\frac{1}{a^8\,d^4\,e^{10}}\right)}^{3/4}}{2048\,a^8\,d^4\,e^{16}+100352\,a^{12}\,d^6\,e^{21}\,\sqrt{-\frac{1}{a^8\,d^4\,e^{10}}}}\right)\,{\left(-\frac{1}{a^8\,d^4\,e^{10}}\right)}^{1/4}}{2}-\mathrm{atan}\left(\frac{a^{10}\,d^5\,e^{18}\,\sqrt{e\,\cot\left(c+d\,x\right)}\,{\left(-\frac{1}{256\,a^8\,d^4\,e^{10}}\right)}^{1/4}\,8192{}\mathrm{i}}{2048\,a^8\,d^4\,e^{16}-1605632\,a^{12}\,d^6\,e^{21}\,\sqrt{-\frac{1}{256\,a^8\,d^4\,e^{10}}}}-\frac{a^{14}\,d^7\,e^{23}\,\sqrt{e\,\cot\left(c+d\,x\right)}\,{\left(-\frac{1}{256\,a^8\,d^4\,e^{10}}\right)}^{3/4}\,6422528{}\mathrm{i}}{2048\,a^8\,d^4\,e^{16}-1605632\,a^{12}\,d^6\,e^{21}\,\sqrt{-\frac{1}{256\,a^8\,d^4\,e^{10}}}}\right)\,{\left(-\frac{1}{256\,a^8\,d^4\,e^{10}}\right)}^{1/4}\,2{}\mathrm{i}-\frac{\frac{9\,{\cot\left(c+d\,x\right)}^2}{2}+\frac{10\,\cot\left(c+d\,x\right)}{3}-\frac{2}{3}}{a^2\,d\,{\left(e\,\cot\left(c+d\,x\right)\right)}^{5/2}+a^2\,d\,e\,{\left(e\,\cot\left(c+d\,x\right)\right)}^{3/2}}-\frac{\mathrm{atan}\left(\frac{\sqrt{e\,\cot\left(c+d\,x\right)}\,\sqrt{-e^5}\,1{}\mathrm{i}}{e^3}\right)\,\sqrt{-e^5}\,7{}\mathrm{i}}{2\,a^2\,d\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))^2),x)
```

```
[Out] - (atan((2048*a^10*d^5*e^18*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d^4*e^10))^(1/4
))/(2048*a^8*d^4*e^16 + 100352*a^12*d^6*e^21*(-1/(a^8*d^4*e^10))^(1/2)) + (
100352*a^14*d^7*e^23*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d^4*e^10))^(3/4))/(204
8*a^8*d^4*e^16 + 100352*a^12*d^6*e^21*(-1/(a^8*d^4*e^10))^(1/2)))*(-1/(a^8*
d^4*e^10))^(1/4))/2 - atan((a^10*d^5*e^18*(e*cot(c + d*x))^(1/2)*(-1/(256*a
^8*d^4*e^10))^(1/4)*8192i)/(2048*a^8*d^4*e^16 - 1605632*a^12*d^6*e^21*(-1/(
256*a^8*d^4*e^10))^(1/2)) - (a^14*d^7*e^23*(e*cot(c + d*x))^(1/2)*(-1/(256*
a^8*d^4*e^10))^(3/4)*6422528i)/(2048*a^8*d^4*e^16 - 1605632*a^12*d^6*e^21*(
-1/(256*a^8*d^4*e^10))^(1/2)))*(-1/(256*a^8*d^4*e^10))^(1/4)*2i - ((10*cot(
c + d*x))/3 + (9*cot(c + d*x)^2)/2 - 2/3)/(a^2*d*(e*cot(c + d*x))^(5/2) + a
^2*d*e*(e*cot(c + d*x))^(3/2)) - (atan(((e*cot(c + d*x))^(1/2)*(-e^5)^(1/2)
*1i)/e^3)*(-e^5)^(1/2)*7i)/(2*a^2*d*e^5)
```

3.35 $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx$

Optimal. Leaf size=164

$$-\frac{e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d}+\frac{e^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d}-\frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(1+\cot(c+dx))}+\frac{e^2\sqrt{e}}{4ad(a+}$$

```
[Out] -1/8*e^(5/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d+1/4*e^(5/2)*arctanh
(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d*2^(1/
2)-5/8*e^2*(e*cot(d*x+c))^(1/2)/a^3/d/(1+cot(d*x+c))+1/4*e^2*(e*cot(d*x+c))
^(1/2)/a/d/(a+a*cot(d*x+c))^2
```

Rubi [A]
time = 0.40, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3646, 3730, 3735, 3613, 214, 3715, 65, 211}

$$-\frac{e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d}+\frac{e^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d}-\frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(\cot(c+dx)+1)}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x])^3,x]
```

```
[Out] -1/8*(e^(5/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(a^3*d) + (e^(5/2)*ArcT
anh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])])/(2*Sq
rt[2]*a^3*d) - (5*e^2*Sqrt[e*Cot[c + d*x]])/(8*a^3*d*(1 + Cot[c + d*x])) +
(e^2*Sqrt[e*Cot[c + d*x]])/(4*a*d*(a + a*Cot[c + d*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
```

```
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}a^2e^3+2a^2e^3\cot(c+dx)-\frac{5}{2}a^2e^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))^2}\,dx}{4a^3}$$

$$= -\frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(1+\cot(c+dx))} + \frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{\int \frac{-\frac{3}{2}a^4e^4+\frac{5}{2}a^4e^4\cot^2(c\cdots}{\sqrt{e\cot(c+dx)}\,(a+a\,c\cdots}}{8a^6e}$$

$$= -\frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(1+\cot(c+dx))} + \frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{\int \frac{-4a^5e^4+4a^5e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,d\cdots}{16a^8e}$$

$$= -\frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(1+\cot(c+dx))} + \frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{e^3\text{Subst}\left(\int \frac{1}{\sqrt{-ex}\,(a-a\cdots}\right.}{16\cdots}$$

$$= \frac{e^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d} - \frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(1+\cot(c+dx))} + \frac{e^2\sqrt{e\cot(\cdots}}{4ad(a+a\cot\cdots}$$

$$= -\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d} - \frac{5\cdots}{8a\cdots}$$

Mathematica [A]

time = 2.23, size = 192, normalized size = 1.17

$$\frac{(e\cot(c+dx))^{5/2}\csc(c+dx)(\cos(c+dx)+\sin(c+dx))^3\left(-\frac{2\csc(c+dx)\left(\text{ArcTan}\left(\sqrt{\cot(c+dx)}\right)+\sqrt{2}\left(\log\left(-1+\sqrt{2}\sqrt{\cot(c+dx)}-\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)\sec(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}+\frac{\sec^4(c+dx)(-3+3\cos(2(c+dx))-5\sin(2(c+dx)))}{(1+\tan(c+dx))^2}\right)}{16a^3d(1+\cot(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x])^3,x]
```

```
[Out] ((e*Cot[c + d*x])^(5/2)*Csc[c + d*x]*(Cos[c + d*x] + Sin[c + d*x])^3*((-2*C
sc[c + d*x]*(ArcTan[Sqrt[Cot[c + d*x]]] + Sqrt[2]*(Log[-1 + Sqrt[2]*Sqrt[Co
t[c + d*x]] - Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c +
d*x]]))*Sec[c + d*x])/Cot[c + d*x]^(3/2) + (Sec[c + d*x]^4*(-3 + 3*Cos[2*(c
 + d*x)] - 5*Sin[2*(c + d*x)]))/(1 + Tan[c + d*x])^2))/(16*a^3*d*(1 + Cot[c
 + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(135) = 270$.
time = 0.59, size = 349, normalized size = 2.13

method	result
derivativedivides	$-\frac{2e^4\left(\frac{\frac{\frac{5(e\cot(dx+c))^{\frac{3}{2}}}{4}+\frac{3e\sqrt{e\cot(dx+c)}}{4}}{(e\cot(dx+c)+e)^2}+\frac{\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{4\sqrt{e}}}{4e}+\frac{-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(}{e\cot(dx+c)-(}\right.\right.}{}}{}\right.}{}$
default	$-\frac{2e^4\left(\frac{\frac{\frac{5(e\cot(dx+c))^{\frac{3}{2}}}{4}+\frac{3e\sqrt{e\cot(dx+c)}}{4}}{(e\cot(dx+c)+e)^2}+\frac{\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{4\sqrt{e}}}{4e}+\frac{-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(}{e\cot(dx+c)-(}\right.\right.}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^3*e^4*(1/4/e*((5/4*(e*cot(d*x+c))^(3/2)+3/4*e*(e*cot(d*x+c))^(1/2))/
(e*cot(d*x+c)+e)^2+1/4/e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2)))+1/4/e*
(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8/(e^2)^(1/4)*2^(1/2)
*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e
*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan
(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.51, size = 136, normalized size = 0.83

$$-\frac{\left(\frac{\frac{3}{\sqrt{\tan(dx+c)}}+\frac{5}{\tan(dx+c)^{\frac{3}{2}}}}{a^3+\frac{2a^3}{\tan(dx+c)}+\frac{a^3}{\tan(dx+c)^2}}-\frac{\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^3}+\frac{\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a^3}\right)e^{\frac{5}{2}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((3/sqrt(tan(d*x + c)) + 5/tan(d*x + c)^(3/2))/(a^3 + 2*a^3/tan(d*x +
c) + a^3/tan(d*x + c)^2) - (sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(
d*x + c) + 1) - sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
1))/a^3 + arctan(1/sqrt(tan(d*x + c)))/a^3)*e^(5/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(119) = 238$.
time = 2.54, size = 246, normalized size = 1.50

$$\frac{2\left(e^{\frac{5}{2}}\sin\left(2\,dx+2\,c\right)+e^{\frac{5}{2}}\right)\arctan\left(\frac{\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\sin\left(2\,dx+2\,c\right)}{\cos\left(2\,dx+2\,c\right)+1}\right)+2\left(\sqrt{2}\,e^{\frac{5}{2}}\sin\left(2\,dx+2\,c\right)+\sqrt{2}\,e^{\frac{5}{2}}\right)\log\left(-\left(\sqrt{2}\,\cos\left(2\,dx+2\,c\right)-\sqrt{2}\,\sin\left(2\,dx+2\,c\right)-\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}+2\,\sin\left(2\,dx+2\,c\right)+1\right)+\left(3\,\cos\left(2\,dx+2\,c\right)e^{\frac{5}{2}}-5\,e^{\frac{5}{2}}\sin\left(2\,dx+2\,c\right)-3\,e^{\frac{5}{2}}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{16\,\left(a^{3}d\sin\left(2\,dx+2\,c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(e^(5/2)*sin(2*d*x + 2*c) + e^(5/2))*arctan(sqrt((cos(2*d*x + 2*c)
+ 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c) + 1)) + 2*(sqrt(2
)*e^(5/2)*sin(2*d*x + 2*c) + sqrt(2)*e^(5/2))*log(-(sqrt(2)*cos(2*d*x + 2*c
) - sqrt(2)*sin(2*d*x + 2*c) - sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d
*x + 2*c)) + 2*sin(2*d*x + 2*c) + 1) + (3*cos(2*d*x + 2*c)*e^(5/2) - 5*e^(5
/2)*sin(2*d*x + 2*c) - 3*e^(5/2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2
*c)))/(a^3*d*sin(2*d*x + 2*c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(e\cot\left(c+dx\right)\right)^{\frac{5}{2}}}{\cot^{3}\left(c+dx\right)+3\cot^{2}\left(c+dx\right)+3\cot\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**3,x)
```

```
[Out] Integral((e*cot(c + d*x))**(5/2)/(cot(c + d*x)**3 + 3*cot(c + d*x)**2 + 3*c
ot(c + d*x) + 1), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)/(a*cot(d*x + c) + a)^3, x)
```

Mupad [B]
time = 1.04, size = 154, normalized size = 0.94

$$\frac{\sqrt{2}\,e^{5/2}\,\mathrm{atanh}\left(\frac{9\sqrt{2}\,e^{33/2}\,\sqrt{e\cot\left(c+d\,x\right)}}{32\left(\frac{9\,e^{17}\cot\left(c+d\,x\right)}{32}+\frac{9\,e^{17}}{32}\right)}\right)}{4\,a^3\,d}-\frac{e^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{8\,a^3\,d}-\frac{\frac{3\,e^4\,\sqrt{e\cot\left(c+d\,x\right)}}{8}+\frac{5\,e^3\,{\left(e\cot\left(c+d\,x\right)\right)}^{3/2}}{8}}{d\,a^3\,e^2\,{\cot\left(c+d\,x\right)}^2+2\,d\,a^3\,e^2\cot\left(c+d\,x\right)+d\,a^3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(5/2)/(a + a*cot(c + d*x))^3,x)
```

```
[Out] (2^(1/2)*e^(5/2)*atanh((9*2^(1/2)*e^(33/2)*(e*cot(c + d*x))^(1/2))/(32*((9*
e^17*cot(c + d*x))/32 + (9*e^17)/32))))/(4*a^3*d) - (e^(5/2)*atan((e*cot(c
+ d*x))^(1/2)/e^(1/2)))/(8*a^3*d) - ((3*e^4*(e*cot(c + d*x))^(1/2))/8 + (5*
e^3*(e*cot(c + d*x))^(3/2))/8)/(a^3*d*e^2 + a^3*d*e^2*cot(c + d*x)^2 + 2*a^
3*d*e^2*cot(c + d*x))
```

3.36 $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx$

Optimal. Leaf size=164

$$\frac{5e^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d}+\frac{e^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d}-\frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2}+\frac{e\sqrt{e}}{8d\,(a^3+}$$

[Out] 5/8*e^(3/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d+1/4*e^(3/2)*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d*2^(1/2)-1/4*e*(e*cot(d*x+c))^(1/2)/a/d/(a+a*cot(d*x+c))^2+1/8*e*(e*cot(d*x+c))^(1/2)/d/(a^3+a^3*cot(d*x+c))

Rubi [A]
time = 0.44, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3648, 3730, 3734, 3613, 211, 3715, 65}

$$\frac{5e^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d}+\frac{e^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d}+\frac{e\sqrt{e\cot(c+dx)}}{8d\,(a^3\cot(c+dx)+a^3)}-\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x])^3,x]
```

```
[Out] (5*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(8*a^3*d) + (e^(3/2)*ArcTan[(Sqrt[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])])/(2*Sqrt[2]*a^3*d) - (e*Sqrt[e*Cot[c + d*x]])/(4*a*d*(a + a*Cot[c + d*x])^2) + (e*Sqrt[e*Cot[c + d*x]])/(8*d*(a^3 + a^3*Cot[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
```

```
!GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = -\frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} - \frac{\int \frac{\frac{ae^2}{2}-2ae^2\cot(c+dx)-\frac{3}{2}ae^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))^2}\,dx}{4a^2}$$

$$= -\frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{e\sqrt{e\cot(c+dx)}}{8d\,(a^3+a^3\cot(c+dx))} + \frac{\int \frac{-\frac{1}{2}a^3e^3+4a^3e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}\,(\ldots}}{8a^5e\ldots}$$

$$= -\frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{e\sqrt{e\cot(c+dx)}}{8d\,(a^3+a^3\cot(c+dx))} + \frac{\int \frac{4a^4e^3+4a^4e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,d\ldots}{16a^7e\ldots}$$

$$= -\frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{e\sqrt{e\cot(c+dx)}}{8d\,(a^3+a^3\cot(c+dx))} - \frac{(5e^2)\,\mathrm{Subst}\left(\int \frac{\ldots}{\sqrt{-ex\ldots}}\ldots\right.}{\ldots}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d} - \frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{e\sqrt{e\cot(\ldots}}{8d\,(a^3+a^3\,\mathrm{c}\ldots}$$

$$= \frac{5e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d} - \frac{e\ldots}{4ad(\ldots}$$

Mathematica [A]
time = 2.15, size = 131, normalized size = 0.80

$$\frac{e\sqrt{e\cot(c+dx)}\left(\frac{2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+5\mathrm{ArcTan}\left(\sqrt{\cot(c+dx)}\right)}{\sqrt{\cot(c+dx)}}+\frac{1-\sec^2(c+dx)+\tan(c+dx)}{(1+\tan(c+dx))^2}\right)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x])^3,x]
```

```
[Out] (e*Sqrt[e*Cot[c + d*x]]*((2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]
- 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 5*ArcTan[Sqrt[Cot[c +
d*x]]])/Sqrt[Cot[c + d*x]] + (1 - Sec[c + d*x]^2 + Tan[c + d*x])/(1 + Tan[c
 + d*x])^2))/(8*a^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(135) = 270.
time = 0.58, size = 349, normalized size = 2.13

method	result
derivativedivides	$-\frac{2e^4\left(\frac{\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$
default	$-\frac{2e^4\left(\frac{\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^3*e^4*(1/4/e^2*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/
8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(
-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/4/e^2*((1/4*(e*cot(d*x+c))
^(3/2)-1/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)+e)^2+5/4/e^(1/2)*arctan((e
*cot(d*x+c))^(1/2)/e^(1/2))))
```

Maxima [A]

time = 0.53, size = 127, normalized size = 0.77

$$-\frac{\left(\frac{\frac{1}{\sqrt{\tan(dx+c)}}-\frac{1}{\tan(dx+c)^{\frac{3}{2}}}}{a^3+\frac{2a^3}{\tan(dx+c)}+\frac{a^3}{\tan(dx+c)^2}}+\frac{2\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)\right)}{a^3}-\frac{5\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a^3}\right)e^{\frac{3}{2}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((1/sqrt(tan(d*x + c)) - 1/tan(d*x + c)^(3/2))/(a^3 + 2*a^3/tan(d*x +
c) + a^3/tan(d*x + c)^2) + 2*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(
tan(d*x + c)))) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c
)))))/a^3 - 5*arctan(1/sqrt(tan(d*x + c)))/a^3)*e^(3/2)/d
```

Fricas [A]
time = 2.87, size = 242, normalized size = 1.48

$$\frac{4\left(\sqrt{2}\,e^{\frac{3}{2}}\sin\left(2\,dx+2\,c\right)+\sqrt{2}\,e^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{2}\cos(2\,dx+2\,c)-\sqrt{2}\sin(2\,dx+2\,c)+\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{2\left(\cos(2\,dx+2\,c)+1\right)}\right)-10\left(e^{\frac{3}{2}}\sin\left(2\,dx+2\,c\right)+e^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)+1}\right)+\left(\cos\left(2\,dx+2\,c\right)e^{\frac{3}{2}}+e^{\frac{3}{2}}\sin\left(2\,dx+2\,c\right)-e^{\frac{3}{2}}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{16\left(a^{3}d\sin\left(2\,dx+2\,c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(sqrt(2)*e^(3/2)*sin(2*d*x + 2*c) + sqrt(2)*e^(3/2))*arctan(-1/2*(s
qrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt((cos(2*d
*x + 2*c) + 1)/sin(2*d*x + 2*c))/(cos(2*d*x + 2*c) + 1)) - 10*(e^(3/2)*sin(
2*d*x + 2*c) + e^(3/2))*arctan(sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)
)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c) + 1)) + (cos(2*d*x + 2*c)*e^(3/2) + e^
(3/2)*sin(2*d*x + 2*c) - e^(3/2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2
*c)))/(a^3*d*sin(2*d*x + 2*c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}{\cot^{3}\left(c+dx\right)+3\cot^{2}\left(c+dx\right)+3\cot\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**3,x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)/(cot(c + d*x)**3 + 3*cot(c + d*x)**2 + 3*c
ot(c + d*x) + 1), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)/(a*cot(d*x + c) + a)^3, x)
```

Mupad [B]
time = 0.94, size = 178, normalized size = 1.09

$$\frac{5\,e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{8\,a^{3}\,d}-\frac{\frac{e^{3}\,\sqrt{e\cot\left(c+d\,x\right)}}{8}-\frac{e^{2}\,\left(e\cot(c+d\,x)\right)^{3/2}}{8}}{d\,a^{3}\,e^{2}\cot\left(c+d\,x\right)^{2}+2\,d\,a^{3}\,e^{2}\cot\left(c+d\,x\right)+d\,a^{3}\,e^{2}}-\frac{\sqrt{2}\,e^{3/2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,\left(e\cot(c+d\,x)\right)^{3/2}}{2\,e^{3/2}}\right)\right)}{8\,a^{3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)/(a + a*cot(c + d*x))^3,x)
```

```
[Out] (5*e^(3/2)*atan((e*cot(c + d*x))^(1/2)/e^(1/2)))/(8*a^3*d) - ((e^3*(e*cot(c
 + d*x))^(1/2))/8 - (e^2*(e*cot(c + d*x))^(3/2))/8)/(a^3*d*e^2 + a^3*d*e^2*
cot(c + d*x)^2 + 2*a^3*d*e^2*cot(c + d*x)) - (2^(1/2)*e^(3/2)*(2*atan((2^(1
/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot(c + d*x))
^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)))))/(8*a^
3*d)
```

3.37 $\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\sqrt{e}\,\mathrm{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d}-\frac{\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\,\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2}+\frac{3\sqrt{}}{8d\,(a^3}$$

```
[Out] -1/8*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^3/d-1/4*arctanh(1/2*(e^
(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*e^(1/2)/a^3/d*2^(1/
2)+1/4*(e*cot(d*x+c))^(1/2)/a/d/(a+a*cot(d*x+c))^2+3/8*(e*cot(d*x+c))^(1/2)
/d/(a^3+a^3*cot(d*x+c))
```

Rubi [A]
time = 0.42, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3649, 3730, 3735, 3613, 214, 3715, 65, 211}

$$-\frac{\sqrt{e}\,\mathrm{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d}+\frac{3\sqrt{e\cot(c+dx)}}{8d\,(a^3\cot(c+dx)+a^3)}-\frac{\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\cot(c+dx)+\sqrt{e}}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^3,x]
```

```
[Out] -1/8*(Sqrt[e]*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(a^3*d) - (Sqrt[e]*ArcT
anh[(Sqrt[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])])/(2*Sq
rt[2]*a^3*d) + Sqrt[e*Cot[c + d*x]]/(4*a*d*(a + a*Cot[c + d*x])^2) + (3*Sqr
t[e*Cot[c + d*x]])/(8*d*(a^3 + a^3*Cot[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
```

```
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx = \frac{\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} - \frac{\int \frac{-\frac{ae}{2}-2ae\cot(c+dx)+\frac{3}{2}ae\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))^2}\,dx}{4a^2}$$

$$= \frac{\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{3\sqrt{e\cot(c+dx)}}{8d\,(a^3+a^3\cot(c+dx))} + \frac{\int \frac{\frac{5a^3e^2}{2}-\frac{3}{2}a^3e^2\cot^2(c+}{\sqrt{e\cot(c+dx)}\,(a+a\,c}}{8a^5e}$$

$$= \frac{\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{3\sqrt{e\cot(c+dx)}}{8d\,(a^3+a^3\cot(c+dx))} + \frac{\int \frac{4a^4e^2-4a^4e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{16a^7e}$$

$$= \frac{\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{3\sqrt{e\cot(c+dx)}}{8d\,(a^3+a^3\cot(c+dx))} + \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{-ex}\,(a-ax)}\right.}{16a}$$

$$= -\frac{\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d} + \frac{\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{3\sqrt{e\,c}}{8d\,(a^3+}$$

$$= -\frac{\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} - \frac{\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\,\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d} + \frac{}{4a}$$

Mathematica [A]
time = 0.86, size = 181, normalized size = 1.12

$$-\frac{\sqrt{e\cot(c+dx)}\left(-2\sqrt{2}\left(\log\left(-1+\sqrt{2}\sqrt{\cot(c+dx)}-\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)(\cos(c+dx)+\sin(c+dx))^2+\sqrt{\cot(c+dx)}\,(-5+5\cos(2(c+dx))-3\sin(2(c+dx)))+2\text{ArcTan}\left(\sqrt{\cot(c+dx)}\right)(1+\sin(2(c+dx)))\right)}{16a^3d\sqrt{\cot(c+dx)}\,(\cos(c+dx)+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^3,x]
```

```
[Out] -1/16*(Sqrt[e*Cot[c + d*x]]*(-2*Sqrt[2]*(Log[-1 + Sqrt[2]*Sqrt[Cot[c + d*x]
] - Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*(Co
s[c + d*x] + Sin[c + d*x])^2 + Sqrt[Cot[c + d*x]]*(-5 + 5*Cos[2*(c + d*x)]
- 3*Sin[2*(c + d*x)]) + 2*ArcTan[Sqrt[Cot[c + d*x]]]*(1 + Sin[2*(c + d*x)])
))/(a^3*d*Sqrt[Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(132) = 264.
time = 0.58, size = 349, normalized size = 2.17

method	result
derivativedivides	$-\frac{2e^4\left(\frac{\frac{(e^2)^{\frac14}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac14}}\right.\right.}{8e}}{}\right.}{}$
default	$-\frac{2e^4\left(\frac{\frac{(e^2)^{\frac14}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac14}}\right.\right.}{8e}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^3*e^4*(1/4/e^3*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/
8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(
-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/4/e^3*((3/4*(e*cot(d*x+c))
^(3/2)+5/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)+e)^2-1/4/e^(1/2)*arctan((e
*cot(d*x+c))^(1/2)/e^(1/2))))
```

Maxima [A]
time = 0.49, size = 137, normalized size = 0.85

$$\frac{\left(\frac{\frac{5}{\sqrt{\tan(dx+c)}}+\frac{3}{\tan(dx+c)^{\frac32}}}{a^3+\frac{2a^3}{\tan(dx+c)}+\frac{a^3}{\tan(dx+c)^2}}-\frac{\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^3}-\frac{\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a^3}\right)e^{\frac12}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

[Out] 1/8*((5/sqrt(tan(d*x + c)) + 3/tan(d*x + c)^(3/2))/(a^3 + 2*a^3/tan(d*x + c
) + a^3/tan(d*x + c)^2) - (sqrt(2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d
*x + c) + 1) - sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1
))/a^3 - arctan(1/sqrt(tan(d*x + c)))/a^3)*e^(1/2)/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(122) = 244$.
time = 2.80, size = 246, normalized size = 1.53

$$\frac{2\left(e^{\frac{1}{2}}\sin\left(2\,dx+2\,c\right)+e^{\frac{1}{2}}\right)\arctan\left(\frac{\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)+1}\right)+2\left(\sqrt{2}\,e^{\frac{1}{2}}\sin\left(2\,dx+2\,c\right)+\sqrt{2}\,e^{\frac{1}{2}}\right)\log\left(\left(\sqrt{2}\,\cos\left(2\,dx+2\,c\right)-\sqrt{2}\,\sin\left(2\,dx+2\,c\right)-\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}+2\,\sin\left(2\,dx+2\,c\right)+1\right)-\left(5\,\cos\left(2\,dx+2\,c\right)e^{\frac{1}{2}}-3\,e^{\frac{1}{2}}\sin\left(2\,dx+2\,c\right)-5\,e^{\frac{1}{2}}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{16\left(a^{3}d\sin\left(2\,dx+2\,c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")

[Out] 1/16*(2*(e^(1/2)*sin(2*d*x + 2*c) + e^(1/2))*arctan(sqrt((cos(2*d*x + 2*c)
+ 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c) + 1)) + 2*(sqrt(2
)*e^(1/2)*sin(2*d*x + 2*c) + sqrt(2)*e^(1/2))*log((sqrt(2)*cos(2*d*x + 2*c)
 - sqrt(2)*sin(2*d*x + 2*c) - sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*
x + 2*c)) + 2*sin(2*d*x + 2*c) + 1) - (5*cos(2*d*x + 2*c)*e^(1/2) - 3*e^(1/
2)*sin(2*d*x + 2*c) - 5*e^(1/2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*
c)))/(a^3*d*sin(2*d*x + 2*c) + a^3*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\cot\left(c+dx\right)}}{\cot^{3}\left(c+dx\right)+3\cot^{2}\left(c+dx\right)+3\cot\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**3,x)

[Out] Integral(sqrt(e*cot(c + d*x))/(cot(c + d*x)**3 + 3*cot(c + d*x)**2 + 3*cot(
c + d*x) + 1), x)/a**3

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")

[Out] integrate(sqrt(e*cot(d*x + c))/(a*cot(d*x + c) + a)^3, x)

Mupad [B]
time = 0.90, size = 151, normalized size = 0.94

$$\frac{\frac{3\,e\,{\left(e\cot\left(c+d\,x\right)\right)}^{3/2}}{8}+\frac{5\,e^2\,\sqrt{e\cot\left(c+d\,x\right)}}{8}}{d\,a^3\,e^2\,{\cot\left(c+d\,x\right)}^2+2\,d\,a^3\,e^2\cot\left(c+d\,x\right)+d\,a^3\,e^2}-\frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{8\,a^3\,d}-\frac{\sqrt{2}\,\sqrt{e}\,\mathrm{atanh}\left(\frac{9\,\sqrt{2}\,e^{17/2}\,\sqrt{e\cot\left(c+d\,x\right)}}{32\,\left(\frac{9\,e^9\cot\left(c+d\,x\right)}{32}+\frac{9\,e^9}{32}\right)}\right)}{4\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)/(a + a*cot(c + d*x))^3,x)
```

```
[Out] ((3*e*(e*cot(c + d*x))^(3/2))/8 + (5*e^2*(e*cot(c + d*x))^(1/2))/8)/(a^3*d*
e^2 + a^3*d*e^2*cot(c + d*x)^2 + 2*a^3*d*e^2*cot(c + d*x)) - (e^(1/2)*atan(
(e*cot(c + d*x))^(1/2)/e^(1/2)))/(8*a^3*d) - (2^(1/2)*e^(1/2)*atanh((9*2^(1
/2)*e^(17/2)*(e*cot(c + d*x))^(1/2))/(32*((9*e^9*cot(c + d*x))/32 + (9*e^9)
/32))))/(4*a^3*d)
```

3.38 $\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))^3}\,dx$

Optimal. Leaf size=165

$$-\frac{11\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d\sqrt{e}}-\frac{\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d\sqrt{e}}-\frac{7\sqrt{e\cot(c+dx)}}{8a^3de(1+\cot(c+dx))}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a+a\cot(c+dx))^2}$$

```
[Out] -11/8*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d/e^(1/2)-1/4*arctan(1/2*(e^
(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d*2^(1/2)/e^(1/
2)-7/8*(e*cot(d*x+c))^(1/2)/a^3/d/e/(1+cot(d*x+c))-1/4*(e*cot(d*x+c))^(1/2)
/a/d/e/(a+a*cot(d*x+c))^2
```

Rubi [A]

time = 0.43, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3650, 3730, 3734, 3613, 211, 3715, 65}

$$-\frac{11\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d\sqrt{e}}-\frac{\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d\sqrt{e}}-\frac{7\sqrt{e\cot(c+dx)}}{8a^3de(\cot(c+dx)+1)}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^3),x]
```

```
[Out] (-11*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(8*a^3*d*Sqrt[e]) - ArcTan[(Sqrt
[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])]/(2*Sqrt[2]*a^3*
d*Sqrt[e]) - (7*Sqrt[e*Cot[c + d*x]])/(8*a^3*d*e*(1 + Cot[c + d*x])) - Sqrt
[e*Cot[c + d*x]]/(4*a*d*e*(a + a*Cot[c + d*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
```

```
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
```

```
!GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))^3}\,dx = -\frac{\sqrt{e\cot(c+dx)}}{4ade(a+a\cot(c+dx))^2} - \frac{\int \frac{-\frac{7a^2e}{2}+2a^2e\cot(c+dx)-\frac{3}{2}a^2e\cot^2(c+\cdots}{\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx)\cdots}}{4a^3e}$$

$$= -\frac{7\sqrt{e\cot(c+dx)}}{8a^3de(1+\cot(c+dx))} - \frac{\sqrt{e\cot(c+dx)}}{4ade(a+a\cot(c+dx))^2} + \frac{\int \frac{7a^4}{2}\cdots}{\cdots}$$

$$= -\frac{7\sqrt{e\cot(c+dx)}}{8a^3de(1+\cot(c+dx))} - \frac{\sqrt{e\cot(c+dx)}}{4ade(a+a\cot(c+dx))^2} + \frac{11\int\cdots}{\cdots}$$

$$= -\frac{7\sqrt{e\cot(c+dx)}}{8a^3de(1+\cot(c+dx))} - \frac{\sqrt{e\cot(c+dx)}}{4ade(a+a\cot(c+dx))^2} + \frac{11\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3d\sqrt{e}} - \frac{7\sqrt{e\cot(c+dx)}}{8a^3de(1+\cot(c+dx))} - \cdots$$

$$= -\frac{11\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d\sqrt{e}} - \frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx\cdots}}\right)}{2\sqrt{2}\,a^3d\sqrt{e}}\cdots$$

Mathematica [**A**]
time = 1.32, size = 217, normalized size = 1.32

$$\frac{\sqrt{\cot(c+dx)}\left(-22\mathrm{ArcTan}\left(\sqrt{\cot(c+dx)}\right)-9\sqrt{\cot(c+dx)}+9\cos(2(c+dx))\sqrt{\cot(c+dx)}-4\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)(\cos(c+dx)+\sin(c+dx))^2+4\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)(\cos(c+dx)+\sin(c+dx))^2-22\mathrm{ArcTan}\left(\sqrt{\cot(c+dx)}\right)\sin(2(c+dx))-7\sqrt{\cot(c+dx)}\sin(2(c+dx))\right)}{16a^3d\sqrt{e\cot(c+dx)}\,(\cos(c+dx)+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^3),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*(-22*ArcTan[Sqrt[Cot[c + d*x]]] - 9*Sqrt[Cot[c + d*x]]
+ 9*Cos[2*(c + d*x)]*Sqrt[Cot[c + d*x]] - 4*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]]]*(Cos[c + d*x] + Sin[c + d*x])^2 + 4*Sqrt[2]*ArcTan[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]]]*(Cos[c + d*x] + Sin[c + d*x])^2 - 22*ArcTan[Sqrt[Co
t[c + d*x]]]*Sin[2*(c + d*x)] - 7*Sqrt[Cot[c + d*x]]*Sin[2*(c + d*x)]))/(16
*a^3*d*Sqrt[e*Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x])^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(136) = 272.
time = 0.60, size = 349, normalized size = 2.12

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2e^4\left(-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^3*e^4*(1/4/e^4*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1
/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2
^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan
(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+1/4/e^4*((7/4*(e*cot(d*x+c)
)^(3/2)+9/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)+e)^2+11/4/e^(1/2)*arctan(
(e*cot(d*x+c))^(1/2)/e^(1/2))))
```

Maxima [A]

time = 0.54, size = 129, normalized size = 0.78

$$-\frac{\left(\frac{\frac{9}{\sqrt{\tan(dx+c)}}+\frac{7}{\tan(dx+c)^{\frac{3}{2}}}}{a^3+\frac{2a^3}{\tan(dx+c)}+\frac{a^3}{\tan(dx+c)^2}}-\frac{2\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)\right)}{a^3}+\frac{11\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a^3}\right)e^{\left(-\frac{1}{2}\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((9/sqrt(tan(d*x + c)) + 7/tan(d*x + c)^(3/2))/(a^3 + 2*a^3/tan(d*x +
c) + a^3/tan(d*x + c)^2) - 2*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(
tan(d*x + c)))) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c
)))))/a^3 + 11*arctan(1/sqrt(tan(d*x + c)))/a^3)*e^(-1/2)/d
```

Fricas [A]
time = 3.50, size = 233, normalized size = 1.41

$$-\frac{4 \left(\sqrt{2}\, \sin \left(2 d x+2 c\right)+\sqrt{2}\right) \arctan \left(-\frac{\left(\sqrt{2}\, \cos \left(2 d x+2 c\right)-\sqrt{2}\, \sin \left(2 d x+2 c\right)+\sqrt{2}\right) \sqrt{\frac{\cos \left(2 d x+2 c\right)+1}{\sin \left(2 d x+2 c\right)}}}{2 \left(\cos \left(2 d x+2 c\right)+1\right)}\right)-22 \left(\sin \left(2 d x+2 c\right)+1\right) \arctan \left(\frac{\sqrt{\frac{\cos \left(2 d x+2 c\right)+1}{\sin \left(2 d x+2 c\right)}}\, \sin \left(2 d x+2 c\right)}{\cos \left(2 d x+2 c\right)+1}\right)-\sqrt{\frac{\cos \left(2 d x+2 c\right)+1}{\sin \left(2 d x+2 c\right)}}\, \left(9 \cos \left(2 d x+2 c\right)-7 \sin \left(2 d x+2 c\right)-9\right)}{16 \left(a^{3} d e^{\frac{1}{2}} \sin \left(2 d x+2 c\right)+a^{3} d e^{\frac{1}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(4*(sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan(-1/2*(sqrt(2)*cos(2*d*
x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/
sin(2*d*x + 2*c))/(cos(2*d*x + 2*c) + 1)) - 22*(sin(2*d*x + 2*c) + 1)*arcta
n(sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c)/(cos(2*d*x
 + 2*c) + 1)) - sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*(9*cos(2*d*x
+ 2*c) - 7*sin(2*d*x + 2*c) - 9))/(a^3*d*e^(1/2)*sin(2*d*x + 2*c) + a^3*d*e
^(1/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e \cot \left(c+d x\right)} \cot ^{3}\left(c+d x\right)+3 \sqrt{e \cot \left(c+d x\right)} \cot ^{2}\left(c+d x\right)+3 \sqrt{e \cot \left(c+d x\right)} \cot \left(c+d x\right)+\sqrt{e \cot \left(c+d x\right)}}\, dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**3,x)
```

```
[Out] Integral(1/(sqrt(e*cot(c + d*x))*cot(c + d*x)**3 + 3*sqrt(e*cot(c + d*x))*c
ot(c + d*x)**2 + 3*sqrt(e*cot(c + d*x))*cot(c + d*x) + sqrt(e*cot(c + d*x))
), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)^3*sqrt(e*cot(d*x + c))), x)
```

Mupad [B]
time = 0.94, size = 173, normalized size = 1.05

$$\frac{\sqrt{2}\, \left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\,\cot \left(c+d\,x\right)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\,\cot \left(c+d\,x\right)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,{\left(e\,\cot \left(c+d\,x\right)\right)}^{3/2}}{2\,e^{3/2}}\right)\right)}{8\,a^3\,d\,\sqrt{e}}-\frac{11\,\mathrm{atan}\left(\frac{\sqrt{e\,\cot \left(c+d\,x\right)}}{\sqrt{e}}\right)}{8\,a^3\,d\,\sqrt{e}}-\frac{\frac{9\,e\,\sqrt{e\,\cot \left(c+d\,x\right)}}{8}+\frac{7\,{\left(e\,\cot \left(c+d\,x\right)\right)}^{3/2}}{8}}{d\,a^3\,e^2\,{\cot \left(c+d\,x\right)}^2+2\,d\,a^3\,e^2\,\cot \left(c+d\,x\right)+d\,a^3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^3),x)
```

```
[Out] (2^(1/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2^
(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/2)
)/(2*e^(3/2)))))/(8*a^3*d*e^(1/2)) - (11*atan((e*cot(c + d*x))^(1/2)/e^(1/2
)))/(8*a^3*d*e^(1/2)) - ((9*e*(e*cot(c + d*x))^(1/2))/8 + (7*(e*cot(c + d*x
))^(3/2))/8)/(a^3*d*e^2 + a^3*d*e^2*cot(c + d*x)^2 + 2*a^3*d*e^2*cot(c + d*
x))
```

3.39 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx$

Optimal. Leaf size=189

$$\frac{31\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3de^{3/2}}+\frac{27}{8a^3de\sqrt{e\cot(c+dx)}}-\frac{}{8a^3de\sqrt{e\cot(c-}}$$

```
[Out] 31/8*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d/e^(3/2)+1/4*arctanh(1/2*(e^
(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d/e^(3/2)*2^(1/
2)+27/8/a^3/d/e/(e*cot(d*x+c))^(1/2)-9/8/a^3/d/e/(1+cot(d*x+c))/(e*cot(d*x+
c))^(1/2)-1/4/a/d/e/(a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3650, 3730, 3735, 3613, 214, 3715, 65, 211}

$$\frac{31\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\cot(c+dx)+\sqrt{e}}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3de^{3/2}}+\frac{27}{8a^3de\sqrt{e\cot(c+dx)}}-\frac{9}{8a^3de(\cot(c+dx)+1)\sqrt{e\cot(c+dx)}}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^3),x]
```

```
[Out] (31*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(8*a^3*d*e^(3/2)) + ArcTanh[(Sqrt
[e] + Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])]/(2*Sqrt[2]*a^3*
d*e^(3/2)) + 27/(8*a^3*d*e*Sqrt[e*Cot[c + d*x]]) - 9/(8*a^3*d*e*Sqrt[e*Cot[
c + d*x]]*(1 + Cot[c + d*x])) - 1/(4*a*d*e*Sqrt[e*Cot[c + d*x]]*(a + a*Cot[
c + d*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
```

```
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = -\frac{1}{4ade\sqrt{e\cot(c+dx)}\,(a+a\cot(c+dx))^2} - \frac{\int \frac{-\frac{9a^2e}{2}+2a^2e\,\mathrm{co}\ldots}{(e\cot(c+dx\ldots}}{\ldots}$$

$$= -\frac{9}{8a^3de\sqrt{e\cot(c+dx)}\,(1+\cot(c+dx))} - \frac{\ldots}{4ade\sqrt{e\cot(c+\ldots}}$$

$$= \frac{27}{8a^3de\sqrt{e\cot(c+dx)}} - \frac{9}{8a^3de\sqrt{e\cot(c+dx)}\,(1+\cot(c+\ldots}$$

$$= \frac{27}{8a^3de\sqrt{e\cot(c+dx)}} - \frac{9}{8a^3de\sqrt{e\cot(c+dx)}\,(1+\cot(c+\ldots}$$

$$= \frac{27}{8a^3de\sqrt{e\cot(c+dx)}} - \frac{9}{8a^3de\sqrt{e\cot(c+dx)}\,(1+\cot(c+\ldots}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}\,a^3de^{3/2}} + \frac{27}{8a^3de\sqrt{e\cot(c+dx)}} - \ldots$$

$$= \frac{31\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx\ldots}{\sqrt{2}\sqrt{e\cot(c+dx\ldots}}\right)}{2\sqrt{2}\,a^3de^{3/2}}\ldots$$

Mathematica [A]
time = 1.23, size = 156, normalized size = 0.83

$$\frac{\cot^{\frac{3}{2}}(c+dx)\left(62\mathrm{ArcTan}\left(\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\left(\log\left(-1+\sqrt{2}\sqrt{\cot(c+dx)}-\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)+\frac{43+11\cos(2(c+dx))+45\sin(2(c+dx))}{\sqrt{\cot(c+dx)}\,(\cos(c+dx)+\sin(c+dx))^2}\right)}{16a^3d(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^3),x]
```

```
[Out] (Cot[c + d*x]^(3/2)*(62*ArcTan[Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*(Log[-1 + Sq
rt[2]*Sqrt[Cot[c + d*x]] - Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x
]] + Cot[c + d*x]]) + (43 + 11*Cos[2*(c + d*x)] + 45*Sin[2*(c + d*x)])/(Sqr
```

```
t[Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x])^2)))/(16*a^3*d*(e*Cot[c + d*x
])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(156) = 312$.
time = 0.59, size = 364, normalized size = 1.93

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot\left(dx+\right.}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2e^4\left(-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot\left(dx+\right.}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^3*e^4*(1/4/e^5*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1
/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2
^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan
(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/e^5/(e*cot(d*x+c))^(1/2)-
1/4/e^5*((11/4*(e*cot(d*x+c))^(3/2)+13/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x
+c)+e)^2+31/4/e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))))
```

Maxima [A]
time = 0.50, size = 146, normalized size = 0.77

$$\frac{\left(\frac{\frac{45}{\tan(dx+c)}+\frac{27}{\tan(dx+c)^2}+16}{\frac{a^3}{\sqrt{\tan\left(dx+c\right)}}+\frac{2a^3}{\tan(dx+c)^{\frac{3}{2}}}+\frac{a^3}{\tan(dx+c)^{\frac{5}{2}}}}+\frac{\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^3}+\frac{31\arctan\left(\frac{1}{\sqrt{\tan\left(dx+c\right)}}\right)}{a^3}\right)e^{\left(-\frac{3}{2}\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*((45/tan(d*x + c) + 27/tan(d*x + c)^2 + 16)/(a^3/sqrt(tan(d*x + c)) + 2
*a^3/tan(d*x + c)^(3/2) + a^3/tan(d*x + c)^(5/2)) + (sqrt(2)*log(sqrt(2)/sq
rt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)/sqrt(tan(d*x
+ c)) + 1/tan(d*x + c) + 1))/a^3 + 31*arctan(1/sqrt(tan(d*x + c)))/a^3)*e^(
-3/2)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(137) = 274$.
time = 2.31, size = 331, normalized size = 1.75

$$-\frac{62\left(\left(\cos\left(2\,dx+2\,c\right)+1\right)\sin\left(2\,dx+2\,c\right)+\cos\left(2\,dx+2\,c\right)+1\right)\arctan\left(\frac{\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\sin\left(2\,dx+2\,c\right)}{\cos\left(2\,dx+2\,c\right)+1}\right)-2\left(\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)+\sqrt{2}\right)\sin\left(2\,dx+2\,c\right)+\sqrt{2}\cos\left(2\,dx+2\,c\right)+\sqrt{2}\right)\log\left(-\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)-\sqrt{2}\sin\left(2\,dx+2\,c\right)-\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}+2\sin\left(2\,dx+2\,c\right)+1\right)+\left(45\cos\left(2\,dx+2\,c\right)^2-\left(11\cos\left(2\,dx+2\,c\right)+43\right)\sin\left(2\,dx+2\,c\right)-45\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{16\left(a^3d\cos\left(2\,dx+2\,c\right)e^{\frac{3}{2}}+a^3de^{\frac{3}{2}}+\left(a^3d\cos\left(2\,dx+2\,c\right)e^{\frac{3}{2}}+a^3de^{\frac{3}{2}}\right)\sin\left(2\,dx+2\,c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(62*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + 1)*
arctan(sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c)/(cos(
2*d*x + 2*c) + 1)) - 2*((sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(2*d*x + 2*
c) + sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(-(sqrt(2)*cos(2*d*x + 2*c) - s
qrt(2)*sin(2*d*x + 2*c) - sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x +
2*c)) + 2*sin(2*d*x + 2*c) + 1) + (45*cos(2*d*x + 2*c)^2 - (11*cos(2*d*x +
2*c) + 43)*sin(2*d*x + 2*c) - 45)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2
*c)))/(a^3*d*cos(2*d*x + 2*c)*e^(3/2) + a^3*d*e^(3/2) + (a^3*d*cos(2*d*x +
2*c)*e^(3/2) + a^3*d*e^(3/2))*sin(2*d*x + 2*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\cot^3(c+dx)+3(e\cot(c+dx))^{\frac{3}{2}}\cot^2(c+dx)+3(e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**3,x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*cot(c + d*x)**3 + 3*(e*cot(c + d*x))**(
3/2)*cot(c + d*x)**2 + 3*(e*cot(c + d*x))**(3/2)*cot(c + d*x) + (e*cot(c +
d*x))**(3/2)), x)/a**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(3/2)), x)
```

Mupad [B]
time = 1.14, size = 175, normalized size = 0.93

$$\frac{\frac{27\,e\cot(c+d\,x)^2}{8}+\frac{45\,e\cot(c+d\,x)}{8}+2\,e}{a^3\,d\,{\left(e\cot\left(c+d\,x\right)\right)}^{5/2}+2\,a^3\,d\,e\,{\left(e\cot\left(c+d\,x\right)\right)}^{3/2}+a^3\,d\,e^2\,\sqrt{e\cot\left(c+d\,x\right)}}+\frac{31\,\mathrm{atan}\left(\frac{\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{8\,a^3\,d\,e^{3/2}}+\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{63504384\,\sqrt{2}\,a^9\,d^3\,e^{15/2}\,\sqrt{e\cot\left(c+d\,x\right)}}{63504384\,a^9\,d^3\,e^8+63504384\,a^9\,d^3\,e^8\cot(c+d\,x)}\right)}{4\,a^3\,d\,e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^3),x)
```

```
[Out] (2*e + (45*e*cot(c + d*x))/8 + (27*e*cot(c + d*x)^2)/8)/(a^3*d*(e*cot(c + d
*x))^(5/2) + 2*a^3*d*e*(e*cot(c + d*x))^(3/2) + a^3*d*e^2*(e*cot(c + d*x))^
(1/2)) + (31*atan((e*cot(c + d*x))^(1/2)/e^(1/2)))/(8*a^3*d*e^(3/2)) + (2^(
1/2)*atanh((63504384*2^(1/2)*a^9*d^3*e^(15/2)*(e*cot(c + d*x))^(1/2))/(6350
4384*a^9*d^3*e^8 + 63504384*a^9*d^3*e^8*cot(c + d*x))))/(4*a^3*d*e^(3/2))
```

3.40 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx$

Optimal. Leaf size=215

$$-\frac{59\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{5/2}}+\frac{\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3de^{5/2}}+\frac{55}{24a^3de(e\cot(c+dx))^{3/2}}-\frac{6}{8a^3de^2\sqrt{e\,c}}$$

```
[Out] -59/8*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d/e^(5/2)+55/24/a^3/d/e/(e*c
ot(d*x+c))^(3/2)-11/8/a^3/d/e/(e*cot(d*x+c))^(3/2)/(1+cot(d*x+c))-1/4/a/d/e
/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2+1/4*arctan(1/2*(e^(1/2)-cot(d*x+c)
*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d/e^(5/2)*2^(1/2)-63/8/a^3/d/e^
2/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.72, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3650, 3730, 3731, 3734, 3613, 211, 3715, 65}

$$-\frac{59\text{ArcTan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{5/2}}+\frac{\text{ArcTan}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3de^{5/2}}-\frac{63}{8a^3de^2\sqrt{e\cot(c+dx)}}-\frac{11}{8a^3de(\cot(c+dx)+1)(e\cot(c+dx))^{3/2}}+\frac{55}{24a^3de(e\cot(c+dx))^{3/2}}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3),x]
```

```
[Out] (-59*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]])/(8*a^3*d*e^(5/2)) + ArcTan[(Sqrt
[e] - Sqrt[e]*Cot[c + d*x])/(Sqrt[2]*Sqrt[e*Cot[c + d*x]])]/(2*Sqrt[2]*a^3*
d*e^(5/2)) + 55/(24*a^3*d*e*(e*Cot[c + d*x])^(3/2)) - 63/(8*a^3*d*e^2*Sqrt[
e*Cot[c + d*x]]) - 11/(8*a^3*d*e*(e*Cot[c + d*x])^(3/2)*(1 + Cot[c + d*x]))
 - 1/(4*a*d*e*(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3613

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*c*d + b*x^2), x], x, (c -
 d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
```

```
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = -\frac{1}{4ade(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2} - \frac{\int \frac{-\frac{11a^2e}{2}+2a^2e\cot\cdots}{(e\cot(c+dx\cdots}}{\cdots}$$

$$= -\frac{11}{8a^3de(e\cot(c+dx))^{3/2}(1+\cot(c+dx))} - \frac{\cdots}{4ade(e\cot(c+\cdots}$$

$$= \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} - \frac{11}{8a^3de(e\cot(c+dx))^{3/2}(1+\cot(c\cdots}$$

$$= \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} - \frac{63}{8a^3de^2\sqrt{e\cot(c+dx)}} - \frac{\cdots}{8a^3de(e\cdots}$$

$$= \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} - \frac{63}{8a^3de^2\sqrt{e\cot(c+dx)}} - \frac{\cdots}{8a^3de(e\cdots}$$

$$= \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} - \frac{63}{8a^3de^2\sqrt{e\cot(c+dx)}} - \frac{\cdots}{8a^3de(e\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3de^{5/2}} + \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} - \cdots$$

$$= -\frac{59\tan^{-1}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{5/2}} + \frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)\cdots}}\right)}{2\sqrt{2}a^3de^{5/2}}\cdots$$

Mathematica [A]
time = 3.31, size = 167, normalized size = 0.78

$$\frac{\cot^{\frac{5}{2}}(c+dx)\left(4\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-4\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)-118\text{ArcTan}\left(\sqrt{\cot(c+dx)}\right)-\frac{\sqrt{\cot(c+dx)}(614+678\cos(2(c+dx))+679\cot(c+dx)+77\cos(3(c+dx))\csc(c+dx))\sec^2(c+dx)}{6(1+\cot(c+dx))^2}\right)}{16a^3d(e\cot(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3),x]
```

```
[Out] (Cot[c + d*x]^(5/2)*(4*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 4*S
qrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] - 118*ArcTan[Sqrt[Cot[c + d*x
]]] - (Sqrt[Cot[c + d*x]]*(614 + 678*Cos[2*(c + d*x)] + 679*Cot[c + d*x] +
77*Cos[3*(c + d*x)]*Csc[c + d*x])*Sec[c + d*x]^2)/(6*(1 + Cot[c + d*x])^2))
)/(16*a^3*d*(e*Cot[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(178) = 356$.
time = 0.61, size = 379, normalized size = 1.76

method	result
derivativedivides	$-\frac{2e^4\left(\frac{\frac{\frac{15(e\cot(dx+c))^{\frac{3}{2}}}{4}+\frac{17e\sqrt{e\cot(dx+c)}}{4}}{(e\cot(dx+c)+e)^2}+\frac{59\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{4\sqrt{e}}}{4e^6}+\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx\cdots}{e\cot(dx\cdots}\right.\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2e^4\left(\frac{\frac{\frac{15(e\cot(dx+c))^{\frac{3}{2}}}{4}+\frac{17e\sqrt{e\cot(dx+c)}}{4}}{(e\cot(dx+c)+e)^2}+\frac{59\arctan\left(\frac{\sqrt{e\cot(dx+c)}}{\sqrt{e}}\right)}{4\sqrt{e}}}{4e^6}+\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx\cdots}{e\cot(dx\cdots}\right.\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^3*e^4*(1/4/e^6*((15/4*(e*cot(d*x+c))^(3/2)+17/4*e*(e*cot(d*x+c))^(1/
2))/(e*cot(d*x+c)+e)^2+59/4/e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2)))+1
/4/e^6*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8/(e^2)^(1/4)*2
^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*
arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/3/e^5/(e*cot(d*x+c))^(3/2)+3/e^6/(e*cot(d
*x+c))^(1/2))
```

<u>Maxima</u> [A]

time = 0.51, size = 149, normalized size = 0.69

$$-\frac{\left(\frac{\frac{112}{\tan(dx+c)}+\frac{323}{\tan(dx+c)^2}+\frac{189}{\tan(dx+c)^3}-16}{\frac{a^3}{\tan(dx+c)^{\frac{3}{2}}}+\frac{2a^3}{\tan(dx+c)^{\frac{5}{2}}}+\frac{a^3}{\tan(dx+c)^{\frac{7}{2}}}}+\frac{6\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)\right)}{a^3}+\frac{177\arctan\left(\frac{1}{\sqrt{\tan(dx+c)}}\right)}{a^3}\right)e^{\left(-\frac{5}{2}\right)}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/24*((112/tan(d*x + c) + 323/tan(d*x + c)^2 + 189/tan(d*x + c)^3 - 16)/(a
^3/tan(d*x + c)^(3/2) + 2*a^3/tan(d*x + c)^(5/2) + a^3/tan(d*x + c)^(7/2))
```

```
+ 6*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))))/a^3 + 177*arctan(1/
sqrt(tan(d*x + c)))/a^3)*e^(-5/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(157) = 314$.
time = 3.39, size = 340, normalized size = 1.58

$$\frac{12\left(\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)+\sqrt{2}\right)\sin\left(2\,dx+2\,c\right)+\sqrt{2}\cos\left(2\,dx+2\,c\right)+\sqrt{2}\right)\arctan\left(-\frac{\left(\sqrt{2}\cos(2\,dx+2\,c)-\sqrt{2}\sin(2\,dx+2\,c)+\sqrt{2}\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{2\left(\cos(2\,dx+2\,c)+1\right)}\right)+354\left(\left(\cos\left(2\,dx+2\,c\right)+1\right)\sin\left(2\,dx+2\,c\right)+\cos\left(2\,dx+2\,c\right)+1\right)\arctan\left(\frac{\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)+1}\right)+\left(339\cos\left(2\,dx+2\,c\right)^{2}-7\left(11\cos\left(2\,dx+2\,c\right)+43\right)\sin\left(2\,dx+2\,c\right)-32\cos\left(2\,dx+2\,c\right)-307\right)\sqrt{\frac{\cos\left(2\,dx+2\,c\right)+1}{\sin\left(2\,dx+2\,c\right)}}}{48\left(a^{3}d\cos\left(2\,dx+2\,c\right)e^{\frac{5}{2}}+a^{3}de^{\frac{5}{2}}+\left(a^{3}d\cos\left(2\,dx+2\,c\right)e^{\frac{5}{2}}+a^{3}de^{\frac{5}{2}}\right)\sin\left(2\,dx+2\,c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(12*((sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(2*d*x + 2*c) + sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*arctan(-1/2*(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*
sin(2*d*x + 2*c) + sqrt(2))*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))/(
cos(2*d*x + 2*c) + 1)) + 354*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos
(2*d*x + 2*c) + 1)*arctan(sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin
(2*d*x + 2*c)/(cos(2*d*x + 2*c) + 1)) + (339*cos(2*d*x + 2*c)^2 - 7*(11*cos
(2*d*x + 2*c) + 43)*sin(2*d*x + 2*c) - 32*cos(2*d*x + 2*c) - 307)*sqrt((cos
(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)))/(a^3*d*cos(2*d*x + 2*c)*e^(5/2) + a^3
*d*e^(5/2) + (a^3*d*cos(2*d*x + 2*c)*e^(5/2) + a^3*d*e^(5/2))*sin(2*d*x + 2
*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\cot^{3}(c+dx)+3(e\cot(c+dx))^{\frac{5}{2}}\cot^{2}(c+dx)+3(e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{5}{2}}}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**3,x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(5/2)*cot(c + d*x)**3 + 3*(e*cot(c + d*x))**(
5/2)*cot(c + d*x)**2 + 3*(e*cot(c + d*x))**(5/2)*cot(c + d*x) + (e*cot(c +
d*x))**(5/2)), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(5/2)), x)
```

Mupad [B]
time = 1.38, size = 193, normalized size = 0.90

$$-\frac{\frac{63\,e\,\cot(c+d\,x)^3}{8}+\frac{323\,e\,\cot(c+d\,x)^2}{24}+\frac{14\,e\,\cot(c+d\,x)}{3}-\frac{2\,e}{3}}{a^3\,d\,\left(e\cot\left(c+d\,x\right)\right)^{7/2}+2\,a^3\,d\,e\,\left(e\cot\left(c+d\,x\right)\right)^{5/2}+a^3\,d\,e^2\,\left(e\cot\left(c+d\,x\right)\right)^{3/2}}-\frac{59\,\mathrm{atan}\left(\frac{\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{8\,a^3\,d\,e^{5/2}}-\frac{\sqrt{2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot\left(c+d\,x\right)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,{\left(e\cot(c+d\,x)\right)}^{3/2}}{2\,e^{3/2}}\right)\right)}{8\,a^3\,d\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))^3),x)
```

```
[Out] - ((14*e*cot(c + d*x))/3 - (2*e)/3 + (323*e*cot(c + d*x)^2)/24 + (63*e*cot(
c + d*x)^3)/8)/(a^3*d*(e*cot(c + d*x))^(7/2) + 2*a^3*d*e*(e*cot(c + d*x))^(
5/2) + a^3*d*e^2*(e*cot(c + d*x))^(3/2)) - (59*atan((e*cot(c + d*x))^(1/2)/
e^(1/2)))/(8*a^3*d*e^(5/2)) - (2^(1/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1
/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (
2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)))))/(8*a^3*d*e^(5/2))
```

3.41 $\int \cot^2(x)\sqrt{1+\cot(x)}\,dx$

Optimal. Leaf size=223

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)+\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(-\right.}}\right.$$

```
[Out] -2/3*(1+cot(x))^(3/2)-1/2*arctan((-2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/
(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2*arctan((2*(1+cot(x))^(1/2)+(2
+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2*ln(1+cot(x
)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)-1/2*ln(
1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3624, 3566, 714, 1141, 1175, 632, 210, 1178, 642}

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{2}{3}(\cot(x)+1)^{3/2}+\frac{\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*Sqrt[1 + Cot[x]],x]
```

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Cot[x]])
/Sqrt[2*(-1 + Sqrt[2])]]) + Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[
2])] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]] - (2*(1 + Cot[x])^(3/2))
/3 + Log[1 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/(2*
Sqrt[2*(1 + Sqrt[2])]) - Log[1 + Sqrt[2] + Cot[x] + Sqrt[2*(1 + Sqrt[2])]*S
qrt[1 + Cot[x]]]/(2*Sqrt[2*(1 + Sqrt[2])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
```

```
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \cot^2(x)\sqrt{1+\cot(x)}\,dx = -\frac{2}{3}(1+\cot(x))^{3/2} - \int \sqrt{1+\cot(x)}\,dx$$
$$= -\frac{2}{3}(1+\cot(x))^{3/2} + \text{Subst}\left(\int \frac{\sqrt{1+x}}{1+x^2}\,dx, x, \cot(x)\right)$$
$$= -\frac{2}{3}(1+\cot(x))^{3/2} + 2\text{Subst}\left(\int \frac{x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+\cot(x)}\right)$$
$$= -\frac{2}{3}(1+\cot(x))^{3/2} - \text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+\cot(x)}\right) + \text{Subst}\Big($$

$$= -\frac{2}{3}(1+\cot(x))^{3/2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+\text{co}\right.$$

$$= -\frac{2}{3}(1+\cot(x))^{3/2} + \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

$$= \frac{\tan^{-1}\left(\frac{-\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1\ldots}}{\sqrt{2\left(-1+\sqrt{2}\right.}}\right.}{\sqrt{2\left(-1+\sqrt{2}\right)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 69, normalized size = 0.31

$$-i\sqrt{1-i}\,\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right) + i\sqrt{1+i}\,\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right) - \frac{2}{3}(1+\cot(x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*Sqrt[1 + Cot[x]],x]
```

[Out] (-I)*Sqrt[1 - I]*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] + I*Sqrt[1 + I]*ArcT
anh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]] - (2*(1 + Cot[x])^(3/2))/3

Maple [A]
time = 0.51, size = 197, normalized size = 0.88

method	result
derivativedivides	$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3}-\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(-\frac{\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}-\cdots\right.}{2}$
default	$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3}-\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(-\frac{\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}-\cdots\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^2*(1+cot(x))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/3*(1+cot(x))^(3/2)-1/2*(2+2*2^(1/2))^(1/2)*(2^(1/2)-1)*(-1/2*ln(1+cot(x)
+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-(2+2*2^(1/2))^(1/2)/(-2+2*2^
(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))
^(1/2)))-1/2*(2+2*2^(1/2))^(1/2)*(2^(1/2)-1)*(1/2*ln(1+cot(x)+2^(1/2)+(1+co
t(x))^(1/2)*(2+2*2^(1/2))^(1/2))-(2+2*2^(1/2))^(1/2)/(-2+2*2^(1/2))^(1/2)*a
rctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2*(1+cot(x))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(cot(x) + 1)*cot(x)^2, x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2*(1+cot(x))^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cot(x) + 1}\,\cot^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2*(1+cot(x))**(1/2),x)
```

```
[Out] Integral(sqrt(cot(x) + 1)*cot(x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(1+cot(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cot(x) + 1)*cot(x)^2, x)
```

Mupad [B]
time = 0.63, size = 119, normalized size = 0.53

$$\operatorname{atanh}\left(4\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}+\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}+2\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)-\frac{2\,(\cot(x)+1)^{3/2}}{3}+\operatorname{atanh}\left(4\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}-\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}-2\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(cot(x) + 1)^(1/2),x)
```

```
[Out] atanh(4*(cot(x) + 1)^(1/2)*((- 2^(1/2)/8 - 1/8)^(1/2) + (2^(1/2)/8 - 1/8)^(
1/2))^3)*(2*(- 2^(1/2)/8 - 1/8)^(1/2) + 2*(2^(1/2)/8 - 1/8)^(1/2)) - (2*(co
t(x) + 1)^(3/2))/3 + atanh(4*(cot(x) + 1)^(1/2)*((- 2^(1/2)/8 - 1/8)^(1/2)
- (2^(1/2)/8 - 1/8)^(1/2))^3)*(2*(- 2^(1/2)/8 - 1/8)^(1/2) - 2*(2^(1/2)/8 -
 1/8)^(1/2))
```

3.42 $\int \cot(x)\sqrt{1+\cot(x)}\,dx$

Optimal. Leaf size=135

$$\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\cot(x)}{2\sqrt{-7+5\sqrt{2}}\,\sqrt{1+\cot(x)}}\right)+\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{4+3\sqrt{2}+\left(2+\right.}{2\sqrt{7+5\sqrt{2}}\,\sqrt{}}\right.$$

```
[Out] -2*(1+cot(x))^(1/2)+1/2*arctan(1/2*(4+cot(x)*(2-2^(1/2))-3*2^(1/2))/(1+cot(
x))^(1/2)/(-7+5*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)+1/2*arctanh(1/2*(4+3*2
^(1/2)+cot(x)*(2+2^(1/2)))/(1+cot(x))^(1/2)/(7+5*2^(1/2))^(1/2))*(2+2*2^(1/
2))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {3609, 3617, 3616, 209, 213}

$$\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\,\text{ArcTan}\left(\frac{\left(2-\sqrt{2}\right)\cot(x)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\,\sqrt{\cot(x)+1}}\right)-2\sqrt{\cot(x)+1}+\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\tanh^{-1}\left(\frac{\left(2+\sqrt{2}\right)\cot(x)+3\sqrt{2}+4}{2\sqrt{7+5\sqrt{2}}\,\sqrt{\cot(x)+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[1 + Cot[x]],x]
```

```
[Out] Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(4 - 3*Sqrt[2] + (2 - Sqrt[2])*Cot[x])/(2*Sqr
t[-7 + 5*Sqrt[2]]*Sqrt[1 + Cot[x]])] + Sqrt[(1 + Sqrt[2])/2]*ArcTanh[(4 + 3
*Sqrt[2] + (2 + Sqrt[2])*Cot[x])/(2*Sqrt[7 + 5*Sqrt[2]]*Sqrt[1 + Cot[x]])]
- 2*Sqrt[1 + Cot[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rubi steps

$$
\begin{aligned}
\int \cot(x)\sqrt{1+\cot(x)}\,dx &= -2\sqrt{1+\cot(x)}-\int \frac{1-\cot(x)}{\sqrt{1+\cot(x)}}\,dx\\
&= -2\sqrt{1+\cot(x)}+\frac{\int \frac{-\sqrt{2}-\left(-2-\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}-\left(-2+\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{2\sqrt{2}}\\
&= -2\sqrt{1+\cot(x)}+\left(-4+3\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-2\sqrt{2}\left(-2+\sqrt{2}\right)-4\left(-2+\sqrt{\right.}}\right.\\
&= \sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\tan^{-1}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\cot(x)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right)+\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 61, normalized size = 0.45

$$\sqrt{1-i}\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right)+\sqrt{1+i}\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right)-2\sqrt{1+\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[1 + Cot[x]],x]
```

```
[Out] Sqrt[1 - I]*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] + Sqrt[1 + I]*ArcTanh[Sqr
t[1 + Cot[x]]/Sqrt[1 + I]] - 2*Sqrt[1 + Cot[x]]
```

Maple [A]
time = 0.28, size = 174, normalized size = 1.29

method	result
derivativedivides	$-2\sqrt{1+\cot\left(x\right)}+\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}+\sqrt{1+\cot\left(x\right)}\,\sqrt{2+2\sqrt{2}}\right)}{4}-\frac{(1-\sqrt{}}{}$
default	$-2\sqrt{1+\cot\left(x\right)}+\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}+\sqrt{1+\cot\left(x\right)}\,\sqrt{2+2\sqrt{2}}\right)}{4}-\frac{(1-\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(1+cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1+cot(x))^(1/2)+1/4*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^
(1/2)*(2+2*2^(1/2))^(1/2))-(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+co
t(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))^(1
/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+(2^(1/2)-1)/(
-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2
^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+cot(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cot(x) + 1)*cot(x), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+cot(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cot{\left(x \right)} + 1} \cot{\left(x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+cot(x))**(1/2),x)
```

```
[Out] Integral(sqrt(cot(x) + 1)*cot(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+cot(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cot(x) + 1)*cot(x), x)
```

Mupad [B]
time = 0.48, size = 210, normalized size = 1.56

$$\mathrm{atanh}\left(\frac{\sqrt{\cot\left(x\right)+1}}{4\,\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}+\frac{\sqrt{\cot\left(x\right)+1}}{4\,\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}-\frac{\sqrt{2}\,\sqrt{\cot\left(x\right)+1}}{8\,\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}+\frac{\sqrt{2}\,\sqrt{\cot\left(x\right)+1}}{8\,\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\,\left(2\,\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}+2\,\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right)-\mathrm{atanh}\left(\frac{\sqrt{\cot\left(x\right)+1}}{4\,\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}-\frac{\sqrt{\cot\left(x\right)+1}}{4\,\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}+\frac{\sqrt{2}\,\sqrt{\cot\left(x\right)+1}}{8\,\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}+\frac{\sqrt{2}\,\sqrt{\cot\left(x\right)+1}}{8\,\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\,\left(2\,\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}-2\,\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right)-2\,\sqrt{\cot\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(cot(x) + 1)^(1/2),x)
```

```
[Out] atanh((cot(x) + 1)^(1/2)/(4*(1/8 - 2^(1/2)/8)^(1/2)) + (cot(x) + 1)^(1/2)/(
4*(2^(1/2)/8 + 1/8)^(1/2)) - (2^(1/2)*(cot(x) + 1)^(1/2))/(8*(1/8 - 2^(1/2)
/8)^(1/2)) + (2^(1/2)*(cot(x) + 1)^(1/2))/(8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(
1/8 - 2^(1/2)/8)^(1/2) + 2*(2^(1/2)/8 + 1/8)^(1/2)) - atanh((cot(x) + 1)^(1
/2)/(4*(2^(1/2)/8 + 1/8)^(1/2)) - (cot(x) + 1)^(1/2)/(4*(1/8 - 2^(1/2)/8)^(
1/2)) + (2^(1/2)*(cot(x) + 1)^(1/2))/(8*(1/8 - 2^(1/2)/8)^(1/2)) + (2^(1/2)
*(cot(x) + 1)^(1/2))/(8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(1/8 - 2^(1/2)/8)^(1/2
) - 2*(2^(1/2)/8 + 1/8)^(1/2)) - 2*(cot(x) + 1)^(1/2)
```

3.43 $\int \cot^2(x)(1+\cot(x))^{3/2}\,dx$

Optimal. Leaf size=139

$$-\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\,\sqrt{1+\cot(x)}}\right)-\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{3+2\sqrt{2}+\left(1+\sqrt{2}\right.}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\sqrt{1+}}\right.$$

```
[Out] -2/5*(1+cot(x))^(5/2)+2*(1+cot(x))^(1/2)-arctan((3+cot(x)*(1-2^(1/2))-2*2^(
1/2))/(1+cot(x))^(1/2)/(-14+10*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)-arctanh((3
+2*2^(1/2)+cot(x)*(1+2^(1/2)))/(1+cot(x))^(1/2)/(14+10*2^(1/2))^(1/2))*(1+2
^(1/2))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3624, 3563, 12, 3617, 3616, 209, 213}

$$-\sqrt{\sqrt{2}-1}\,\text{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\,\sqrt{\cot(x)+1}}\right)-\frac{2}{5}(\cot(x)+1)^{5/2}+2\sqrt{\cot(x)+1}-\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\sqrt{\cot(x)+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*(1 + Cot[x])^(3/2),x]
```

```
[Out] -(Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Cot[x])/(Sqrt[2*
(-7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])]) - Sqrt[1 + Sqrt[2]]*ArcTanh[(3 + 2*Sq
rt[2] + (1 + Sqrt[2])*Cot[x])/(Sqrt[2*(7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])] +
 2*Sqrt[1 + Cot[x]] - (2*(1 + Cot[x])^(5/2))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = -\frac{2}{5}(1+\cot(x))^{5/2} - \int (1+\cot(x))^{3/2}\,dx$$
$$= 2\sqrt{1+\cot(x)} - \frac{2}{5}(1+\cot(x))^{5/2} - \int \frac{2\cot(x)}{\sqrt{1+\cot(x)}}\,dx$$
$$= 2\sqrt{1+\cot(x)} - \frac{2}{5}(1+\cot(x))^{5/2} - 2\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx$$
$$= 2\sqrt{1+\cot(x)} - \frac{2}{5}(1+\cot(x))^{5/2} - \frac{\int \frac{-1-\left(-1-\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{\sqrt{2}} + \frac{\int \frac{-1-\left(-1+\sqrt{\right.}}{\sqrt{1+}}}{\sqrt{}}$$
$$= 2\sqrt{1+\cot(x)} - \frac{2}{5}(1+\cot(x))^{5/2} - \left(-4+3\sqrt{2}\right)\text{Subst}\left(\int \frac{}{2\left(-1+\sqrt{2}\right.}\right.$$
$$= -\sqrt{-1+\sqrt{2}}\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right) - \sqrt{1+\sqrt{2}}\,\text{ta}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.34, size = 96, normalized size = 0.69

$$\frac{\sin(x)\left(-2\left(\frac{\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}}+\frac{\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}}\right)(1+\cot(x))^2\sin(x)-\frac{2}{5}(1+\cot(x))^{5/2}\left(-5+2\cot(x)+\csc^2(x)\right)\sin(x)\right)}{(\cos(x)+\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*(1 + Cot[x])^(3/2),x]
```

```
[Out] (Sin[x]*(-2*(ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]]/Sqrt[1 - I] + ArcTanh[Sq
rt[1 + Cot[x]]/Sqrt[1 + I]]/Sqrt[1 + I])*(1 + Cot[x])^2*Sin[x] - (2*(1 + Co
t[x])^(5/2)*(-5 + 2*Cot[x] + Csc[x]^2)*Sin[x])/5))/(Cos[x] + Sin[x])^2
```

Maple [A]
time = 0.26, size = 197, normalized size = 1.42

method	result

derivativedivides	$-\frac{2(1+\cot(x))^{\frac{5}{2}}}{5}+2\sqrt{1+\cot(x)}-\frac{\sqrt{2}\left(-\frac{\sqrt{2+2\sqrt{2}}\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\sqrt{2}\right.}{2}\right.}{}$
default	$-\frac{2(1+\cot(x))^{\frac{5}{2}}}{5}+2\sqrt{1+\cot(x)}-\frac{\sqrt{2}\left(-\frac{\sqrt{2+2\sqrt{2}}\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\sqrt{2}\right.}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(1+cot(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(1+cot(x))^(5/2)+2*(1+cot(x))^(1/2)-1/2*2^(1/2)*(-1/2*(2+2*2^(1/2))^(1
/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+2*(1-2^(1/2))
/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2
*2^(1/2))^(1/2)))-1/2*2^(1/2)*(1/2*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+
(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*ar
ctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(1+cot(x))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(1+cot(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int(\cot(x)+1)^{\frac{3}{2}}\cot^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2*(1+cot(x))**(3/2),x)
```

```
[Out] Integral((cot(x) + 1)**(3/2)*cot(x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(1+cot(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((cot(x) + 1)^(3/2)*cot(x)^2, x)
```

Mupad [B]
time = 1.00, size = 254, normalized size = 1.83

$$\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64}-\frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\,\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64}\right)\left(\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,2\mathrm{i}+\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\,2\mathrm{i}\right)-\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}+64}+\frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\,\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}+64}\right)\left(\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,2\mathrm{i}-\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\,2\mathrm{i}\right)+2\,\sqrt{\cot(x)+1}-\frac{2\,{\left(\cot(x)+1\right)}^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(cot(x) + 1)^(3/2),x)
```

```
[Out] atan((2^(1/2)*(1/4 - 2^(1/2)/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(1/4 - 2
^(1/2)/4)^(1/2)*(2^(1/2)/4 + 1/4)^(1/2) - 64) - (2^(1/2)*(2^(1/2)/4 + 1/4)^
(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(1/4 - 2^(1/2)/4)^(1/2)*(2^(1/2)/4 + 1/4
)^(1/2) - 64))*((1/4 - 2^(1/2)/4)^(1/2)*2i + (2^(1/2)/4 + 1/4)^(1/2)*2i) -
atan((2^(1/2)*(1/4 - 2^(1/2)/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(1/4 - 2
^(1/2)/4)^(1/2)*(2^(1/2)/4 + 1/4)^(1/2) + 64) + (2^(1/2)*(2^(1/2)/4 + 1/4)^
(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(1/4 - 2^(1/2)/4)^(1/2)*(2^(1/2)/4 + 1/4
)^(1/2) + 64))*((1/4 - 2^(1/2)/4)^(1/2)*2i - (2^(1/2)/4 + 1/4)^(1/2)*2i) +
2*(cot(x) + 1)^(1/2) - (2*(cot(x) + 1)^(5/2))/5
```

3.44 $\int \cot(x)(1+\cot(x))^{3/2}\,dx$

Optimal. Leaf size=221

$$-\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)+\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1-}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right.$$

```
[Out] -2/3*(1+cot(x))^(3/2)-2*(1+cot(x))^(1/2)-1/2*ln(1+cot(x)+2^(1/2)-(1+cot(x))
^(1/2)*(2+2*2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)+1/2*ln(1+cot(x)+2^(1/2)+(1+co
t(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)-arctan((-2*(1+cot(x))^(1
/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)+arctan((2*
(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/
2)
```

Rubi [A]
time = 0.15, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3609, 12, 3566, 722, 1108, 648, 632, 210, 642}

$$-\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}-\frac{\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\log\left(\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*(1 + Cot[x])^(3/2),x]
```

```
[Out] -(Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Cot[x]])/Sqr
t[2*(-1 + Sqrt[2])]]) + Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2
*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]] - 2*Sqrt[1 + Cot[x]] - (2*(1 + C
ot[x])^(3/2))/3 - Log[1 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 +
 Cot[x]]]/(2*Sqrt[1 + Sqrt[2]]) + Log[1 + Sqrt[2] + Cot[x] + Sqrt[2*(1 + Sq
rt[2])]*Sqrt[1 + Cot[x]]]/(2*Sqrt[1 + Sqrt[2]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cot(x)(1+\cot(x))^{3/2}\,dx &= -\frac{2}{3}(1+\cot(x))^{3/2} - \int (1-\cot(x))\sqrt{1+\cot(x)}\,dx \\
&= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} - \int \frac{2}{\sqrt{1+\cot(x)}}\,dx \\
&= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} - 2\int \frac{1}{\sqrt{1+\cot(x)}}\,dx \\
&= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} + 2\text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx, x, \cot(\right. \\
&= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} + 4\text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+\text{co}} \right. \\
&= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, \right.}{\sqrt{1+\sqrt{2}}} \\
&= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, \right.}{\sqrt{2}} \\
&= -2\sqrt{1+\cot(x)} - \frac{2}{3}(1+\cot(x))^{3/2} - \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right.}\right.}{2\sqrt{1+\sqrt{2}}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1}}{\sqrt{2\left(-1+\sqrt{2}\right.}}\right.}{\sqrt{-1+\sqrt{2}}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 98, normalized size = 0.44

$$
\frac{\sin(x)\left((1+i)\left(-i\sqrt{1-i}\,\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right)+\sqrt{1+i}\,\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right)\right)(1+\cot(x))^2\sin(x)-\frac{2}{3}(1+\cot(x))^{3/2}(4+\cot(x))(\cos(x)+\sin(x))\right)}{(\cos(x)+\sin(x))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*(1 + Cot[x])^(3/2),x]
```

```
[Out] (Sin[x]*((1 + I)*((-I)*Sqrt[1 - I]*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] +
Sqrt[1 + I]*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]])*(1 + Cot[x])^2*Sin[x] -
(2*(1 + Cot[x])^(3/2)*(4 + Cot[x])*(Cos[x] + Sin[x]))/3))/(Cos[x] + Sin[x])
^2
```

Maple [A]
time = 0.21, size = 290, normalized size = 1.31

method	result
derivativedivides	$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\cot(x)}+\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(1+\cot(x)+\sqrt{2}+\right.}{4}$
default	$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\cot(x)}+\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(1+\cot(x)+\sqrt{2}+\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(1+cot(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+cot(x))^(3/2)-2*(1+cot(x))^(1/2)+1/4*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+
2*(2+2*2^(1/2))^(1/2))*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(
1/2))+(2*2^(1/2)-1/2*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*(
2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^
(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/4*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+
2*2^(1/2))^(1/2))*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))
-(-2*2^(1/2)+1/2*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*(2+2*
2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2
))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+cot(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((cot(x) + 1)^(3/2)*cot(x), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+cot(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\cot (x) + 1)^{\frac{3}{2}} \cot (x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+cot(x))**(3/2),x)
```

```
[Out] Integral((cot(x) + 1)**(3/2)*cot(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+cot(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((cot(x) + 1)^(3/2)*cot(x), x)
```

Mupad [B]
time = 0.67, size = 254, normalized size = 1.15

$$-\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{\cot (x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}-\frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{\cot (x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}\right)\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}+\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}\right)+\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{\cot (x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64}+\frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{\cot (x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64}\right)\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}-\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}\right)-2\,\sqrt{\cot (x)+1}-\frac{2\,{\left(\cot (x)+1\right)}^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(cot(x) + 1)^(3/2),x)
```

```
[Out] atan((2^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(2^(1/
2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) + 64) + (2^(1/2)*(2^(1/2)/4 - 1
/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4
 - 1/4)^(1/2) + 64))*((- 2^(1/2)/4 - 1/4)^(1/2)*2i - (2^(1/2)/4 - 1/4)^(1/2
)*2i) - atan((2^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(25
6*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) - 64) - (2^(1/2)*(2^(1/
2)/4 - 1/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2
^(1/2)/4 - 1/4)^(1/2) - 64))*((- 2^(1/2)/4 - 1/4)^(1/2)*2i + (2^(1/2)/4 - 1
/4)^(1/2)*2i) - 2*(cot(x) + 1)^(1/2) - (2*(cot(x) + 1)^(3/2))/3
```

3.45 $\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx$

Optimal. Leaf size=214

$$-\frac{1}{2}\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)+\frac{1}{2}\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2}{\sqrt{2\left(-1+\right.}}\right.$$

```
[Out] -2*(1+cot(x))^(1/2)-1/4*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^
(1/2))/(1+2^(1/2))^(1/2)+1/4*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1
/2))^(1/2))/(1+2^(1/2))^(1/2)-1/2*arctan((-2*(1+cot(x))^(1/2)+(2+2*2^(1/2))
^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)+1/2*arctan((2*(1+cot(x))^(1
/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3624, 3566, 722, 1108, 648, 632, 210, 642}

$$-\frac{1}{2}\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-2\sqrt{\cot(x)+1}-\frac{\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\log\left(\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/Sqrt[1 + Cot[x]],x]
```

```
[Out] -1/2*(Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Cot[x]])
/Sqrt[2*(-1 + Sqrt[2])]]) + (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])
] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]])/2 - 2*Sqrt[1 + Cot[x]] - L
og[1 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/(4*Sqrt[1
 + Sqrt[2]]) + Log[1 + Sqrt[2] + Cot[x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Co
t[x]]]/(4*Sqrt[1 + Sqrt[2]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx &= -2\sqrt{1+\cot(x)} - \int \frac{1}{\sqrt{1+\cot(x)}}\,dx \\
&= -2\sqrt{1+\cot(x)} + \text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx, x, \cot(x)\right) \\
&= -2\sqrt{1+\cot(x)} + 2\text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+\cot(x)}\right) \\
&= -2\sqrt{1+\cot(x)} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\text{Su}\ldots}{\ldots} \\
&= -2\sqrt{1+\cot(x)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{2\sqrt{2}} + \frac{\text{Su}\ldots}{\ldots} \\
&= -2\sqrt{1+\cot(x)} - \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\,\sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\log\ldots}{\ldots} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{-1+\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot\ldots}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{-1+\sqrt{2}}}\ldots
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 67, normalized size = 0.31

$$
\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right)+\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right)-2\sqrt{1+\cot(x)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/Sqrt[1 + Cot[x]],x]
[Out] ((1 - I)^(3/2)*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]])/2 + ((1 + I)^(3/2)*Ar
cTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]])/2 - 2*Sqrt[1 + Cot[x]]
```

Maple [A]
time = 0.26, size = 283, normalized size = 1.32

method	result
derivativedivides	$-2\sqrt{1+\cot(x)}-\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\right.}{8}$
default	$-2\sqrt{1+\cot(x)}-\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(1+cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1+cot(x))^(1/2)-1/8*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2)
)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/2*(-2*2^(1/2)
+1/2*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/
2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-
2+2*2^(1/2))^(1/2))+1/8*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2)
)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/2*(2*2^(1/2)-
1/2*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2
))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2
+2*2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(1+cot(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)^2/sqrt(cot(x) + 1), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(1+cot(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{\sqrt{\cot(x)+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(1+cot(x))**(1/2),x)
[Out] Integral(cot(x)**2/sqrt(cot(x) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(1+cot(x))^(1/2),x, algorithm="giac")
[Out] integrate(cot(x)^2/sqrt(cot(x) + 1), x)
```

Mupad [B]
time = 0.44, size = 238, normalized size = 1.11

$$\operatorname{atanh}\left(\frac{16\sqrt{2}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}-8}-\frac{16\sqrt{2}\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}-8}\right)\left(2\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}+2\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\right)-\operatorname{atanh}\left(\frac{16\sqrt{2}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}+8}+\frac{16\sqrt{2}\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}+8}\right)\left(2\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}-2\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\right)-2\sqrt{\cot(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(cot(x) + 1)^(1/2),x)
[Out] atanh((16*2^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(2^(
1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2) - 8) - (16*2^(1/2)*(2^(1/
2)/16 - 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(2^(1/2)/16 - 1/16)^(1/2)*(- 2
^(1/2)/16 - 1/16)^(1/2) - 8))*(2*(- 2^(1/2)/16 - 1/16)^(1/2) + 2*(2^(1/2)/1
6 - 1/16)^(1/2)) - atanh((16*2^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2)*(cot(x) +
1)^(1/2))/(128*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2) + 8) +
 (16*2^(1/2)*(2^(1/2)/16 - 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(2^(1/2)/16
 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2) + 8))*(2*(- 2^(1/2)/16 - 1/16)^(
1/2) - 2*(2^(1/2)/16 - 1/16)^(1/2)) - 2*(cot(x) + 1)^(1/2)
```

3.46 $\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx$

Optimal. Leaf size=121

$$\frac{1}{2}\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\,\sqrt{1+\cot(x)}}\right)+\frac{1}{2}\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{3+2\sqrt{2}+\left(1+\sqrt{2}\right.}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\sqrt{1+}}\right.$$

```
[Out] 1/2*arctan((3+cot(x)*(1-2^(1/2))-2*2^(1/2))/(1+cot(x))^(1/2)/(-14+10*2^(1/2
))^(1/2))*(2^(1/2)-1)^(1/2)+1/2*arctanh((3+2*2^(1/2)+cot(x)*(1+2^(1/2)))/(1
+cot(x))^(1/2)/(14+10*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3617, 3616, 209, 213}

$$\frac{1}{2}\sqrt{\sqrt{2}-1}\,\text{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\,\sqrt{\cot(x)+1}}\right)+\frac{1}{2}\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\,\sqrt{\cot(x)+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[1 + Cot[x]],x]
```

```
[Out] (Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Cot[x])/(Sqrt[2*(
-7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])])/2 + (Sqrt[1 + Sqrt[2]]*ArcTanh[(3 + 2*
Sqrt[2] + (1 + Sqrt[2])*Cot[x])/(Sqrt[2*(7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])]
)/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
```

```
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx &= \frac{\int \frac{-1-\left(-1-\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{2\sqrt{2}} - \frac{\int \frac{-1-\left(-1+\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{2\sqrt{2}}\\ &= \frac{1}{2}\left(-4+3\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{2\left(-1+\sqrt{2}\right)-4\left(-1+\sqrt{2}\right)^2+x^2}\,dx, x, \frac{1-2\left(-1+\right.}{}\right.\\ &= \frac{1}{2}\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right)+\frac{1}{2}\sqrt{1+\sqrt{2}}\,\tanh^{-1}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 51, normalized size = 0.42

$$\frac{\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}}+\frac{\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[1 + Cot[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]]/Sqrt[1 - I] + ArcTanh[Sqrt[1 + Cot[x]
]/Sqrt[1 + I]]/Sqrt[1 + I]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(85) = 170$.
time = 0.48, size = 181, normalized size = 1.50

method	result
derivativedivides	$\frac{\sqrt{2}\left(-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(1-\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cdots}}{\cdots}\right)}{\sqrt{-2+\cdots}}\cdots\right.}{4}$
default	$\frac{\sqrt{2}\left(-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(1-\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cdots}}{\cdots}\right)}{\sqrt{-2+\cdots}}\cdots\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(1+cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)*(-1/2*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*
(2+2*2^(1/2))^(1/2))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x)
)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))+1/4*2^(1/2)*(1/2*(2+2*2
^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+2*(
1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1
/2))/(-2+2*2^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(1+cot(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)/sqrt(cot(x) + 1), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(1+cot(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{\cot(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(1+cot(x))**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(cot(x) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(1+cot(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(x)/sqrt(cot(x) + 1), x)
```

Mupad [B]
time = 0.41, size = 230, normalized size = 1.90

$$\operatorname{atanh}\left(\frac{16\sqrt{2}\,\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\,\sqrt{\cot(x)+1}}{128\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\,\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}-8}-\frac{16\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}\,\sqrt{\cot(x)+1}}{128\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\,\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}-8}\right)\left(2\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}+2\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}\right)-\operatorname{atanh}\left(\frac{16\sqrt{2}\,\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\,\sqrt{\cot(x)+1}}{128\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\,\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}+8}+\frac{16\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}\,\sqrt{\cot(x)+1}}{128\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\,\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}+8}\right)\left(2\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}-2\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(cot(x) + 1)^(1/2),x)
```

```
[Out] atanh((16*2^(1/2)*(1/16 - 2^(1/2)/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(1/16
- 2^(1/2)/16)^(1/2)*(2^(1/2)/16 + 1/16)^(1/2) - 8) - (16*2^(1/2)*(2^(1/2)/1
6 + 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(1/16 - 2^(1/2)/16)^(1/2)*(2^(1/2)
/16 + 1/16)^(1/2) - 8))*(2*(1/16 - 2^(1/2)/16)^(1/2) + 2*(2^(1/2)/16 + 1/16
)^(1/2)) - atanh((16*2^(1/2)*(1/16 - 2^(1/2)/16)^(1/2)*(cot(x) + 1)^(1/2))/
(128*(1/16 - 2^(1/2)/16)^(1/2)*(2^(1/2)/16 + 1/16)^(1/2) + 8) + (16*2^(1/2)
*(2^(1/2)/16 + 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(1/16 - 2^(1/2)/16)^(1/
2)*(2^(1/2)/16 + 1/16)^(1/2) + 8))*(2*(1/16 - 2^(1/2)/16)^(1/2) - 2*(2^(1/2
)/16 + 1/16)^(1/2))
```

3.47 $\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{1}{2}\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\ \text{ArcTan}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\cot(x)}{2\sqrt{-7+5\sqrt{2}}\ \sqrt{1+\cot(x)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\ \tanh^{-1}\left(\frac{4+3\sqrt{2}+\left(2\right.}{2\sqrt{7+5\sqrt{2}}}\right.$$

```
[Out] 1/(1+cot(x))^(1/2)+1/4*arctan(1/2*(4+cot(x)*(2-2^(1/2))-3*2^(1/2))/(1+cot(x
))^(1/2)/(-7+5*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)+1/4*arctanh(1/2*(4+3*2^
(1/2)+cot(x)*(2+2^(1/2)))/(1+cot(x))^(1/2)/(7+5*2^(1/2))^(1/2))*(2+2*2^(1/2
))^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3623, 3617, 3616, 209, 213}

$$\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\ \text{ArcTan}\left(\frac{\left(2-\sqrt{2}\right)\cot(x)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\ \sqrt{\cot(x)+1}}\right)+\frac{1}{\sqrt{\cot(x)+1}}+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\ \tanh^{-1}\left(\frac{\left(2+\sqrt{2}\right)\cot(x)+3\sqrt{2}+4}{2\sqrt{7+5\sqrt{2}}\ \sqrt{\cot(x)+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/(1 + Cot[x])^(3/2),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(4 - 3*Sqrt[2] + (2 - Sqrt[2])*Cot[x])/(2*Sq
rt[-7 + 5*Sqrt[2]]*Sqrt[1 + Cot[x]])])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTanh[(
4 + 3*Sqrt[2] + (2 + Sqrt[2])*Cot[x])/(2*Sqrt[7 + 5*Sqrt[2]]*Sqrt[1 + Cot[x
]])])/2 + 1/Sqrt[1 + Cot[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
```

```
x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx &= \frac{1}{\sqrt{1+\cot(x)}} + \frac{1}{2}\int \frac{-1+\cot(x)}{\sqrt{1+\cot(x)}}\,dx \\
&= \frac{1}{\sqrt{1+\cot(x)}} + \frac{\int \frac{-\sqrt{2}-\left(2-\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{4\sqrt{2}} - \frac{\int \frac{\sqrt{2}-\left(2+\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{4\sqrt{2}} \\
&= \frac{1}{\sqrt{1+\cot(x)}} - \frac{1}{2}\left(-4+3\sqrt{2}\right)\mathrm{Subst}\left(\int \frac{1}{2\sqrt{2}\left(2-\sqrt{2}\right)-4\left(2-\sqrt{2}\right)^2+x^2}\right. \\
&= \frac{1}{2}\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\tan^{-1}\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\cot(x)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.14, size = 65, normalized size = 0.47

$$\frac{1}{2}\sqrt{1-i}\,\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right)+\frac{1}{2}\sqrt{1+i}\,\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right)+\frac{1}{\sqrt{1+\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/(1 + Cot[x])^(3/2),x]
```

```
[Out] (Sqrt[1 - I]*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]])/2 + (Sqrt[1 + I]*ArcTan
h[Sqrt[1 + Cot[x]]/Sqrt[1 + I]])/2 + 1/Sqrt[1 + Cot[x]]
```

Maple [**A**]
time = 0.23, size = 173, normalized size = 1.24

method	result
derivativedivides	$-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{8}+\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}}{\sqrt{}}\right)}{2\sqrt{-2+}}$
default	$-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{8}+\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}}{\sqrt{}}\right)}{2\sqrt{-2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(1+cot(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))
^(1/2))+1/2*(2^(1/2)-1)/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+
2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)
+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/2*(1-2^(1/2))/(-2+2*2^(1/2
))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/
2))+1/(1+cot(x))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(1+cot(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)^2/(cot(x) + 1)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(1+cot(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{(\cot(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(1+cot(x))**(3/2),x)
```

```
[Out] Integral(cot(x)**2/(cot(x) + 1)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(1+cot(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(x)^2/(cot(x) + 1)^(3/2), x)
```

Mupad [B]
time = 0.50, size = 208, normalized size = 1.50

$$\frac{1}{\sqrt{\cot(x)+1}}-\operatorname{atanh}\left(\frac{\sqrt{\cot(x)+1}}{8\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}}-\frac{\sqrt{\cot(x)+1}}{8\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}}+\frac{\sqrt{2}\,\sqrt{\cot(x)+1}}{16\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}}+\frac{\sqrt{2}\,\sqrt{\cot(x)+1}}{16\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}}\right)\left(2\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}-2\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}\right)+\operatorname{atanh}\left(\frac{\sqrt{\cot(x)+1}}{8\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}}+\frac{\sqrt{\cot(x)+1}}{8\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}}-\frac{\sqrt{2}\,\sqrt{\cot(x)+1}}{16\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}}+\frac{\sqrt{2}\,\sqrt{\cot(x)+1}}{16\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}}\right)\left(2\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}+2\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(cot(x) + 1)^(3/2),x)
```

```
[Out] 1/(cot(x) + 1)^(1/2) - atanh((cot(x) + 1)^(1/2)/(8*(2^(1/2)/32 + 1/32)^(1/2
)) - (cot(x) + 1)^(1/2)/(8*(1/32 - 2^(1/2)/32)^(1/2)) + (2^(1/2)*(cot(x) +
1)^(1/2))/(16*(1/32 - 2^(1/2)/32)^(1/2)) + (2^(1/2)*(cot(x) + 1)^(1/2))/(16
*(2^(1/2)/32 + 1/32)^(1/2)))*(2*(1/32 - 2^(1/2)/32)^(1/2) - 2*(2^(1/2)/32 +
 1/32)^(1/2)) + atanh((cot(x) + 1)^(1/2)/(8*(1/32 - 2^(1/2)/32)^(1/2)) + (c
ot(x) + 1)^(1/2)/(8*(2^(1/2)/32 + 1/32)^(1/2)) - (2^(1/2)*(cot(x) + 1)^(1/2
))/(16*(1/32 - 2^(1/2)/32)^(1/2)) + (2^(1/2)*(cot(x) + 1)^(1/2))/(16*(2^(1/
2)/32 + 1/32)^(1/2)))*(2*(1/32 - 2^(1/2)/32)^(1/2) + 2*(2^(1/2)/32 + 1/32)^
(1/2))
```

3.48 $\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\, dx$

Optimal. Leaf size=226

$$\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right.}}{\sqrt{2\left(\right.}}\right.$$

```
[Out] -1/(1+cot(x))^(1/2)+1/4*arctan((-2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-
2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)-1/4*arctan((2*(1+cot(x))^(1/2)+(2+2
*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)-1/4*ln(1+cot(x)+
2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)+1/4*ln(1+
cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {3610, 21, 3566, 714, 1141, 1175, 632, 210, 1178, 642}

$$\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{\sqrt{\cot(x)+1}}-\frac{\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\log\left(\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(1 + Cot[x])^(3/2),x]
```

```
[Out] (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Cot[x]])/
Sqrt[2*(-1 + Sqrt[2])]])/2 - (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqr
t[2])] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]])/2 - 1/Sqrt[1 + Cot[x]
] - Log[1 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/(4*S
qrt[2*(1 + Sqrt[2])]) + Log[1 + Sqrt[2] + Cot[x] + Sqrt[2*(1 + Sqrt[2])]*Sq
rt[1 + Cot[x]]]/(4*Sqrt[2*(1 + Sqrt[2])])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
```

```
d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx = -\frac{1}{\sqrt{1+\cot(x)}} - \frac{1}{2}\int \frac{-1-\cot(x)}{\sqrt{1+\cot(x)}}\,dx$$

$$= -\frac{1}{\sqrt{1+\cot(x)}} + \frac{1}{2}\int \sqrt{1+\cot(x)}\,dx$$

$$= -\frac{1}{\sqrt{1+\cot(x)}} - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1+x}}{1+x^2}\,dx, x, \cot(x)\right)$$

$$= -\frac{1}{\sqrt{1+\cot(x)}} - \text{Subst}\left(\int \frac{x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+\cot(x)}\right)$$

$$= -\frac{1}{\sqrt{1+\cot(x)}} + \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+\cot(x)}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\cdots}{2-\cdots}\right.$$

$$= -\frac{1}{\sqrt{1+\cot(x)}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)$$

$$= -\frac{1}{\sqrt{1+\cot(x)}} - \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{\log\left(\cdots\right.}{\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}\cdots}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 71, normalized size = 0.31

$$\frac{1}{2}i\sqrt{1-i}\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right)-\frac{1}{2}i\sqrt{1+i}\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right)-\frac{1}{\sqrt{1+\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(1 + Cot[x])^(3/2),x]
```

```
[Out] (I/2)*Sqrt[1 - I]*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] - (I/2)*Sqrt[1 + I]
*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]] - 1/Sqrt[1 + Cot[x]]
```

Maple [A]
time = 0.26, size = 197, normalized size = 0.87

method	result
derivativedivides	$-\frac{1}{\sqrt{1+\cot(x)}}+\frac{\sqrt{2+2\sqrt{2}}\,(\sqrt{2}-1)\left(\frac{\ln\left(1+\cot(x)+\sqrt{2}+\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}\right.}{4}$
default	$-\frac{1}{\sqrt{1+\cot(x)}}+\frac{\sqrt{2+2\sqrt{2}}\,(\sqrt{2}-1)\left(\frac{\ln\left(1+\cot(x)+\sqrt{2}+\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(1+cot(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(1+cot(x))^(1/2)+1/4*(2+2*2^(1/2))^(1/2)*(2^(1/2)-1)*(1/2*ln(1+cot(x)+2^
(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-(2+2*2^(1/2))^(1/2)/(-2+2*2^(1/
2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1
/2)))+1/4*(2+2*2^(1/2))^(1/2)*(2^(1/2)-1)*(-1/2*ln(1+cot(x)+2^(1/2)-(1+cot(
x))^(1/2)*(2+2*2^(1/2))^(1/2))-(2+2*2^(1/2))^(1/2)/(-2+2*2^(1/2))^(1/2)*arc
tan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(1+cot(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)/(cot(x) + 1)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(1+cot(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(\cot(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(1+cot(x))**(3/2),x)
```

```
[Out] Integral(cot(x)/(cot(x) + 1)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(1+cot(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(x)/(cot(x) + 1)^(3/2), x)
```

Mupad [**B**]
time = 0.40, size = 121, normalized size = 0.54

$$-\operatorname{atanh}\left(32\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)-\frac{1}{\sqrt{\cot(x)+1}}-\operatorname{atanh}\left(32\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(cot(x) + 1)^(3/2),x)
```

```
[Out] - atanh(32*(cot(x) + 1)^(1/2)*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 -
1/32)^(1/2))^3)*(2*(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1/2
)) - 1/(cot(x) + 1)^(1/2) - atanh(32*(cot(x) + 1)^(1/2)*((- 2^(1/2)/32 - 1/
32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/32 - 1/32)^(1/2) -
2*(2^(1/2)/32 - 1/32)^(1/2))
```

3.49 $\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{1}{4}\sqrt{-1+\sqrt{2}}\ \text{ArcTan}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\ \sqrt{1+\cot(x)}}\right)+\frac{1}{4}\sqrt{1+\sqrt{2}}\ \tanh^{-1}\left(\frac{3+2\sqrt{2}+\left(1+\sqrt{2}\right.}{\sqrt{2\left(7+5\sqrt{2}\right)}\ \sqrt{1}}\right.$$

```
[Out] 1/3/(1+cot(x))^(3/2)-1/(1+cot(x))^(1/2)+1/4*arctan((3+cot(x)*(1-2^(1/2))-2*
2^(1/2))/(1+cot(x))^(1/2)/(-14+10*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)+1/4*arc
tanh((3+2*2^(1/2)+cot(x)*(1+2^(1/2)))/(1+cot(x))^(1/2)/(14+10*2^(1/2))^(1/2
))*(1+2^(1/2))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3623, 3610, 12, 3617, 3616, 209, 213}

$$\frac{1}{4}\sqrt{\sqrt{2}-1}\ \text{ArcTan}\left(\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\ \sqrt{\cot(x)+1}}\right)-\frac{1}{\sqrt{\cot(x)+1}}+\frac{1}{3(\cot(x)+1)^{3/2}}+\frac{1}{4}\sqrt{1+\sqrt{2}}\ \tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\ \sqrt{\cot(x)+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/(1 + Cot[x])^(5/2),x]
```

```
[Out] (Sqrt[-1 + Sqrt[2]]*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Cot[x])/(Sqrt[2*(
-7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])])/4 + (Sqrt[1 + Sqrt[2]]*ArcTanh[(3 + 2*
Sqrt[2] + (1 + Sqrt[2])*Cot[x])/(Sqrt[2*(7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])]
)/4 + 1/(3*(1 + Cot[x])^(3/2)) - 1/Sqrt[1 + Cot[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3616

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[-2*(d^2/f), Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 3617

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \frac{1}{3(1+\cot(x))^{3/2}} + \frac{1}{2}\int \frac{-1+\cot(x)}{(1+\cot(x))^{3/2}}\,dx$$

$$= \frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{\sqrt{1+\cot(x)}} + \frac{1}{4}\int \frac{2\cot(x)}{\sqrt{1+\cot(x)}}\,dx$$

$$= \frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{\sqrt{1+\cot(x)}} + \frac{1}{2}\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx$$

$$= \frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{\sqrt{1+\cot(x)}} + \frac{\int \frac{-1-\left(-1-\sqrt{2}\right)\cot(x)}{\sqrt{1+\cot(x)}}\,dx}{4\sqrt{2}} - \frac{\int \frac{-1-\left(-1+\sqrt{2}\right)\cot(}{\sqrt{1+\cot(x)}}}{4\sqrt{2}}$$

$$= \frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{\sqrt{1+\cot(x)}} + \frac{1}{4}\left(-4+3\sqrt{2}\right)\operatorname{Subst}\left(\int \frac{1}{2\left(-1+\sqrt{2}\right)-4}\right.$$

$$= \frac{1}{4}\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right) + \frac{1}{4}\sqrt{1+\sqrt{2}}\,\tanh^{-1}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.43, size = 75, normalized size = 0.52

$$\frac{\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right)}{2\sqrt{1-i}} + \frac{\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right)}{2\sqrt{1+i}} + \frac{-2-3\cot(x)}{3(1+\cot(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/(1 + Cot[x])^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]]/(2*Sqrt[1 - I]) + ArcTanh[Sqrt[1 + Co
t[x]]/Sqrt[1 + I]]/(2*Sqrt[1 + I]) + (-2 - 3*Cot[x])/(3*(1 + Cot[x])^(3/2))
```

Maple [A]
time = 0.24, size = 197, normalized size = 1.38

method	result

derivativedivides	$\frac{\sqrt{2}\left(\frac{\sqrt{2+2\sqrt{2}}\ln\left(1+\cot(x)+\sqrt{2}+\sqrt{1+\cot(x)}\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(1-\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot \cdots}}{\sqrt{\cdots}}\right)}{\sqrt{-2+\cdots}}\right.}{8}$
default	$\frac{\sqrt{2}\left(\frac{\sqrt{2+2\sqrt{2}}\ln\left(1+\cot(x)+\sqrt{2}+\sqrt{1+\cot(x)}\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(1-\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot \cdots}}{\sqrt{\cdots}}\right)}{\sqrt{-2+\cdots}}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(1+cot(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*2^(1/2)*(1/2*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(
2+2*2^(1/2))^(1/2))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))
^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))+1/8*2^(1/2)*(-1/2*(2+2*2
^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+2*(
1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1
/2))/(-2+2*2^(1/2))^(1/2)))+1/3/(1+cot(x))^(3/2)-1/(1+cot(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(1+cot(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)^2/(cot(x) + 1)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(1+cot(x))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2{(x)}}{(\cot{(x)}+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(1+cot(x))**(5/2),x)
```

```
[Out] Integral(cot(x)**2/(cot(x) + 1)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(1+cot(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(x)^2/(cot(x) + 1)^(5/2), x)
```

Mupad [B]
time = 0.80, size = 242, normalized size = 1.69

$$\mathrm{atanh}\left(\frac{4\sqrt{2}\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\cot{(x)}+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}-\frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\sqrt{\cot{(x)}+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}+2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right)-\mathrm{atanh}\left(\frac{4\sqrt{2}\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\cot{(x)}+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}+1}+\frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\sqrt{\cot{(x)}+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}+1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}-2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right)-\frac{\cot{(x)}+\frac{2}{3}}{(\cot{(x)}+1)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(cot(x) + 1)^(5/2),x)
```

```
[Out] atanh((4*2^(1/2)*(1/64 - 2^(1/2)/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(1/64 -
2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2) - 1) - (4*2^(1/2)*(2^(1/2)/64 +
 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/64
+ 1/64)^(1/2) - 1))*(2*(1/64 - 2^(1/2)/64)^(1/2) + 2*(2^(1/2)/64 + 1/64)^(1
/2)) - atanh((4*2^(1/2)*(1/64 - 2^(1/2)/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(
1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2) + 1) + (4*2^(1/2)*(2^(1/
2)/64 + 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1
/2)/64 + 1/64)^(1/2) + 1))*(2*(1/64 - 2^(1/2)/64)^(1/2) - 2*(2^(1/2)/64 + 1
/64)^(1/2)) - (cot(x) + 2/3)/(cot(x) + 1)^(3/2)
```

3.50 $\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx$

Optimal. Leaf size=216

$$\frac{1}{4}\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)-\frac{1}{4}\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1}}{\sqrt{2\left(-1+\sqrt{2}\right.}}\right.$$

```
[Out] -1/3/(1+cot(x))^(3/2)+1/8*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2)
)^(1/2))/(1+2^(1/2))^(1/2)-1/8*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^
(1/2))^(1/2))/(1+2^(1/2))^(1/2)+1/4*arctan((-2*(1+cot(x))^(1/2)+(2+2*2^(1/2
))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)-1/4*arctan((2*(1+cot(x))^
(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3610, 21, 3566, 722, 1108, 648, 632, 210, 642}

$$\frac{1}{4}\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{4}\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{3(\cot(x)+1)^{3/2}}+\frac{\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{8\sqrt{1+\sqrt{2}}}-\frac{\log\left(\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{8\sqrt{1+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(1 + Cot[x])^(5/2),x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + Cot[x]])/Sqrt
[2*(-1 + Sqrt[2])]])/4 - (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] +
 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]])/4 - 1/(3*(1 + Cot[x])^(3/2))
+ Log[1 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/(8*Sqr
t[1 + Sqrt[2]]) - Log[1 + Sqrt[2] + Cot[x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 +
 Cot[x]]]/(8*Sqrt[1 + Sqrt[2]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx &= -\frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{2}\int \frac{-1-\cot(x)}{(1+\cot(x))^{3/2}}\,dx \\
&= -\frac{1}{3(1+\cot(x))^{3/2}} + \frac{1}{2}\int \frac{1}{\sqrt{1+\cot(x)}}\,dx \\
&= -\frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx, x, \cot(x)\right) \\
&= -\frac{1}{3(1+\cot(x))^{3/2}} - \text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+\cot(x)}\right) \\
&= -\frac{1}{3(1+\cot(x))^{3/2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}} - \cdots \text{S} \\
&= -\frac{1}{3(1+\cot(x))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+\cot(x)}\right)}{4\sqrt{2}} - \cdots \text{S} \\
&= -\frac{1}{3(1+\cot(x))^{3/2}} + \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{8\sqrt{1+\sqrt{2}}} - \cdots \text{lo} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{4\sqrt{-1+\sqrt{2}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{4\sqrt{-1+\sqrt{2}}} \cdots
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.26, size = 69, normalized size = 0.32

$$-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right) - \frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right) - \frac{1}{3(1+\cot(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(1 + Cot[x])^(5/2),x]
```

```
[Out] -1/4*((1 - I)^(3/2)*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]]) - ((1 + I)^(3/2)
*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]])/4 - 1/(3*(1 + Cot[x])^(3/2))
```

Maple [A]
time = 0.25, size = 283, normalized size = 1.31

method	result
derivativedivides	$-\frac{1}{3(1+\cot(x))^{\frac{3}{2}}}-\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(1+\cot(x)+\sqrt{2}+\sqrt{1+\cot(x)}\,\sqrt{2\right.}}{16}$
default	$-\frac{1}{3(1+\cot(x))^{\frac{3}{2}}}-\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(1+\cot(x)+\sqrt{2}+\sqrt{1+\cot(x)}\,\sqrt{2\right.}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(1+cot(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(1+cot(x))^(3/2)-1/16*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1
/2))*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/4*(2*2^(1/
2)-1/2*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(
1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/
(-2+2*2^(1/2))^(1/2))+1/16*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1
/2))*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/4*(-2*2^(1
/2)+1/2*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^
(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))
/(-2+2*2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(1+cot(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)/(cot(x) + 1)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(1+cot(x))^(5/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(\cot(x)+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(1+cot(x))**(5/2),x)

[Out] Integral(cot(x)/(cot(x) + 1)**(5/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(1+cot(x))^(5/2),x, algorithm="giac")

[Out] integrate(cot(x)/(cot(x) + 1)^(5/2), x)

Mupad [B]
time = 0.71, size = 238, normalized size = 1.10

$$\operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1}+\frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)-\operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1}-\frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)-\frac{1}{3(\cot(x)+1)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(cot(x) + 1)^(5/2),x)

[Out] atanh((4*2^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) + 1) + (4*2^(1/2)*(2^(1/2)/64 - 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) + 1))*(2*(- 2^(1/2)/64 - 1/64)^(1/2) - 2*(2^(1/2)/64 - 1/64)^(1/2)) - atanh((4*2^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) - 1) - (4*2^(1/2)*(2^(1/2)/64 - 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) - 1))*(2*(- 2^(1/2)/64 - 1/64)^(1/2) + 2*(2^(1/2)/64 - 1/64)^(1/2)) - 1/(3*(cot(x) + 1)^(3/2))

3.51 $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx$

Optimal. Leaf size=247

$$-\frac{(a+b)e^{3/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{(a+b)e^{3/2}\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{2ae\sqrt{e}}{}$$

```
[Out] -2/3*b*(e*cot(d*x+c))^(3/2)/d-1/2*(a+b)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x
+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/2*(a+b)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x
+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/4*(a-b)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/
2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)+1/4*(a-b)*e^(3/2)*ln(e^(1/2)+cot
(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)-2*a*e*(e*cot(d*x+c)
)^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{e^{3/2}(a+b)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{e^{3/2}(a+b)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d}-\frac{e^{3/2}(a-b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}+\frac{e^{3/2}(a-b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}-\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x]),x]
```

```
[Out] -(((a + b)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*d)) + ((a + b)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[
e]])/(Sqrt[2]*d) - (2*a*e*Sqrt[e*Cot[c + d*x]])/d - (2*b*(e*Cot[c + d*x])^(
3/2))/(3*d) - ((a - b)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]
*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d) + ((a - b)*e^(3/2)*Log[Sqrt[e] + Sqrt
[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx = -\frac{2b(e\cot(c+dx))^{3/2}}{3d} + \int \sqrt{e\cot(c+dx)}\,(-be+ae\cot(c+$$

$$= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2b(e\cot(c+dx))^{3/2}}{3d} + \int \frac{-ae^2-be^2}{\sqrt{e\cot(}}$$

$$= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2b(e\cot(c+dx))^{3/2}}{3d} + \frac{2\text{Subst}\left(\int \frac{ae^3+}{e^2}\right.}{}$$

$$= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2b(e\cot(c+dx))^{3/2}}{3d} + \frac{((a-b)e^2)\,\text{Su}}{}$$

$$= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2b(e\cot(c+dx))^{3/2}}{3d} - \frac{\left((a-b)e^{3/2}\right)\,\text{S}}{}$$

$$= -\frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2b(e\cot(c+dx))^{3/2}}{3d} - \frac{(a-b)e^{3/2}\log}{}$$

$$= -\frac{(a+b)e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d} + \frac{(a+b)e^{3/2}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.15, size = 68, normalized size = 0.28

$$-\frac{2e\sqrt{e\cot(c+dx)}\left(b\cot(c+dx)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(c+dx)\right)+3a\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x]),x]
```

```
[Out] (-2*e*Sqrt[e*Cot[c + d*x]]*(b*Cot[c + d*x]*Hypergeometric2F1[-3/4, 1, 1/4,
-Tan[c + d*x]^2] + 3*a*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2]))/(
3*d)
```

Maple [A]
time = 0.42, size = 306, normalized size = 1.24

method	result

derivativedivides	$\frac{-\frac{2b(e\cot(dx+c))^{\frac{3}{2}}}{3}-2ae\sqrt{e\cot(dx+c)}+2e^2\left(\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+}\right.\right.}{}\right.}{}$
default	$\frac{-\frac{2b(e\cot(dx+c))^{\frac{3}{2}}}{3}-2ae\sqrt{e\cot(dx+c)}+2e^2\left(\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/3*b*(e*cot(d*x+c))^(3/2)-2*a*e*(e*cot(d*x+c))^(1/2)+2*e^2*(1/8*a/e*
(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^
2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2
^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*b/(e^2)^(1/4)*2^(1/2)*(ln((
e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d
*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/
2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.56, size = 153, normalized size = 0.62

$$\frac{\left(6\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+3\sqrt{2}(a-b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-3\sqrt{2}(a-b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{24a}{\sqrt{\tan(dx+c)}}-\frac{8b}{\tan(dx+c)^{\frac{3}{2}}}\right)e^{\frac{3}{2}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))
) + 6*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
 + 3*sqrt(2)*(a - b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
 3*sqrt(2)*(a - b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
24*a/sqrt(tan(d*x + c)) - 8*b/tan(d*x + c)^(3/2))*e^(3/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cot(c+dx))^{\frac{3}{2}} (a+b\cot(c+dx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+b*cot(d*x+c)),x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)*(e*cot(d*x + c))^(3/2), x)
```

Mupad [**B**]
time = 1.40, size = 153, normalized size = 0.62

$$\frac{(-1)^{1/4}\, b\, e^{3/2} \operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{2\, a\, e\, \sqrt{e\cot(c+dx)}}{d} - \frac{2\, b\, (e\cot(c+dx))^{3/2}}{3\, d} - \frac{(-1)^{1/4}\, b\, e^{3/2} \operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{(-1)^{1/4}\, a\, e^{3/2} \operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) 1\mathrm{i}}{d} - \frac{(-1)^{1/4}\, a\, e^{3/2} \operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) 1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x)),x)
```

```
[Out] ((-1)^(1/4)*b*e^(3/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/d
- (2*a*e*(e*cot(c + d*x))^(1/2))/d - ((-1)^(1/4)*a*e^(3/2)*atan(((-1)^(1/4)
*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*a*e^(3/2)*atanh(((-1)
^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - (2*b*(e*cot(c + d*x))^(3/2)
)/(3*d) - ((-1)^(1/4)*b*e^(3/2)*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e
^(1/2)))/d
```

3.52 $\int \sqrt{e \cot(c+dx)}\,(a+b\cot(c+dx))\,dx$

Optimal. Leaf size=226

$$\frac{(a-b)\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{(a-b)\sqrt{e}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{2b\sqrt{e\cot}}{d}$$

```
[Out] 1/2*(a-b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-
1/2*(a-b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-
1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/
2)/d*2^(1/2)+1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))
^(1/2))*e^(1/2)/d*2^(1/2)-2*b*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt{e}\,(a-b)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{\sqrt{e}\,(a-b)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d}-\frac{\sqrt{e}\,(a+b)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}+\frac{\sqrt{e}\,(a+b)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}-\frac{2b\sqrt{e\cot(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x]),x]
```

```
[Out] ((a - b)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d) - ((a - b)*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]
)/(Sqrt[2]*d) - (2*b*Sqrt[e*Cot[c + d*x]])/d - ((a + b)*Sqrt[e]*Log[Sqrt[e]
 + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d) + ((
a + b)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c +
d*x]]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))\,dx = -\frac{2b\sqrt{e\cot(c+dx)}}{d} + \int \frac{-be+ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$$

$$= -\frac{2b\sqrt{e\cot(c+dx)}}{d} + \frac{2\mathrm{Subst}\left(\int \frac{be^2-aex^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right.}{d}$$

$$= -\frac{2b\sqrt{e\cot(c+dx)}}{d} - \frac{((a-b)e)\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c\right.}{d}$$

$$= -\frac{2b\sqrt{e\cot(c+dx)}}{d} - \frac{\left((a+b)\sqrt{e}\right)\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}\,x-x^2}\,d\right.}{2\sqrt{2}\,d}$$

$$= -\frac{2b\sqrt{e\cot(c+dx)}}{d} - \frac{(a+b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)\right.}{2\sqrt{2}\,d}$$

$$= \frac{(a-b)\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d} - \frac{(a-b)\sqrt{e}\tan}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.31, size = 155, normalized size = 0.69

$$-\frac{\sqrt{e\cot(c+dx)}\left(8b\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(c+dx)\right)+\sqrt{2}\,a\left(2\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)\sqrt{\tan(c+dx)}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x]),x]
```

```
[Out] -1/4*(Sqrt[e*Cot[c + d*x]]*(8*b*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*
x]^2] + Sqrt[2]*a*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c + d*
x]]))/d
```

Maple [A]
time = 0.41, size = 290, normalized size = 1.28

method	result

derivativedivides	$\frac{-2b\sqrt{e\cot(dx+c)}-2e\left(-\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\right.}{8}\right.}{}$
default	$\frac{-2b\sqrt{e\cot(dx+c)}-2e\left(-\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\right.}{8}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b*(e*cot(d*x+c))^(1/2)-2*e*(-1/8*b/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot
(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)
-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1))+1/8*a/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.51, size = 141, normalized size = 0.62

$$-\frac{\left(2\sqrt{2}(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}(a+b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}(a+b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\frac{8b}{\sqrt{\tan(dx+c)}}\right)e^{\frac{1}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))
) + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
 - sqrt(2)*(a + b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + s
qrt(2)*(a + b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 8*b/
sqrt(tan(d*x + c)))*e^(1/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e \cot (c + dx)}\, (a + b \cot (c + dx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)*(a+b*cot(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)*sqrt(e*cot(d*x + c)), x)
```

Mupad [B]
time = 0.73, size = 128, normalized size = 0.57

$$-\frac{2 b \sqrt{e \cot (c + d x)}}{d} - \frac{(-1)^{1/4} a \sqrt{e} \left(\mathrm{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot (c + d x)}}{\sqrt{e}}\right) - \mathrm{atanh}\left(\frac{(-1)^{1/4} \sqrt{e \cot (c + d x)}}{\sqrt{e}}\right)\right)}{d} - \frac{(-1)^{1/4} b \sqrt{e}\, \mathrm{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot (c + d x)}}{\sqrt{e}}\right) 1i}{d} - \frac{(-1)^{1/4} b \sqrt{e}\, \mathrm{atanh}\left(\frac{(-1)^{1/4} \sqrt{e \cot (c + d x)}}{\sqrt{e}}\right) 1i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x)),x)
```

```
[Out] - (2*b*(e*cot(c + d*x))^(1/2))/d - ((-1)^(1/4)*b*e^(1/2)*atan(((-1)^(1/4)*(
e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*b*e^(1/2)*atanh(((-1)^(
1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*a*e^(1/2)*(atan((
(-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)) - atanh(((-1)^(1/4)*(e*cot(c +
d*x))^(1/2))/e^(1/2))))/d
```

3.53 $\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal. Leaf size=208

$$\frac{(a+b)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}-\frac{(a+b)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{(a-b)\log\left(\sqrt{e}\right.}{}$$

```
[Out] 1/2*(a+b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)-
1/2*(a+b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+
1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(
1/2)/e^(1/2)-1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))
^(1/2))/d*2^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a+b)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}-\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{(a-b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}-\frac{(a-b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] ((a + b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqr
t[e]) - ((a + b)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d*Sqrt[e]) + ((a - b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[
e*Cot[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) - ((a - b)*Log[Sqrt[e] + Sqrt[e]*Co
t[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \frac{2\mathrm{Subst}\left(\int \frac{-ae-bx^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d}$$

$$= -\frac{(a-b)\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} - \frac{(a+b)\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{\right.}}{d}$$

$$= -\frac{(a+b)\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}\,x+x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2d} - \frac{(a+b)\mathrm{Subst}\left(\int \frac{}{e+}\right.}{}$$

$$= \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,d\sqrt{e}} - \frac{(a-b)\log\left(\sqrt{e}\right.}{}$$

$$= \frac{(a+b)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}} - \frac{(a+b)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+}}{\sqrt{e}}\right.}{\sqrt{2}\,d\sqrt{e}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.21, size = 166, normalized size = 0.80

$$\frac{3\sqrt{2}\,b\left(-2\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)+8a\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{12d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (3*Sqrt[2]*b*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sqrt
[2]*Sqrt[Tan[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]
] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) + 8*a*Hypergeometri
c2F1[3/4, 1, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(3/2))/(12*d*Sqrt[e*Cot[c +
 d*x]]*Sqrt[Tan[c + d*x]])
```

Maple [A]
time = 0.52, size = 273, normalized size = 1.31

method	result
derivativedivides	$-\frac{\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+\right.\right.}{4e}}{}$

default	$-\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right.\right.}{4e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4*b/(e^2)^(1/4)
*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(
1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+
2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Maxima [A]
time = 0.50, size = 130, normalized size = 0.62

$$-\frac{\left(2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}(a-b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}(a-b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)e^{\left(-\frac{1}{2}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))
) + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
 + sqrt(2)*(a - b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - s
qrt(2)*(a - b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*e^(-1
/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))**(1/2),x)
[Out] Integral((a + b*cot(c + d*x))/sqrt(e*cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)/sqrt(e*cot(d*x + c)), x)
```

Mupad [B]
time = 0.65, size = 118, normalized size = 0.57

$$\frac{(-1)^{1/4}\,b\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{d\,\sqrt{e}}-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{d\,\sqrt{e}}+\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d\,\sqrt{e}}+\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x))/(e*cot(c + d*x))^(1/2),x)
[Out] ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/(d*e^(1
/2)) + ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)
/(d*e^(1/2)) - ((-1)^(1/4)*b*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/
2)))/(d*e^(1/2)) + ((-1)^(1/4)*b*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/
e^(1/2)))/(d*e^(1/2))
```

3.54 $\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{(a-b)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}+\frac{(a-b)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}+\frac{2a}{de\sqrt{e\cot(c+d}}$$

```
[Out] -1/2*(a-b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)
+1/2*(a-b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)
+1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^
(3/2)*2^(1/2)-1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c)
)^(1/2))/d/e^(3/2)*2^(1/2)+2*a/d/e/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a-b)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}+\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{3/2}}+\frac{(a+b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}}-\frac{(a+b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}}+\frac{2a}{de\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] -(((a - b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e
^(3/2))) + ((a - b)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d*e^(3/2)) + (2*a)/(d*e*Sqrt[e*Cot[c + d*x]]) + ((a + b)*Log[Sqrt[e]
+ Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2
)) - ((a + b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d
*x]]])/(2*Sqrt[2]*d*e^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{\int \frac{be-ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^2}$$

$$= \frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{-be^2+aex^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2}$$

$$= \frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{(a-b)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de} - \frac{(a+b)\text{Sub}}{}$$

$$= \frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{(a+b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} +$$

$$= \frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}$$

$$= -\frac{(a-b)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} + \frac{(a-b)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.36, size = 196, normalized size = 0.86

$$\frac{3a\left(2\left(\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+8\sqrt{\tan(c+dx)}\right)+8b\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{12d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (3*a*(2*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - Sqrt[2]*ArcTan[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]]]) + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]] + 8*Sqrt[Tan[c + d*x]]) + 8*b*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*
x]^2]*Tan[c + d*x]^(3/2))/(12*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```

Maple [A]
time = 0.40, size = 295, normalized size = 1.29

method	result

derivativedivides	$\frac{\frac{2a}{e\sqrt{e\cot(dx+c)}}-2\left(\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\sqrt{\cdots}\right.\right.}{8e}\right.}{}$
default	$\frac{\frac{2a}{e\sqrt{e\cot(dx+c)}}-2\left(\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\sqrt{\cdots}\right.\right.}{8e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a/e/(e*cot(d*x+c))^(1/2)-2/e*(1/8*b/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot
(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)
-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1))-1/8*a/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.51, size = 141, normalized size = 0.62

$$\frac{\left(2\sqrt{2}(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}(a+b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}(a+b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+8a\sqrt{\tan(dx+c)}\right)e^{\left(-\frac{3}{2}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))))
 + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
- sqrt(2)*(a + b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sq
rt(2)*(a + b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 8*a*s
qrt(tan(d*x + c)))*e^(-3/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\cot{(c + dx)}}{(e\cot{(c + dx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))/(e*cot(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)/(e*cot(d*x + c))^(3/2), x)
```

Mupad [B]
time = 0.80, size = 137, normalized size = 0.60

$$\frac{2\,a}{d\,e\,\sqrt{e\,\cot\left(c+d\,x\right)}}+\frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{d\,e^{3/2}}-\frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)}{d\,e^{3/2}}+\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,1{}\mathrm{i}}{d\,e^{3/2}}+\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\,\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,1{}\mathrm{i}}{d\,e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x))/(e*cot(c + d*x))^(3/2),x)
```

```
[Out] (2*a)/(d*e*(e*cot(c + d*x))^(1/2)) + ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(
c + d*x))^(1/2))/e^(1/2)))/(d*e^(3/2)) - ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e
*cot(c + d*x))^(1/2))/e^(1/2)))/(d*e^(3/2)) + ((-1)^(1/4)*b*atan(((-1)^(1/4
)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/(d*e^(3/2)) + ((-1)^(1/4)*b*atanh(((
-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/(d*e^(3/2))
```

3.55 $\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=252

$$-\frac{(a+b)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}+\frac{(a+b)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}+\frac{2a}{3de(e\cot(c+}$$

```
[Out] 2/3*a/d/e/(e*cot(d*x+c))^(3/2)-1/2*(a+b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1
/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/2*(a+b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1
/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1
/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)+1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)
*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)+2*b/d/e^2/(e*cot(d
*x+c))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a+b)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}+\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{5/2}}-\frac{(a-b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}+\frac{(a-b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}+\frac{2b}{de^2\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] -(((a + b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e
^(5/2))) + ((a + b)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d*e^(5/2)) + (2*a)/(3*d*e*(e*Cot[c + d*x])^(3/2)) + (2*b)/(d*e^2*Sqrt
[e*Cot[c + d*x]]) - ((a - b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*S
qrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) + ((a - b)*Log[Sqrt[e] + Sqrt[e
]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{\int \frac{be-ae\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{e^2}$$

$$= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2b}{de^2\sqrt{e\cot(c+dx)}} + \frac{\int \frac{-ae^2-be^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^4}$$

$$= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2b}{de^2\sqrt{e\cot(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{ae^3+be^2x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(\ldots}\right.}{de^4}$$

$$= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2b}{de^2\sqrt{e\cot(c+dx)}} + \frac{(a-b)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\,c\ldots}\right.}{de^2}$$

$$= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2b}{de^2\sqrt{e\cot(c+dx)}} - \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2} \ldots\right.}{2\sqrt{2}de^{5/\ldots}}$$

$$= \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2b}{de^2\sqrt{e\cot(c+dx)}} - \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+d\ldots\right.}{2\sqrt{2}de^{\ldots}}$$

$$= -\frac{(a+b)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{(a+b)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c\ldots}}{\sqrt{e}}\right.}{\sqrt{2}de^{5/2}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.71, size = 196, normalized size = 0.78

$$\frac{3b\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+8\sqrt{\tan(c+dx)}\right)-8a\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\right)\tan^{\frac{3}{2}}(c+dx)}{12d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] (3*b*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*Sqrt[2]*ArcTan[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]] + 8*Sqrt[Tan[c + d*x]]) - 8*a*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Tan[
c + d*x]^2])*Tan[c + d*x]^(3/2))/(12*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^
(5/2))
```

Maple [**A**]
time = 0.40, size = 311, normalized size = 1.23

method	result
derivativedivides	$-\frac{2\left(-\frac{a\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot\left(dx+c\right)}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2\left(-\frac{a\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot\left(dx+c\right)}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(-2/e^2*(-1/8*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+2/3*a/e/(e*cot(d*x+c))^(3/2)+2*b/e^2/(e*cot(d*x+c))^(1/2))

Maxima [A]
time = 0.55, size = 154, normalized size = 0.61

$$\frac{\left(6\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+3\sqrt{2}(a-b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-3\sqrt{2}(a-b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+8\left(a+\frac{3b}{\tan(dx+c)}\right)\tan(dx+c)^{\frac{3}{2}}\right)e^{\left(-\frac{5}{2}\right)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/12*(6*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 6*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 3*sqrt(2)*(a - b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 3*sqrt(2)*(a - b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 8*(a + 3*b/tan(d*x + c))*tan(d*x + c)^(3/2))*e^(-5/2)/d

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))/(e*cot(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)/(e*cot(d*x + c))^(5/2), x)
```

Mupad [B]
time = 1.24, size = 158, normalized size = 0.63

$$\frac{2a}{3de(e\cot(c+dx))^{3/2}}+\frac{2b}{de^2\sqrt{e\cot(c+dx)}}+\frac{(-1)^{1/4}b\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}-\frac{(-1)^{1/4}b\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}-\frac{(-1)^{1/4}a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)1\mathrm{i}}{de^{5/2}}-\frac{(-1)^{1/4}a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)1\mathrm{i}}{de^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x))/(e*cot(c + d*x))^(5/2),x)
```

```
[Out] (2*a)/(3*d*e*(e*cot(c + d*x))^(3/2)) + (2*b)/(d*e^2*(e*cot(c + d*x))^(1/2))
 - ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/(d*e
^(5/2)) - ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*
1i)/(d*e^(5/2)) + ((-1)^(1/4)*b*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^
(1/2)))/(d*e^(5/2)) - ((-1)^(1/4)*b*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2
))/e^(1/2)))/(d*e^(5/2))
```

3.56 $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx$

Optimal. Leaf size=317

$$-\frac{(a^2+2ab-b^2)\,e^{3/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{(a^2+2ab-b^2)\,e^{3/2}\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+}}{\sqrt{e}}\right.}{\sqrt{2}\,d}$$

```
[Out] -4/3*a*b*(e*cot(d*x+c))^(3/2)/d-2/5*b^2*(e*cot(d*x+c))^(5/2)/d/e-1/2*(a^2+2
*a*b-b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)+
1/2*(a^2+2*a*b-b^2)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/
d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)
*(e*cot(d*x+c))^(1/2))/d*2^(1/2)+1/4*(a^2-2*a*b-b^2)*e^(3/2)*ln(e^(1/2)+cot
(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)-2*(a^2-b^2)*e*(e*co
t(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.23, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3624, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{e^{3/2}(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{e^{3/2}(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d}-\frac{e^{3/2}(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}+\frac{e^{3/2}(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}-\frac{2e(a^2-b^2)\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 2*a*b - b^2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sq
rt[e]])/(Sqrt[2]*d)) + ((a^2 + 2*a*b - b^2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqr
t[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d) - (2*(a^2 - b^2)*e*Sqrt[e*Cot[c +
d*x]])/d - (4*a*b*(e*Cot[c + d*x])^(3/2))/(3*d) - (2*b^2*(e*Cot[c + d*x])^(
5/2))/(5*d*e) - ((a^2 - 2*a*b - b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c +
d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d) + ((a^2 - 2*a*b - b^2)*
e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])
/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = -\frac{2b^2(e\cot(c+dx))^{5/2}}{5de} + \int (e\cot(c+dx))^{3/2}\left(a^2-b^2+2ab\,\mathrm{co}\right.$$

$$= -\frac{4ab(e\cot(c+dx))^{3/2}}{3d} - \frac{2b^2(e\cot(c+dx))^{5/2}}{5de} + \int \sqrt{e\cot(c}$$

$$= -\frac{2(a^2-b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{4ab(e\cot(c+dx))^{3/2}}{3d} - \frac{2b^2(e}{}$$

$$= -\frac{2(a^2-b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{4ab(e\cot(c+dx))^{3/2}}{3d} - \frac{2b^2(e}{}$$

$$= -\frac{2(a^2-b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{4ab(e\cot(c+dx))^{3/2}}{3d} - \frac{2b^2(e}{}$$

$$= -\frac{2(a^2-b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{4ab(e\cot(c+dx))^{3/2}}{3d} - \frac{2b^2(e}{}$$

$$= -\frac{2(a^2-b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{4ab(e\cot(c+dx))^{3/2}}{3d} - \frac{2b^2(e}{}$$

$$= -\frac{(a^2+2ab-b^2)\,e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d} + \frac{(a^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.10, size = 224, normalized size = 0.71

$$-\frac{(e\cot(c+dx))^{3/2}\left(\frac{2}{5}b^2\cot^{\frac{5}{2}}(c+dx)-\frac{4}{3}ab\cot^{\frac{3}{2}}(c+dx)\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)+\frac{1}{4}(a^2-b^2)\left(2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{d\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -(((e*Cot[c + d*x])^(3/2)*((2*b^2*Cot[c + d*x]^(5/2))/5 - (4*a*b*Cot[c + d*
x]^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/3 + ((a^2
```

```
- b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan
[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - S
qrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Co
t[c + d*x]] + Cot[c + d*x]]))/4))/(d*Cot[c + d*x]^(3/2)))
```

Maple [A]
time = 0.54, size = 360, normalized size = 1.14

method	result
derivativedivides	$-\frac{2\left(\frac{b^2(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2aeb(e\cot(dx+c))^{\frac{3}{2}}}{3}+a^2e^2\sqrt{e\cot(dx+c)}-b^2e^2\sqrt{e\cot(dx+c)}-e^3\left(\frac{(a^2e-b^2e)(\cdots}{\cdots}\right.\right.}{\cdots}$
default	$-\frac{2\left(\frac{b^2(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2aeb(e\cot(dx+c))^{\frac{3}{2}}}{3}+a^2e^2\sqrt{e\cot(dx+c)}-b^2e^2\sqrt{e\cot(dx+c)}-e^3\left(\frac{(a^2e-b^2e)(\cdots}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e*(1/5*b^2*(e*cot(d*x+c))^(5/2)+2/3*a*e*b*(e*cot(d*x+c))^(3/2)+a^2*e^2
*(e*cot(d*x+c))^(1/2)-b^2*e^2*(e*cot(d*x+c))^(1/2)-e^3*(1/8*(a^2*e-b^2*e)*(
e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2
^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan
(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/4*a*b/(e^2)^(1/4)*2^(1/2)*
(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*
cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(
2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.51, size = 211, normalized size = 0.67

$$\frac{\left(30\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+30\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+15\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-15\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{80\,ab}{\tan(dx+c)^{\frac{3}{2}}}-\frac{120\,(a^2-b^2)}{\sqrt{\tan(dx+c)}}-\frac{24\,b^2}{\tan(dx+c)^{\frac{5}{2}}}\right)e^{\frac{3}{2}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/60*(30*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(t
an(d*x + c)))) + 30*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2
) - 2/sqrt(tan(d*x + c)))) + 15*sqrt(2)*(a^2 - 2*a*b - b^2)*log(sqrt(2)/sqr
```

```
t(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 15*sqrt(2)*(a^2 - 2*a*b - b^2)*log(
-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 80*a*b/tan(d*x + c)^(3/
2) - 120*(a^2 - b^2)/sqrt(tan(d*x + c)) - 24*b^2/tan(d*x + c)^(5/2))*e^(3/2
)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e \cot (c + dx))^{\frac{3}{2}} (a + b \cot (c + dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+b*cot(d*x+c))**2,x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(3/2), x)
```

Mupad [**B**]
time = 2.47, size = 1274, normalized size = 4.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))^2,x)
```

```
[Out] atan((a^4*e^6*(e*cot(c + d*x))^(1/2)*((a*b^3*e^3)/d^2 - (b^4*e^3*1i)/(4*d^2
) - (a^4*e^3*1i)/(4*d^2) - (a^3*b*e^3)/d^2 + (a^2*b^2*e^3*3i)/(2*d^2))^(1/2
)*32i)/((a^6*e^8*16i)/d - (b^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*a^5*b*e^
```

8)/d + (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d - (a^4*b^2*e^8*112i)/d) + (b^4*e^6*(e*cot(c + d*x))^(1/2)*((a*b^3*e^3)/d^2 - (b^4*e^3*1i)/(4*d^2) - (a^4*e^3*1i)/(4*d^2) - (a^3*b*e^3)/d^2 + (a^2*b^2*e^3*3i)/(2*d^2))^(1/2)*32i)/((a^6*e^8*16i)/d - (b^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*a^5*b*e^8)/d + (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d - (a^4*b^2*e^8*112i)/d) - (a^2*b^2*e^6*(e*cot(c + d*x))^(1/2)*((a*b^3*e^3)/d^2 - (b^4*e^3*1i)/(4*d^2) - (a^4*e^3*1i)/(4*d^2) - (a^3*b*e^3)/d^2 + (a^2*b^2*e^3*3i)/(2*d^2))^(1/2)*192i)/((a^6*e^8*16i)/d - (b^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*a^5*b*e^8)/d + (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d - (a^4*b^2*e^8*112i)/d))*(-(a^4*e^3*1i + b^4*e^3*1i - 4*a*b^3*e^3 + 4*a^3*b*e^3 - a^2*b^2*e^3*6i)/(4*d^2))^(1/2)*2i + atan((a^4*e^6*(e*cot(c + d*x))^(1/2)*((a^4*e^3*1i)/(4*d^2) + (b^4*e^3*1i)/(4*d^2) + (a*b^3*e^3)/d^2 - (a^3*b*e^3)/d^2 - (a^2*b^2*e^3*3i)/(2*d^2))^(1/2)*32i)/((b^6*e^8*16i)/d - (a^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*a^5*b*e^8)/d - (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d + (a^4*b^2*e^8*112i)/d) + (b^4*e^6*(e*cot(c + d*x))^(1/2)*((a^4*e^3*1i)/(4*d^2) + (b^4*e^3*1i)/(4*d^2) + (a*b^3*e^3)/d^2 - (a^3*b*e^3)/d^2 - (a^2*b^2*e^3*3i)/(2*d^2))^(1/2)*32i)/((b^6*e^8*16i)/d - (a^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*a^5*b*e^8)/d - (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d + (a^4*b^2*e^8*112i)/d) - (a^2*b^2*e^6*(e*cot(c + d*x))^(1/2)*((a^4*e^3*1i)/(4*d^2) + (b^4*e^3*1i)/(4*d^2) + (a*b^3*e^3)/d^2 - (a^3*b*e^3)/d^2 - (a^2*b^2*e^3*3i)/(2*d^2))^(1/2)*192i)/((b^6*e^8*16i)/d - (a^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*a^5*b*e^8)/d - (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d + (a^4*b^2*e^8*112i)/d))*((a^4*e^3*1i + b^4*e^3*1i + 4*a*b^3*e^3 - 4*a^3*b*e^3 - a^2*b^2*e^3*6i)/(4*d^2))^(1/2)*2i - (e*cot(c + d*x))^(1/2)*((2*a^2*e)/d - (2*b^2*e)/d) - (2*b^2*(e*cot(c + d*x))^(5/2))/(5*d*e) - (4*a*b*(e*cot(c + d*x))^(3/2))/(3*d)

3.57 $\int \sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^2\,dx$

Optimal. Leaf size=288

$$\frac{(a^2-2ab-b^2)\sqrt{e}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{(a^2-2ab-b^2)\sqrt{e}\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right.}{\sqrt{2}\,d}$$

```
[Out] -2/3*b^2*(e*cot(d*x+c))^(3/2)/d/e+1/2*(a^2-2*a*b-b^2)*arctan(1-2^(1/2)*(e*c
ot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-1/2*(a^2-2*a*b-b^2)*arctan(1+2^
(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*l
n(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2
)+1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^
(1/2))*e^(1/2)/d*2^(1/2)-4*a*b*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3624, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt{e}(a^2-2ab-b^2)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{\sqrt{e}(a^2-2ab-b^2)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d}-\frac{\sqrt{e}(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}+\frac{\sqrt{e}(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}-\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2,x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt
[e]])/(Sqrt[2]*d) - ((a^2 - 2*a*b - b^2)*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e
*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d) - (4*a*b*Sqrt[e*Cot[c + d*x]])/d - (2
*b^2*(e*Cot[c + d*x])^(3/2))/(3*d*e) - ((a^2 + 2*a*b - b^2)*Sqrt[e]*Log[Sqr
t[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d)
+ ((a^2 + 2*a*b - b^2)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]
*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
```

```
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^2\,dx &= -\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}+\int \sqrt{e\cot(c+dx)}\,\left(a^2-b^2+2ab\cot(\right.\\
&= -\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}+\int \frac{-2abe+(a}{\sqrt{e}}\\
&= -\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}+\frac{2\text{Subst}\left(\int \frac{2abe^2}{}\right.}{}\\
&= -\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}-\frac{((a^2-2ab-b}{}\\
&= -\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}-\frac{((a^2+2ab-b}{}\\
&= -\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}-\frac{(a^2+2ab-b^2}{}\\
&= \frac{(a^2-2ab-b^2)\sqrt{e}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{(a^2-}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.57, size = 220, normalized size = 0.76

$$-\frac{\sqrt{e\cot(c+dx)}\left(4(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)+b\left(6\sqrt{2}\,a\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-6\sqrt{2}\,a\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+24a\sqrt{\cot(c+dx)}+4b\cot^{\frac{3}{2}}(c+dx)+3\sqrt{2}\,a\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-3\sqrt{2}\,a\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{6d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -1/6*(Sqrt[e*Cot[c + d*x]]*(4*(a^2 - b^2)*Cot[c + d*x]^(3/2)*Hypergeometric
2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + b*(6*Sqrt[2]*a*ArcTan[1 - Sqrt[2]*Sqrt[
Cot[c + d*x]]] - 6*Sqrt[2]*a*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24*a*
Sqrt[Cot[c + d*x]] + 4*b*Cot[c + d*x]^(3/2) + 3*Sqrt[2]*a*Log[1 - Sqrt[2]*S
qrt[Cot[c + d*x]] + Cot[c + d*x]] - 3*Sqrt[2]*a*Log[1 + Sqrt[2]*Sqrt[Cot[c
+ d*x]] + Cot[c + d*x]])))/(d*Sqrt[Cot[c + d*x]])
```

Maple [A]
time = 0.57, size = 321, normalized size = 1.11

method	result
derivativedivides	$-\frac{2\left(\frac{b^2 (e\cot(dx+c))^{\frac{3}{2}}}{3}+2abe\sqrt{e\cot\left(dx+c\right)}+e^2\left(-\frac{ab\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right.}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right.}}\right.\right.}{}\right.\right.}{}$
default	$-\frac{2\left(\frac{b^2 (e\cot(dx+c))^{\frac{3}{2}}}{3}+2abe\sqrt{e\cot\left(dx+c\right)}+e^2\left(-\frac{ab\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right.}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right.}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e*(1/3*b^2*(e*cot(d*x+c))^(3/2)+2*a*b*e*(e*cot(d*x+c))^(1/2)+e^2*(-1/4
*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*ar
ctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(a^2-b^2)/(e^2)^(1/4
)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^
(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))
+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.52, size = 192, normalized size = 0.67

$$-\frac{\left(6\sqrt{2}\left(a^2-2ab-b^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+6\sqrt{2}\left(a^2-2ab-b^2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-3\sqrt{2}\left(a^2+2ab-b^2\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+3\sqrt{2}\left(a^2+2ab-b^2\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\frac{48ab}{\sqrt{\tan\left(dx+c\right)}}+\frac{8b^2}{\tan(dx+c)^{\frac{3}{2}}}\right)e^{\frac{1}{2}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(6*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(t
an(d*x + c)))) + 6*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2)
 - 2/sqrt(tan(d*x + c)))) - 3*sqrt(2)*(a^2 + 2*a*b - b^2)*log(sqrt(2)/sqrt(
tan(d*x + c)) + 1/tan(d*x + c) + 1) + 3*sqrt(2)*(a^2 + 2*a*b - b^2)*log(-sq
rt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 48*a*b/sqrt(tan(d*x + c))
+ 8*b^2/tan(d*x + c)^(3/2))*e^(1/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e \cot{\left(c + dx \right)}} \left(a + b \cot{\left(c + dx \right)}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)*(a+b*cot(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^2*sqrt(e*cot(d*x + c)), x)
```

Mupad [B]
time = 1.21, size = 1157, normalized size = 4.02

$$-\frac{2b^2\,{\left(e\cot\left(c+d\,x\right)\right)}^{3/2}}{3\,d\,e}-\frac{4\,a\,b\,\sqrt{e\cot\left(c+d\,x\right)}}{d}-\mathrm{atan}\left(\frac{a^4\,e^4\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b\,e}{d^2}-\frac{a\,b^3\,e}{d^2}-\frac{a^4\,e\,1i}{4\,d^2}-\frac{b^4\,e\,1i}{4\,d^2}+\frac{a^2\,b^2\,e\,3i}{2\,d^2}}\,32i}{\frac{16\,a^6\,e^5}{d}-\frac{16\,b^6\,e^5}{d}+\frac{112\,a^2\,b^4\,e^5}{d}-\frac{112\,a^4\,b^2\,e^5}{d}+\frac{a\,b^5\,e^5\,32i}{d}+\frac{a^5\,b\,e^5\,32i}{d}-\frac{a^3\,b^3\,e^5\,192i}{d}}+\frac{b^4\,e^4\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b\,e}{d^2}-\frac{a\,b^3\,e}{d^2}-\frac{a^4\,e\,1i}{4\,d^2}-\frac{b^4\,e\,1i}{4\,d^2}+\frac{a^2\,b^2\,e\,3i}{2\,d^2}}\,32i}{\frac{16\,a^6\,e^5}{d}-\frac{16\,b^6\,e^5}{d}+\frac{112\,a^2\,b^4\,e^5}{d}-\frac{112\,a^4\,b^2\,e^5}{d}+\frac{a\,b^5\,e^5\,32i}{d}+\frac{a^5\,b\,e^5\,32i}{d}-\frac{a^3\,b^3\,e^5\,192i}{d}}-\frac{a^2\,b^2\,e^4\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b\,e}{d^2}-\frac{a\,b^3\,e}{d^2}-\frac{a^4\,e\,1i}{4\,d^2}-\frac{b^4\,e\,1i}{4\,d^2}+\frac{a^2\,b^2\,e\,3i}{2\,d^2}}\,192i}{\frac{16\,a^6\,e^5}{d}-\frac{16\,b^6\,e^5}{d}+\frac{112\,a^2\,b^4\,e^5}{d}-\frac{112\,a^4\,b^2\,e^5}{d}+\frac{a\,b^5\,e^5\,32i}{d}+\frac{a^5\,b\,e^5\,32i}{d}-\frac{a^3\,b^3\,e^5\,192i}{d}}\right)\,\sqrt{-\frac{1i\,e\,a^4-4\,e\,a^3\,b-6i\,e\,a^2\,b^2+4\,e\,a\,b^3+1i\,e\,b^4}{4\,d^2}}\,2i+\mathrm{atan}\left(\frac{a^4\,e^4\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b\,e}{d^2}-\frac{a\,b^3\,e}{d^2}+\frac{a^4\,e\,1i}{4\,d^2}+\frac{b^4\,e\,1i}{4\,d^2}-\frac{a^2\,b^2\,e\,3i}{2\,d^2}}\,32i}{\frac{16\,b^6\,e^5}{d}-\frac{16\,a^6\,e^5}{d}-\frac{112\,a^2\,b^4\,e^5}{d}+\frac{112\,a^4\,b^2\,e^5}{d}+\frac{a\,b^5\,e^5\,32i}{d}+\frac{a^5\,b\,e^5\,32i}{d}-\frac{a^3\,b^3\,e^5\,192i}{d}}+\frac{b^4\,e^4\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b\,e}{d^2}-\frac{a\,b^3\,e}{d^2}+\frac{a^4\,e\,1i}{4\,d^2}+\frac{b^4\,e\,1i}{4\,d^2}-\frac{a^2\,b^2\,e\,3i}{2\,d^2}}\,32i}{\frac{16\,b^6\,e^5}{d}-\frac{16\,a^6\,e^5}{d}-\frac{112\,a^2\,b^4\,e^5}{d}+\frac{112\,a^4\,b^2\,e^5}{d}+\frac{a\,b^5\,e^5\,32i}{d}+\frac{a^5\,b\,e^5\,32i}{d}-\frac{a^3\,b^3\,e^5\,192i}{d}}-\frac{a^2\,b^2\,e^4\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b\,e}{d^2}-\frac{a\,b^3\,e}{d^2}+\frac{a^4\,e\,1i}{4\,d^2}+\frac{b^4\,e\,1i}{4\,d^2}-\frac{a^2\,b^2\,e\,3i}{2\,d^2}}\,192i}{\frac{16\,b^6\,e^5}{d}-\frac{16\,a^6\,e^5}{d}-\frac{112\,a^2\,b^4\,e^5}{d}+\frac{112\,a^4\,b^2\,e^5}{d}+\frac{a\,b^5\,e^5\,32i}{d}+\frac{a^5\,b\,e^5\,32i}{d}-\frac{a^3\,b^3\,e^5\,192i}{d}}\right)\,\sqrt{\frac{1i\,e\,a^4+4\,e\,a^3\,b-6i\,e\,a^2\,b^2-4\,e\,a\,b^3+1i\,e\,b^4}{4\,d^2}}\,2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))^2,x)
```

```
[Out] atan((a^4*e^4*(e*cot(c + d*x))^(1/2)*((a^4*e*1i)/(4*d^2) + (b^4*e*1i)/(4*d^
2) - (a^2*b^2*e*3i)/(2*d^2) - (a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*32i)/((1
6*b^6*e^5)/d - (16*a^6*e^5)/d + (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d - (11
2*a^2*b^4*e^5)/d - (a^3*b^3*e^5*192i)/d + (112*a^4*b^2*e^5)/d) + (b^4*e^4*(
e*cot(c + d*x))^(1/2)*((a^4*e*1i)/(4*d^2) + (b^4*e*1i)/(4*d^2) - (a^2*b^2*e
*3i)/(2*d^2) - (a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*32i)/((16*b^6*e^5)/d -
(16*a^6*e^5)/d + (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d - (112*a^2*b^4*e^5)/
d - (a^3*b^3*e^5*192i)/d + (112*a^4*b^2*e^5)/d) - (a^2*b^2*e^4*(e*cot(c + d
*x))^(1/2)*((a^4*e*1i)/(4*d^2) + (b^4*e*1i)/(4*d^2) - (a^2*b^2*e*3i)/(2*d^2
) - (a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*192i)/((16*b^6*e^5)/d - (16*a^6*e^
5)/d + (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d - (112*a^2*b^4*e^5)/d - (a^3*b
^3*e^5*192i)/d + (112*a^4*b^2*e^5)/d))*((a^4*e*1i + b^4*e*1i - a^2*b^2*e*6i
```

```
 - 4*a*b^3*e + 4*a^3*b*e)/(4*d^2))^(1/2)*2i - atan((a^4*e^4*(e*cot(c + d*x)
)^(1/2)*((a^2*b^2*e*3i)/(2*d^2) - (b^4*e*1i)/(4*d^2) - (a^4*e*1i)/(4*d^2) -
 (a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*32i)/((16*a^6*e^5)/d - (16*b^6*e^5)/d
 + (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d + (112*a^2*b^4*e^5)/d - (a^3*b^3*e
^5*192i)/d - (112*a^4*b^2*e^5)/d) + (b^4*e^4*(e*cot(c + d*x))^(1/2)*((a^2*b
^2*e*3i)/(2*d^2) - (b^4*e*1i)/(4*d^2) - (a^4*e*1i)/(4*d^2) - (a*b^3*e)/d^2
+ (a^3*b*e)/d^2)^(1/2)*32i)/((16*a^6*e^5)/d - (16*b^6*e^5)/d + (a*b^5*e^5*3
2i)/d + (a^5*b*e^5*32i)/d + (112*a^2*b^4*e^5)/d - (a^3*b^3*e^5*192i)/d - (1
12*a^4*b^2*e^5)/d) - (a^2*b^2*e^4*(e*cot(c + d*x))^(1/2)*((a^2*b^2*e*3i)/(2
*d^2) - (b^4*e*1i)/(4*d^2) - (a^4*e*1i)/(4*d^2) - (a*b^3*e)/d^2 + (a^3*b*e)
/d^2)^(1/2)*192i)/((16*a^6*e^5)/d - (16*b^6*e^5)/d + (a*b^5*e^5*32i)/d + (a
^5*b*e^5*32i)/d + (112*a^2*b^4*e^5)/d - (a^3*b^3*e^5*192i)/d - (112*a^4*b^2
*e^5)/d))*(-(a^4*e*1i + b^4*e*1i - a^2*b^2*e*6i + 4*a*b^3*e - 4*a^3*b*e)/(4
*d^2))^(1/2)*2i - (2*b^2*(e*cot(c + d*x))^(3/2))/(3*d*e) - (4*a*b*(e*cot(c
+ d*x))^(1/2))/d
```

3.58 $\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal. Leaf size=267

$$\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}-\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}-$$

```
[Out] 1/2*(a^2+2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2
)/e^(1/2)-1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2)
)/d*2^(1/2)/e^(1/2)+1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/
2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)/e^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+c
ot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)/e^(1/2)-2*b^2*(e*
cot(d*x+c))^(1/2)/d/e
```

Rubi [A]

time = 0.17, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3624, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}-\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}-\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}-\frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] ((a^2 + 2*a*b - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(S
qrt[2]*d*Sqrt[e]) - ((a^2 + 2*a*b - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c +
 d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[e]) - (2*b^2*Sqrt[e*Cot[c + d*x]])/(d*e)
+ ((a^2 - 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*
Cot[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) - ((a^2 - 2*a*b - b^2)*Log[Sqrt[e] +
Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx &= -\frac{2b^2\sqrt{e\cot(c+dx)}}{de} + \int \frac{a^2-b^2+2ab\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx \\
&= -\frac{2b^2\sqrt{e\cot(c+dx)}}{de} + \frac{2\mathrm{Subst}\left(\int \frac{-(a^2-b^2)e-2abx^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} \\
&= -\frac{2b^2\sqrt{e\cot(c+dx)}}{de} - \frac{(a^2-2ab-b^2)\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{d} \\
&= -\frac{2b^2\sqrt{e\cot(c+dx)}}{de} - \frac{(a^2+2ab-b^2)\,\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}\,x+x^2}\,dx, x, \sqrt{e\cot(\right.}{2d} \\
&= -\frac{2b^2\sqrt{e\cot(c+dx)}}{de} + \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\right.}{2\sqrt{2}\,d\sqrt{e}} \\
&= \frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}} - \frac{(a^2+2ab-b^2)\tan^{-1}\left(1\right.}{\sqrt{2}\,d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.91, size = 192, normalized size = 0.72

$$
-\frac{\sqrt{\cot(c+dx)}\left(2b^2\sqrt{\cot(c+dx)}+\frac{4}{3}ab\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)-\frac{(a^2-b^2)\left(2\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{2\sqrt{2}}\right)}{d\sqrt{e\cot(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] -((Sqrt[Cot[c + d*x]]*(2*b^2*Sqrt[Cot[c + d*x]] + (4*a*b*Cot[c + d*x]^(3/2)
*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])/3 - ((a^2 - b^2)*(2*ArcTa
n[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]] + Cot[c + d*x]]))/(2*Sqrt[2])))/(d*Sqrt[e*Cot[c + d*x]]))
```

Maple [A]
time = 0.47, size = 306, normalized size = 1.15

method	result

derivativedivides	$-\frac{2\left(b^2\sqrt{e\cot(dx+c)}+e\left(\frac{(a^2e-b^2e)(e^2)^{\frac14}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right.\right.}{}\right.\right.}{}$
default	$-\frac{2\left(b^2\sqrt{e\cot(dx+c)}+e\left(\frac{(a^2e-b^2e)(e^2)^{\frac14}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac14}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e*(b^2*(e*cot(d*x+c))^(1/2)+e*(1/8*(a^2*e-b^2*e)*(e^2)^(1/4)/e^2*2^(1/
2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/
(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arct
an(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)+1))+1/4*a*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1))))
```

Maxima [A]
time = 0.50, size = 179, normalized size = 0.67

$$-\frac{\left(2\sqrt{2}\,(a^2+2ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\,(a^2+2ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\,(a^2-2ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\,(a^2-2ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\frac{8b^2}{\sqrt{\tan(dx+c)}}\right)e^{\left(-\frac{1}{2}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(ta
n(d*x + c)))) + 2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2)
- 2/sqrt(tan(d*x + c)))) + sqrt(2)*(a^2 - 2*a*b - b^2)*log(sqrt(2)/sqrt(tan
(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(a^2 - 2*a*b - b^2)*log(-sqrt(2)
/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 8*b^2/sqrt(tan(d*x + c)))*e^(-1
/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\cot{(c + dx)})^2}{\sqrt{e\cot{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))**2/sqrt(e*cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^2/sqrt(e*cot(d*x + c)), x)
```

Mupad [B]
time = 1.01, size = 1234, normalized size = 4.62

$$2\,\mathrm{atanh}\left(\frac{32\,a^4\,e^2\,\sqrt{e\,\cot(c+d\,x)}\,\sqrt{\frac{a\,b^3}{d^2\,e}-\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e}-\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e}-\frac{a^3\,b}{d^2\,e}+\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e}}}{\frac{a^6\,e^2\,16{}\mathrm{i}}{d}-\frac{b^6\,e^2\,16{}\mathrm{i}}{d}+\frac{32\,a\,b^5\,e^2}{d}+\frac{32\,a^5\,b\,e^2}{d}+\frac{a^2\,b^4\,e^2\,112{}\mathrm{i}}{d}-\frac{192\,a^3\,b^3\,e^2}{d}-\frac{a^4\,b^2\,e^2\,112{}\mathrm{i}}{d}}+\frac{32\,b^4\,e^2\,\sqrt{e\,\cot(c+d\,x)}\,\sqrt{\frac{a\,b^3}{d^2\,e}-\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e}-\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e}-\frac{a^3\,b}{d^2\,e}+\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e}}}{\frac{a^6\,e^2\,16{}\mathrm{i}}{d}-\frac{b^6\,e^2\,16{}\mathrm{i}}{d}+\frac{32\,a\,b^5\,e^2}{d}+\frac{32\,a^5\,b\,e^2}{d}+\frac{a^2\,b^4\,e^2\,112{}\mathrm{i}}{d}-\frac{192\,a^3\,b^3\,e^2}{d}-\frac{a^4\,b^2\,e^2\,112{}\mathrm{i}}{d}}-\frac{192\,a^2\,b^2\,e^2\,\sqrt{e\,\cot(c+d\,x)}\,\sqrt{\frac{a\,b^3}{d^2\,e}-\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e}-\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e}-\frac{a^3\,b}{d^2\,e}+\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e}}}{\frac{a^6\,e^2\,16{}\mathrm{i}}{d}-\frac{b^6\,e^2\,16{}\mathrm{i}}{d}+\frac{32\,a\,b^5\,e^2}{d}+\frac{32\,a^5\,b\,e^2}{d}+\frac{a^2\,b^4\,e^2\,112{}\mathrm{i}}{d}-\frac{192\,a^3\,b^3\,e^2}{d}-\frac{a^4\,b^2\,e^2\,112{}\mathrm{i}}{d}}\right)\,\sqrt{\frac{a\,b^3}{d^2\,e}-\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e}-\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e}-\frac{a^3\,b}{d^2\,e}+\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e}}+2\,\mathrm{atanh}\left(\frac{32\,a^4\,e^2\,\sqrt{e\,\cot(c+d\,x)}\,\sqrt{\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e}+\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e}+\frac{a\,b^3}{d^2\,e}-\frac{a^3\,b}{d^2\,e}-\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e}}}{\frac{32\,a\,b^5\,e^2}{d}+\frac{b^6\,e^2\,16{}\mathrm{i}}{d}-\frac{a^6\,e^2\,16{}\mathrm{i}}{d}+\frac{32\,a^5\,b\,e^2}{d}-\frac{a^2\,b^4\,e^2\,112{}\mathrm{i}}{d}-\frac{192\,a^3\,b^3\,e^2}{d}+\frac{a^4\,b^2\,e^2\,112{}\mathrm{i}}{d}}+\frac{32\,b^4\,e^2\,\sqrt{e\,\cot(c+d\,x)}\,\sqrt{\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e}+\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e}+\frac{a\,b^3}{d^2\,e}-\frac{a^3\,b}{d^2\,e}-\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e}}}{\frac{32\,a\,b^5\,e^2}{d}+\frac{b^6\,e^2\,16{}\mathrm{i}}{d}-\frac{a^6\,e^2\,16{}\mathrm{i}}{d}+\frac{32\,a^5\,b\,e^2}{d}-\frac{a^2\,b^4\,e^2\,112{}\mathrm{i}}{d}-\frac{192\,a^3\,b^3\,e^2}{d}+\frac{a^4\,b^2\,e^2\,112{}\mathrm{i}}{d}}-\frac{192\,a^2\,b^2\,e^2\,\sqrt{e\,\cot(c+d\,x)}\,\sqrt{\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e}+\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e}+\frac{a\,b^3}{d^2\,e}-\frac{a^3\,b}{d^2\,e}-\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e}}}{\frac{32\,a\,b^5\,e^2}{d}+\frac{b^6\,e^2\,16{}\mathrm{i}}{d}-\frac{a^6\,e^2\,16{}\mathrm{i}}{d}+\frac{32\,a^5\,b\,e^2}{d}-\frac{a^2\,b^4\,e^2\,112{}\mathrm{i}}{d}-\frac{192\,a^3\,b^3\,e^2}{d}+\frac{a^4\,b^2\,e^2\,112{}\mathrm{i}}{d}}\right)\,\sqrt{\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e}+\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e}+\frac{a\,b^3}{d^2\,e}-\frac{a^3\,b}{d^2\,e}-\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e}}-\frac{2\,b^2\,\sqrt{e\,\cot(c+d\,x)}}{d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x))^2/(e*cot(c + d*x))^(1/2),x)
```

```
[Out] 2*atanh((32*a^4*e^2*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e) - (b^4*1i)/(4*d
^2*e) - (a^4*1i)/(4*d^2*e) - (a^3*b)/(d^2*e) + (a^2*b^2*3i)/(2*d^2*e))^(1/2
))/((a^6*e^2*16i)/d - (b^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2)/d
 + (a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d - (a^4*b^2*e^2*112i)/d) + (32
*b^4*e^2*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e) - (b^4*1i)/(4*d^2*e) - (a^
4*1i)/(4*d^2*e) - (a^3*b)/(d^2*e) + (a^2*b^2*3i)/(2*d^2*e))^(1/2))/((a^6*e^
2*16i)/d - (b^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2)/d + (a^2*b^4
*e^2*112i)/d - (192*a^3*b^3*e^2)/d - (a^4*b^2*e^2*112i)/d) - (192*a^2*b^2*e
^2*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e) - (b^4*1i)/(4*d^2*e) - (a^4*1i)/
(4*d^2*e) - (a^3*b)/(d^2*e) + (a^2*b^2*3i)/(2*d^2*e))^(1/2))/((a^6*e^2*16i)
/d - (b^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2)/d + (a^2*b^4*e^2*1
12i)/d - (192*a^3*b^3*e^2)/d - (a^4*b^2*e^2*112i)/d))*((a*b^3)/(d^2*e) - (b
```

```
^4*1i)/(4*d^2*e) - (a^4*1i)/(4*d^2*e) - (a^3*b)/(d^2*e) + (a^2*b^2*3i)/(2*d
^2*e))^(1/2) + 2*atanh((32*a^4*e^2*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*
e) + (b^4*1i)/(4*d^2*e) + (a*b^3)/(d^2*e) - (a^3*b)/(d^2*e) - (a^2*b^2*3i)/
(2*d^2*e))^(1/2))/((b^6*e^2*16i)/d - (a^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (
32*a^5*b*e^2)/d - (a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d + (a^4*b^2*e^2
*112i)/d) + (32*b^4*e^2*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e) + (b^4*1
i)/(4*d^2*e) + (a*b^3)/(d^2*e) - (a^3*b)/(d^2*e) - (a^2*b^2*3i)/(2*d^2*e))^
(1/2))/((b^6*e^2*16i)/d - (a^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^
2)/d - (a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d + (a^4*b^2*e^2*112i)/d) -
 (192*a^2*b^2*e^2*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e) + (b^4*1i)/(4*
d^2*e) + (a*b^3)/(d^2*e) - (a^3*b)/(d^2*e) - (a^2*b^2*3i)/(2*d^2*e))^(1/2))
/((b^6*e^2*16i)/d - (a^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2)/d -
 (a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d + (a^4*b^2*e^2*112i)/d))*((a^4*
1i)/(4*d^2*e) + (b^4*1i)/(4*d^2*e) + (a*b^3)/(d^2*e) - (a^3*b)/(d^2*e) - (a
^2*b^2*3i)/(2*d^2*e))^(1/2) - (2*b^2*(e*cot(c + d*x))^(1/2))/(d*e)
```

3.59 $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}+\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}$$

```
[Out] -1/2*(a^2-2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(3/
2)*2^(1/2)+1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2
))/d/e^(3/2)*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1
/2)*(e*cot(d*x+c))^(1/2))/d/e^(3/2)*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+
cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(3/2)*2^(1/2)+2*a^2/d/
e/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3623, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}+\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{3/2}}+\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}}-\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}}+\frac{2a^2}{de\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] -(((a^2 - 2*a*b - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
(Sqrt[2]*d*e^(3/2))) + ((a^2 - 2*a*b - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[
c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)) + (2*a^2)/(d*e*Sqrt[e*Cot[c + d*x]
]) + ((a^2 + 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt
[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - ((a^2 + 2*a*b - b^2)*Log[Sqrt[e]
 + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(3/
2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx &= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{\int \frac{2abe-(a^2-b^2)e\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^2} \\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{-2abe^2+(a^2-b^2)ex^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de^2} \\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{(a^2-2ab-b^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{de} - \\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{(a^2+2ab-b^2)\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cot(c}{2\sqrt{2}de^{3/2}} \\
&= \frac{2a^2}{de\sqrt{e\cot(c+dx)}} + \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot}\right.}{2\sqrt{2}de^{3/2}} \\
&= -\frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} + \frac{(a^2-2ab-b^2)\tan^{-1}\left(\right.}{\sqrt{2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.33, size = 218, normalized size = 0.82

$$
-\frac{\cot^{\frac{3}{2}}(c+dx)\left(-\frac{2b^2}{\sqrt{\cot(c+dx)}}-\frac{2(a^2-b^2)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)}{\sqrt{\cot(c+dx)}}+4ab\left(\frac{1}{2}\left(-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}+\frac{\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(-\frac{\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)}{2\sqrt{2}}\right)\right)\right)}{d(e\cot(c+dx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] -((Cot[c + d*x]^(3/2)*((-2*b^2)/Sqrt[Cot[c + d*x]] - (2*(a^2 - b^2)*Hyperge
ometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2])/Sqrt[Cot[c + d*x]] + 4*a*b*((-(A
rcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Co
t[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
+ d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqr
t[2]))/2)))/(d*(e*Cot[c + d*x])^(3/2)))
```

Maple [A]
time = 0.46, size = 301, normalized size = 1.13

method	result

derivativedivides	$-\frac{2\left(-\frac{a^2}{\sqrt{e\cot(dx+c)}}+\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\right.}{4e}\right.}{}$
default	$-\frac{2\left(-\frac{a^2}{\sqrt{e\cot(dx+c)}}+\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\right.}{4e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e*(-a^2/(e*cot(d*x+c))^(1/2)+1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(
d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)+1))+1/8*(-a^2+b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Maxima [A]
time = 0.50, size = 179, normalized size = 0.67

$$\frac{\left(2\sqrt{2}(a^2-2ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a^2-2ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}(a^2+2ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}(a^2+2ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+8a^2\sqrt{\tan(dx+c)}\right)e^{\left(-\frac{3}{2}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan
(d*x + c)))) + 2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) -
 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^2 + 2*a*b - b^2)*log(sqrt(2)/sqrt(tan(
d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^2 + 2*a*b - b^2)*log(-sqrt(2)/
sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 8*a^2*sqrt(tan(d*x + c)))*e^(-3/
2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(d*x+c))**2/(e*cot(d*x+c))**(3/2),x)

[Out] Integral((a + b*cot(c + d*x))**2/(e*cot(c + d*x))**(3/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(3/2), x)

Mupad [B]
time = 0.94, size = 1196, normalized size = 4.48

$$\frac{2a^2}{d\,e\,\sqrt{e\cot(c+dx)}}+2\,\mathrm{atanh}\left(\frac{32\,a^4\,d^3\,e^5\,\sqrt{e\cot(c+dx)}\,\sqrt{\frac{a^4\,1i}{4\,d^2\,e^3}+\frac{b^4\,1i}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}+\frac{a^3\,b}{d^2\,e^3}-\frac{a^2\,b^2\,3i}{2\,d^2\,e^3}}}{-16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32i+112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192i-112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32i+16\,b^6\,d^2\,e^4}+\frac{32\,b^4\,d^3\,e^5\,\sqrt{e\cot(c+dx)}\,\sqrt{\frac{a^4\,1i}{4\,d^2\,e^3}+\frac{b^4\,1i}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}+\frac{a^3\,b}{d^2\,e^3}-\frac{a^2\,b^2\,3i}{2\,d^2\,e^3}}}{-16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32i+112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192i-112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32i+16\,b^6\,d^2\,e^4}-\frac{192\,a^2\,b^2\,d^3\,e^5\,\sqrt{e\cot(c+dx)}\,\sqrt{\frac{a^4\,1i}{4\,d^2\,e^3}+\frac{b^4\,1i}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}+\frac{a^3\,b}{d^2\,e^3}-\frac{a^2\,b^2\,3i}{2\,d^2\,e^3}}}{-16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32i+112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192i-112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32i+16\,b^6\,d^2\,e^4}\right)\sqrt{\frac{(a^4-a^3\,b\,4i-6\,a^2\,b^2+a\,b^3\,4i+b^4)\,1i}{4\,d^2\,e^3}}-2\,\mathrm{atanh}\left(\frac{32\,a^4\,d^3\,e^5\,\sqrt{e\cot(c+dx)}\,\sqrt{\frac{a^3\,b}{d^2\,e^3}-\frac{b^4\,1i}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}-\frac{a^4\,1i}{4\,d^2\,e^3}+\frac{a^2\,b^2\,3i}{2\,d^2\,e^3}}}{16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32i-112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192i+112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32i-16\,b^6\,d^2\,e^4}+\frac{32\,b^4\,d^3\,e^5\,\sqrt{e\cot(c+dx)}\,\sqrt{\frac{a^3\,b}{d^2\,e^3}-\frac{b^4\,1i}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}-\frac{a^4\,1i}{4\,d^2\,e^3}+\frac{a^2\,b^2\,3i}{2\,d^2\,e^3}}}{16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32i-112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192i+112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32i-16\,b^6\,d^2\,e^4}-\frac{192\,a^2\,b^2\,d^3\,e^5\,\sqrt{e\cot(c+dx)}\,\sqrt{\frac{a^3\,b}{d^2\,e^3}-\frac{b^4\,1i}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}-\frac{a^4\,1i}{4\,d^2\,e^3}+\frac{a^2\,b^2\,3i}{2\,d^2\,e^3}}}{16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32i-112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192i+112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32i-16\,b^6\,d^2\,e^4}\right)\sqrt{-\frac{(a^4+a^3\,b\,4i-6\,a^2\,b^2-a\,b^3\,4i+b^4)\,1i}{4\,d^2\,e^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cot(c + d*x))^2/(e*cot(c + d*x))^(3/2),x)

[Out] 2*atanh((32*a^4*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^3) + (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) + (a^3*b)/(d^2*e^3) - (a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*b^6*d^2*e^4 - 16*a^6*d^2*e^4 + a*b^5*d^2*e^4*32i + a^5*b*d^2*e^4*32i - 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*192i + 112*a^4*b^2*d^2*e^4) + (32*b^4*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^3) + (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) + (a^3*b)/(d^2*e^3) - (a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*b^6*d^2*e^4 - 16*a^6*d^2*e^4 + a*b^5*d^2*e^4*32i + a^5*b*d^2*e^4*32i - 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*192i + 112*a^4*b^2*d^2*e^4) - (192*a^2*b^2*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^3) + (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) + (a^3*b)/(d^2*e^3) - (a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*b^6*d^2*e^4 - 16*a^6*d^2*e^4 + a*b^5*d^2*e^4*32i + a^5*b*d^2*e^4*32i - 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*192i + 112*a^4*b^2*d^2*e^4))*(((a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2)*1i)/(4*d^

```
2*e^3))^(1/2) - 2*atanh((32*a^4*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^3*b)/(d^
2*e^3) - (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) - (a^4*1i)/(4*d^2*e^3) +
(a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*a^6*d^2*e^4 - 16*b^6*d^2*e^4 + a*b^5*d
^2*e^4*32i + a^5*b*d^2*e^4*32i + 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*192i
 - 112*a^4*b^2*d^2*e^4) + (32*b^4*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^3*b)/(
d^2*e^3) - (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) - (a^4*1i)/(4*d^2*e^3)
+ (a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*a^6*d^2*e^4 - 16*b^6*d^2*e^4 + a*b^5
*d^2*e^4*32i + a^5*b*d^2*e^4*32i + 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*19
2i - 112*a^4*b^2*d^2*e^4) - (192*a^2*b^2*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a
^3*b)/(d^2*e^3) - (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) - (a^4*1i)/(4*d^
2*e^3) + (a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*a^6*d^2*e^4 - 16*b^6*d^2*e^4
+ a*b^5*d^2*e^4*32i + a^5*b*d^2*e^4*32i + 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2
*e^4*192i - 112*a^4*b^2*d^2*e^4))*(-((a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a
^2*b^2)*1i)/(4*d^2*e^3))^(1/2) + (2*a^2)/(d*e*(e*cot(c + d*x))^(1/2))
```

3.60 $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{(a^2+2ab-b^2)\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}+\frac{(a^2+2ab-b^2)\operatorname{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}$$

```
[Out] 2/3*a^2/d/e/(e*cot(d*x+c))^(3/2)-1/2*(a^2+2*a*b-b^2)*arctan(1-2^(1/2)*(e*co
t(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/2*(a^2+2*a*b-b^2)*arctan(1+2^(
1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln
(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)
+1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(
1/2))/d/e^(5/2)*2^(1/2)+4*a*b/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3623, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a^2+2ab-b^2)\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}+\frac{(a^2+2ab-b^2)\operatorname{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{5/2}}-\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}+\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}+\frac{4ab}{de^2\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] -(((a^2 + 2*a*b - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
(Sqrt[2]*d*e^(5/2))) + ((a^2 + 2*a*b - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[
c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)) + (2*a^2)/(3*d*e*(e*Cot[c + d*x])^
(3/2)) + (4*a*b)/(d*e^2*Sqrt[e*Cot[c + d*x]]) - ((a^2 - 2*a*b - b^2)*Log[Sq
rt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*
e^(5/2)) + ((a^2 - 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2
]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
```

```
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{\int \frac{2abe-(a^2-b^2)e\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{e^2}$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4ab}{de^2\sqrt{e\cot(c+dx)}} + \frac{\int \frac{-(a^2-b^2)e^2-2abe^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^4}$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4ab}{de^2\sqrt{e\cot(c+dx)}} + \frac{2\mathrm{Subst}\left(\int \frac{(a^2-b^2)e^3+2abe^2x^2}{e^2+x^4}\,dx, x,\right.}{de^4}$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4ab}{de^2\sqrt{e\cot(c+dx)}} + \frac{(a^2-2ab-b^2)\,\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,d\right.}{de^2}$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4ab}{de^2\sqrt{e\cot(c+dx)}} - \frac{(a^2-2ab-b^2)\,\mathrm{Subst}\left(\int \frac{\sqrt{2}}{-e-\sqrt{2}}\right.}{2\sqrt{2}}$$

$$= \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4ab}{de^2\sqrt{e\cot(c+dx)}} - \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\right.}{2\sqrt{}}$$

$$= -\frac{(a^2+2ab-b^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}} + \frac{(a^2+2ab-b^2)\tan^{-1}\left(\right.}{\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.32, size = 82, normalized size = 0.28

$$\frac{2\left((a^2-b^2)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)+b\left(b+6a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)\right)\right)}{3de(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(5/2),x]
[Out] (2*((a^2 - b^2)*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2] + b*(b + 6
*a*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2]))/(3*d*e*
(e*Cot[c + d*x])^(3/2))
```

Maple [A]
time = 0.46, size = 326, normalized size = 1.12

method	result
derivativedivides	$-\frac{2\left(-\frac{a^2}{3(e\cot(dx+c))^{\frac{3}{2}}}-\frac{2ab}{e\sqrt{e\cot(dx+c)}}+\frac{(-a^2e+b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+}\right.\right.}{}\right.}{}$
default	$-\frac{2\left(-\frac{a^2}{3(e\cot(dx+c))^{\frac{3}{2}}}-\frac{2ab}{e\sqrt{e\cot(dx+c)}}+\frac{(-a^2e+b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e*(-1/3*a^2/(e*cot(d*x+c))^(3/2)-2*a*b/e/(e*cot(d*x+c))^(1/2)+1/e*(1/8
*(-a^2*e+b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4*a*b/(e^
2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)
+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^
(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1
/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.54, size = 193, normalized size = 0.66

$$\frac{\left(6\sqrt{2}(a^2+2ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}(a^2+2ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+3\sqrt{2}(a^2-2ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-3\sqrt{2}(a^2-2ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+8\left(a^2+\frac{6ab}{\tan(dx+c)}\right)\tan(dx+c)^{\frac{3}{2}}\right)e^{\left(-\frac{5}{2}\right)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(6*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(ta
n(d*x + c)))) + 6*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2)
- 2/sqrt(tan(d*x + c)))) + 3*sqrt(2)*(a^2 - 2*a*b - b^2)*log(sqrt(2)/sqrt(t
an(d*x + c)) + 1/tan(d*x + c) + 1) - 3*sqrt(2)*(a^2 - 2*a*b - b^2)*log(-sqr
```

```
t(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 8*(a^2 + 6*a*b/tan(d*x + c)
)*tan(d*x + c)^(3/2))*e^(-5/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\cot{(c + dx)})^2}{(e\cot{(c + dx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))**2/(e*cot(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))**2/(e*cot(c + d*x))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(5/2), x)
```

Mupad [**B**]
time = 1.51, size = 1214, normalized size = 4.17

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x))^2/(e*cot(c + d*x))^(5/2),x)
```

```
[Out] ((2*a^2)/3 + 4*a*b*cot(c + d*x))/(d*e*(e*cot(c + d*x))^(3/2)) - 2*atanh((32
*a^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^5) + (b^4*1i)/(4*d^2
*e^5) + (a*b^3)/(d^2*e^5) - (a^3*b)/(d^2*e^5) - (a^2*b^2*3i)/(2*d^2*e^5))^(
1/2))/(b^6*d^2*e^6*16i - a^6*d^2*e^6*16i + 32*a*b^5*d^2*e^6 + 32*a^5*b*d^2*
```

e^6 - a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 + a^4*b^2*d^2*e^6*112i) +
(32*b^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^5) + (b^4*1i)/(4*
d^2*e^5) + (a*b^3)/(d^2*e^5) - (a^3*b)/(d^2*e^5) - (a^2*b^2*3i)/(2*d^2*e^5)
)^(1/2))/(b^6*d^2*e^6*16i - a^6*d^2*e^6*16i + 32*a*b^5*d^2*e^6 + 32*a^5*b*d
^2*e^6 - a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 + a^4*b^2*d^2*e^6*112i)
 - (192*a^2*b^2*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^5) + (b^4
*1i)/(4*d^2*e^5) + (a*b^3)/(d^2*e^5) - (a^3*b)/(d^2*e^5) - (a^2*b^2*3i)/(2*
d^2*e^5))^(1/2))/(b^6*d^2*e^6*16i - a^6*d^2*e^6*16i + 32*a*b^5*d^2*e^6 + 32
*a^5*b*d^2*e^6 - a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 + a^4*b^2*d^2*e
^6*112i))*(((a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2)*1i)/(4*d^2*e^5))^
(1/2) - 2*atanh((32*a^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e^5) -
 (b^4*1i)/(4*d^2*e^5) - (a^4*1i)/(4*d^2*e^5) - (a^3*b)/(d^2*e^5) + (a^2*b^2
*3i)/(2*d^2*e^5))^(1/2))/(a^6*d^2*e^6*16i - b^6*d^2*e^6*16i + 32*a*b^5*d^2*
e^6 + 32*a^5*b*d^2*e^6 + a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 - a^4*b
^2*d^2*e^6*112i) + (32*b^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e^5
) - (b^4*1i)/(4*d^2*e^5) - (a^4*1i)/(4*d^2*e^5) - (a^3*b)/(d^2*e^5) + (a^2*
b^2*3i)/(2*d^2*e^5))^(1/2))/(a^6*d^2*e^6*16i - b^6*d^2*e^6*16i + 32*a*b^5*d
^2*e^6 + 32*a^5*b*d^2*e^6 + a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 - a^
4*b^2*d^2*e^6*112i) - (192*a^2*b^2*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a*b^3)/
(d^2*e^5) - (b^4*1i)/(4*d^2*e^5) - (a^4*1i)/(4*d^2*e^5) - (a^3*b)/(d^2*e^5)
 + (a^2*b^2*3i)/(2*d^2*e^5))^(1/2))/(a^6*d^2*e^6*16i - b^6*d^2*e^6*16i + 32
*a*b^5*d^2*e^6 + 32*a^5*b*d^2*e^6 + a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*
e^6 - a^4*b^2*d^2*e^6*112i))*(-((a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^
2)*1i)/(4*d^2*e^5))^(1/2)

3.61 $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=322

$$\frac{(a^2-2ab-b^2)\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}}-\frac{(a^2-2ab-b^2)\operatorname{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}}+$$

```
[Out] 2/5*a^2/d/e/(e*cot(d*x+c))^(5/2)+4/3*a*b/d/e^2/(e*cot(d*x+c))^(3/2)+1/2*(a^
2-2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/
2)-1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(
7/2)*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*c
ot(d*x+c))^(1/2))/d/e^(7/2)*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+
c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(7/2)*2^(1/2)-2*(a^2-b^2)/d/e^
3/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3623, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2-2ab-b^2)\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}}-\frac{(a^2-2ab-b^2)\operatorname{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{7/2}}-\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{7/2}}+\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{7/2}}-\frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}+\frac{4ab}{3de^2(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(S
qrt[2]*d*e^(7/2)) - ((a^2 - 2*a*b - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c +
 d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/2)) + (2*a^2)/(5*d*e*(e*Cot[c + d*x])^(5/
2)) + (4*a*b)/(3*d*e^2*(e*Cot[c + d*x])^(3/2)) - (2*(a^2 - b^2))/(d*e^3*Sqr
t[e*Cot[c + d*x]]) - ((a^2 + 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x
] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) + ((a^2 + 2*a*b -
b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2
*Sqrt[2]*d*e^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3623

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
 f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{\int \frac{2abe-(a^2-b^2)e\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx}{e^2}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4ab}{3de^2(e\cot(c+dx))^{3/2}} + \frac{\int \frac{-(a^2-b^2)e^2-2abe^2\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx}{e^4}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4ab}{3de^2(e\cot(c+dx))^{3/2}} - \frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{\int \frac{-2abe}{\sqrt{}}}{}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4ab}{3de^2(e\cot(c+dx))^{3/2}} - \frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2\text{Subst}}{}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4ab}{3de^2(e\cot(c+dx))^{3/2}} - \frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}} - \frac{(a^2-2}{}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4ab}{3de^2(e\cot(c+dx))^{3/2}} - \frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}} - \frac{(a^2+2}{}$$

$$= \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} + \frac{4ab}{3de^2(e\cot(c+dx))^{3/2}} - \frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}} - \frac{(a^2+2}{}$$

$$= \frac{(a^2-2ab-b^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}} - \frac{(a^2-2ab-b^2)\tan^{-1}\left(1\right.}{\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.39, size = 85, normalized size = 0.26

$$\frac{2\left(3(a^2-b^2)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(c+dx)\right)+b\left(3b+10a\cot(c+dx)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)\right)\right)}{15de(e\cot(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] (2*(3*(a^2 - b^2)*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2] + b*(3*
b + 10*a*Cot[c + d*x]*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2])))/(
15*d*e*(e*Cot[c + d*x])^(5/2))
```

Maple [A]
time = 0.47, size = 347, normalized size = 1.08

method	result
derivativedivides	$-\frac{2\left(-\frac{ab\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+ \ldots}{\left(e^2\right)^{\frac{1}{4}}}\ldots\right)\ldots}{4e}\ldots\right.}{\ldots}$ …
default	$-\frac{2\left(-\frac{ab\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+ \ldots}{\left(e^2\right)^{\frac{1}{4}}}\ldots\right)\ldots}{4e}\ldots\right.}{\ldots}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e*(1/e^2*(-1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(
a^2-b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*
arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/5*a^2/(e*cot(d*x+c)
)^(5/2)-(-a^2+b^2)/e^2/(e*cot(d*x+c))^(1/2)-2/3*a*b/e/(e*cot(d*x+c))^(3/2))
```

Maxima [A]
time = 0.51, size = 214, normalized size = 0.66

$$\frac{\left(8\left(3a^2+\frac{10ab}{\tan(dx+c)}-\frac{15(a^2-b^2)}{\tan(dx+c)^2}\right)\tan(dx+c)^{\frac{5}{2}}-30\sqrt{2}(a^2-2ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-30\sqrt{2}(a^2-2ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+15\sqrt{2}(a^2+2ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-15\sqrt{2}(a^2+2ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)e^{\left(-\frac{7}{2}\right)}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/60*(8*(3*a^2 + 10*a*b/tan(d*x + c) - 15*(a^2 - b^2)/tan(d*x + c)^2)*tan(d
*x + c)^(5/2) - 30*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2)
+ 2/sqrt(tan(d*x + c)))) - 30*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(-1/2*sqrt(
2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 15*sqrt(2)*(a^2 + 2*a*b - b^2)*log(s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 15*sqrt(2)*(a^2 + 2*a*b -
 b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*e^(-7/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \cot{\left(c + d x \right)}\right)^{2}}{\left(e \cot{\left(c + d x \right)}\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))**2/(e*cot(d*x+c))**(7/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))**2/(e*cot(c + d*x))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(7/2), x)
```

Mupad [B]
time = 2.32, size = 1227, normalized size = 3.81

$$2\,\mathrm{atanh}\left(\frac{32\,a^4\,d^3\,e^{11}\,\sqrt{e\,\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b}{d^2\,e^7}-\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e^7}-\frac{a\,b^3}{d^2\,e^7}-\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e^7}+\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e^7}}}{16\,a^6\,d^2\,e^8+a^5\,b\,d^2\,e^8\,32{}\mathrm{i}-112\,a^4\,b^2\,d^2\,e^8-a^3\,b^3\,d^2\,e^8\,192{}\mathrm{i}+112\,a^2\,b^4\,d^2\,e^8+a\,b^5\,d^2\,e^8\,32{}\mathrm{i}-16\,b^6\,d^2\,e^8}+\frac{32\,b^4\,d^3\,e^{11}\,\sqrt{e\,\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b}{d^2\,e^7}-\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e^7}-\frac{a\,b^3}{d^2\,e^7}-\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e^7}+\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e^7}}}{16\,a^6\,d^2\,e^8+a^5\,b\,d^2\,e^8\,32{}\mathrm{i}-112\,a^4\,b^2\,d^2\,e^8-a^3\,b^3\,d^2\,e^8\,192{}\mathrm{i}+112\,a^2\,b^4\,d^2\,e^8+a\,b^5\,d^2\,e^8\,32{}\mathrm{i}-16\,b^6\,d^2\,e^8}-\frac{192\,a^2\,b^2\,d^3\,e^{11}\,\sqrt{e\,\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b}{d^2\,e^7}-\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e^7}-\frac{a\,b^3}{d^2\,e^7}-\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e^7}+\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e^7}}}{16\,a^6\,d^2\,e^8+a^5\,b\,d^2\,e^8\,32{}\mathrm{i}-112\,a^4\,b^2\,d^2\,e^8-a^3\,b^3\,d^2\,e^8\,192{}\mathrm{i}+112\,a^2\,b^4\,d^2\,e^8+a\,b^5\,d^2\,e^8\,32{}\mathrm{i}-16\,b^6\,d^2\,e^8}\right)\,\sqrt{-\frac{\left(a^4+a^3\,b\,4{}\mathrm{i}-6\,a^2\,b^2-a\,b^3\,4{}\mathrm{i}+b^4\right)\,1{}\mathrm{i}}{4\,d^2\,e^7}}-2\,\mathrm{atanh}\left(\frac{32\,a^4\,d^3\,e^{11}\,\sqrt{e\,\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e^7}+\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e^7}-\frac{a\,b^3}{d^2\,e^7}+\frac{a^3\,b}{d^2\,e^7}-\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e^7}}}{-16\,a^6\,d^2\,e^8+a^5\,b\,d^2\,e^8\,32{}\mathrm{i}+112\,a^4\,b^2\,d^2\,e^8-a^3\,b^3\,d^2\,e^8\,192{}\mathrm{i}-112\,a^2\,b^4\,d^2\,e^8+a\,b^5\,d^2\,e^8\,32{}\mathrm{i}+16\,b^6\,d^2\,e^8}+\frac{32\,b^4\,d^3\,e^{11}\,\sqrt{e\,\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e^7}+\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e^7}-\frac{a\,b^3}{d^2\,e^7}+\frac{a^3\,b}{d^2\,e^7}-\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e^7}}}{-16\,a^6\,d^2\,e^8+a^5\,b\,d^2\,e^8\,32{}\mathrm{i}+112\,a^4\,b^2\,d^2\,e^8-a^3\,b^3\,d^2\,e^8\,192{}\mathrm{i}-112\,a^2\,b^4\,d^2\,e^8+a\,b^5\,d^2\,e^8\,32{}\mathrm{i}+16\,b^6\,d^2\,e^8}-\frac{192\,a^2\,b^2\,d^3\,e^{11}\,\sqrt{e\,\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^4\,1{}\mathrm{i}}{4\,d^2\,e^7}+\frac{b^4\,1{}\mathrm{i}}{4\,d^2\,e^7}-\frac{a\,b^3}{d^2\,e^7}+\frac{a^3\,b}{d^2\,e^7}-\frac{a^2\,b^2\,3{}\mathrm{i}}{2\,d^2\,e^7}}}{-16\,a^6\,d^2\,e^8+a^5\,b\,d^2\,e^8\,32{}\mathrm{i}+112\,a^4\,b^2\,d^2\,e^8-a^3\,b^3\,d^2\,e^8\,192{}\mathrm{i}-112\,a^2\,b^4\,d^2\,e^8+a\,b^5\,d^2\,e^8\,32{}\mathrm{i}+16\,b^6\,d^2\,e^8}\right)\,\sqrt{\frac{\left(a^4-a^3\,b\,4{}\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4{}\mathrm{i}+b^4\right)\,1{}\mathrm{i}}{4\,d^2\,e^7}}+\frac{\frac{2\,a^2}{5}-2\,\mathrm{cot}\left(c+d\,x\right)^2\,\left(a^2-b^2\right)+\frac{4\,a\,b\,\mathrm{cot}\left(c+d\,x\right)}{3}}{d\,e\,{\left(e\,\mathrm{cot}\left(c+d\,x\right)\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x))^2/(e*cot(c + d*x))^(7/2),x)

[Out] 2*atanh((32*a^4*d^3*e^11*(e*cot(c + d*x))^(1/2)*((a^3*b)/(d^2*e^7) - (b^4*1
i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) - (a^4*1i)/(4*d^2*e^7) + (a^2*b^2*3i)/(2
*d^2*e^7))^(1/2))/(16*a^6*d^2*e^8 - 16*b^6*d^2*e^8 + a*b^5*d^2*e^8*32i + a^
5*b*d^2*e^8*32i + 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i - 112*a^4*b^2*
d^2*e^8) + (32*b^4*d^3*e^11*(e*cot(c + d*x))^(1/2)*((a^3*b)/(d^2*e^7) - (b^
4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) - (a^4*1i)/(4*d^2*e^7) + (a^2*b^2*3i)
/(2*d^2*e^7))^(1/2))/(16*a^6*d^2*e^8 - 16*b^6*d^2*e^8 + a*b^5*d^2*e^8*32i +
 a^5*b*d^2*e^8*32i + 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i - 112*a^4*b
^2*d^2*e^8) - (192*a^2*b^2*d^3*e^11*(e*cot(c + d*x))^(1/2)*((a^3*b)/(d^2*e^
7) - (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) - (a^4*1i)/(4*d^2*e^7) + (a^2
*b^2*3i)/(2*d^2*e^7))^(1/2))/(16*a^6*d^2*e^8 - 16*b^6*d^2*e^8 + a*b^5*d^2*e
^8*32i + a^5*b*d^2*e^8*32i + 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i - 1
12*a^4*b^2*d^2*e^8))*(-((a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2)*1i)/(
4*d^2*e^7))^(1/2) - 2*atanh((32*a^4*d^3*e^11*(e*cot(c + d*x))^(1/2)*((a^4*1
i)/(4*d^2*e^7) + (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) + (a^3*b)/(d^2*e^
7) - (a^2*b^2*3i)/(2*d^2*e^7))^(1/2))/(16*b^6*d^2*e^8 - 16*a^6*d^2*e^8 + a*
b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i - 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8
*192i + 112*a^4*b^2*d^2*e^8) + (32*b^4*d^3*e^11*(e*cot(c + d*x))^(1/2)*((a^
4*1i)/(4*d^2*e^7) + (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) + (a^3*b)/(d^2
*e^7) - (a^2*b^2*3i)/(2*d^2*e^7))^(1/2))/(16*b^6*d^2*e^8 - 16*a^6*d^2*e^8 +
 a*b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i - 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*
e^8*192i + 112*a^4*b^2*d^2*e^8) - (192*a^2*b^2*d^3*e^11*(e*cot(c + d*x))^(1
/2)*((a^4*1i)/(4*d^2*e^7) + (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) + (a^3
*b)/(d^2*e^7) - (a^2*b^2*3i)/(2*d^2*e^7))^(1/2))/(16*b^6*d^2*e^8 - 16*a^6*d
^2*e^8 + a*b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i - 112*a^2*b^4*d^2*e^8 - a^3*
b^3*d^2*e^8*192i + 112*a^4*b^2*d^2*e^8))*(((a*b^3*4i - a^3*b*4i + a^4 + b^4
 - 6*a^2*b^2)*1i)/(4*d^2*e^7))^(1/2) + ((2*a^2)/5 - 2*cot(c + d*x)^2*(a^2 -
 b^2) + (4*a*b*cot(c + d*x))/3)/(d*e*(e*cot(c + d*x))^(5/2))
```

3.62 $\int (e \cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx$

Optimal. Leaf size=372

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{(a-b)\left(a^2+4ab+b^2\right)e^{3/2}\text{ArcTan}\left(1+\right.}{\sqrt{2}\,d}$$

```
[Out] -2/3*b*(3*a^2-b^2)*(e*cot(d*x+c))^(3/2)/d-32/35*a*b^2*(e*cot(d*x+c))^(5/2)/
d/e-2/7*b^2*(e*cot(d*x+c))^(5/2)*(a+b*cot(d*x+c))/d/e-1/2*(a-b)*(a^2+4*a*b+
b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/2*(
a-b)*(a^2+4*a*b+b^2)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))
/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-
2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*e^(3/2)*l
n(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)-2*a*(a
^2-3*b^2)*e*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.40, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3647, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{e^{3/2}(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}+\frac{e^{3/2}(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d}-\frac{e^{3/2}(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}+\frac{e^{3/2}(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}-\frac{2b\left(3a^2-b^2\right)(e\cot(c+dx))^{3/2}}{3d}-\frac{2ae\left(a^2-3b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}-\frac{32ab^2(e\cot(c+dx))^{5/2}}{35de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -(((a - b)*(a^2 + 4*a*b + b^2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d
*x]])/Sqrt[e]])/(Sqrt[2]*d)) + ((a - b)*(a^2 + 4*a*b + b^2)*e^(3/2)*ArcTan[
1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d) - (2*a*(a^2 - 3*b^
2)*e*Sqrt[e*Cot[c + d*x]])/d - (2*b*(3*a^2 - b^2)*(e*Cot[c + d*x])^(3/2))/(
3*d) - (32*a*b^2*(e*Cot[c + d*x])^(5/2))/(35*d*e) - (2*b^2*(e*Cot[c + d*x])
^(5/2)*(a + b*Cot[c + d*x]))/(7*d*e) - ((a + b)*(a^2 - 4*a*b + b^2)*e^(3/2)
*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqr
t[2]*d) + ((a + b)*(a^2 - 4*a*b + b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c
+ d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx = -\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de} - \frac{2\int(e\cot(c+dx)}{}$$

$$= -\frac{32ab^2(e\cot(c+dx))^{5/2}}{35de} - \frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c}{7de}$$

$$= -\frac{2b(3a^2-b^2)\,(e\cot(c+dx))^{3/2}}{3d} - \frac{32ab^2(e\cot(c+dx))^{5/2}}{35de} -$$

$$= -\frac{2a(a^2-3b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{2b(3a^2-b^2)\,(e\cot(c+dx}{3d}$$

$$= -\frac{2a(a^2-3b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{2b(3a^2-b^2)\,(e\cot(c+dx}{3d}$$

$$= -\frac{2a(a^2-3b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{2b(3a^2-b^2)\,(e\cot(c+dx}{3d}$$

$$= -\frac{2a(a^2-3b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{2b(3a^2-b^2)\,(e\cot(c+dx}{3d}$$

$$= -\frac{2a(a^2-3b^2)\,e\sqrt{e\cot(c+dx)}}{d} - \frac{2b(3a^2-b^2)\,(e\cot(c+dx}{3d}$$

$$= -\frac{(a-b)\,(a^2+4ab+b^2)\,e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right.}{\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 3.13, size = 251, normalized size = 0.67

$$-\frac{(e\cot(c+dx))^{3/2}\left(\frac{6}{5}ab^2\cot^{\frac{5}{2}}(c+dx)+\frac{2}{7}b^3\cot^{\frac{7}{2}}(c+dx)+\frac{2}{3}b(-3a^2+b^2)\cot^{\frac{3}{2}}(c+dx)\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)+\frac{1}{4}a(a^2-3b^2)\left(2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{d\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -(((e*Cot[c + d*x])^(3/2)*((6*a*b^2*Cot[c + d*x]^(5/2))/5 + (2*b^3*Cot[c +
d*x]^(7/2))/7 + (2*b*(-3*a^2 + b^2)*Cot[c + d*x]^(3/2)*(-1 + Hypergeometric
2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/3 + (a*(a^2 - 3*b^2)*(2*Sqrt[2]*ArcTan[
1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c +
 d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
 + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x
]]))/4))/(d*Cot[c + d*x]^(3/2)))
```

Maple [A]
time = 0.58, size = 410, normalized size = 1.10

method	result
derivativedivides	$-\frac{2\left(\frac{b^{3}(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{3ae\,b^{2}(e\cot(dx+c))^{\frac{5}{2}}}{5}+a^{2}e^{2}b(e\cot(dx+c))^{\frac{3}{2}}-\frac{b^{3}e^{2}(e\cot(dx+c))^{\frac{3}{2}}}{3}+a^{3}e^{3}\sqrt{e\cot(dx+c)}-3a\right.}{}$
default	$-\frac{2\left(\frac{b^{3}(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{3ae\,b^{2}(e\cot(dx+c))^{\frac{5}{2}}}{5}+a^{2}e^{2}b(e\cot(dx+c))^{\frac{3}{2}}-\frac{b^{3}e^{2}(e\cot(dx+c))^{\frac{3}{2}}}{3}+a^{3}e^{3}\sqrt{e\cot(dx+c)}-3a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e^2*(1/7*b^3*(e*cot(d*x+c))^(7/2)+3/5*a*e*b^2*(e*cot(d*x+c))^(5/2)+a^2
*e^2*b*(e*cot(d*x+c))^(3/2)-1/3*b^3*e^2*(e*cot(d*x+c))^(3/2)+a^3*e^3*(e*cot
(d*x+c))^(1/2)-3*a*b^2*e^3*(e*cot(d*x+c))^(1/2)-e^4*(1/8*(a^3*e-3*a*b^2*e)*
(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*
2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)
+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arcta
n(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(3*a^2*b-b^3)/(e^2)^(1/
4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)
^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))
)+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.52, size = 264, normalized size = 0.71

$$\frac{\left(210\sqrt{2}\left(a^{3}+3a^{2}b-3ab^{2}-b^{3}\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+210\sqrt{2}\left(a^{3}+3a^{2}b-3ab^{2}-b^{3}\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+105\sqrt{2}\left(a^{3}-3a^{2}b-3ab^{2}+b^{3}\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-105\sqrt{2}\left(a^{3}-3a^{2}b-3ab^{2}+b^{3}\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{504ab^{2}}{\tan(dx+c)^{\frac{5}{2}}}-\frac{840\left(a^{3}-3ab^{2}\right)}{\sqrt{\tan(dx+c)}}-\frac{120b^{3}}{\tan(dx+c)^{\frac{7}{2}}}-\frac{280\left(3a^{2}b-b^{3}\right)}{\tan(dx+c)^{\frac{3}{2}}}\right)e^{\frac{3}{2}}}{420d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/420*(210*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt
(2) + 2/sqrt(tan(d*x + c)))) + 210*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 105*sqrt(2)*(a^3 -
3*a^2*b - 3*a*b^2 + b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
1) - 105*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x
+ c)) + 1/tan(d*x + c) + 1) - 504*a*b^2/tan(d*x + c)^(5/2) - 840*(a^3 - 3*a
```

```
*b^2)/sqrt(tan(d*x + c)) - 120*b^3/tan(d*x + c)^(7/2) - 280*(3*a^2*b - b^3)
/tan(d*x + c)^(3/2))*e^(3/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cot(c+dx))^{\frac{3}{2}}(a+b\cot(c+dx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+b*cot(d*x+c))**3,x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(3/2), x)
```

Mupad [B]
time = 5.47, size = 2317, normalized size = 6.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))^3,x)
```

```
[Out] (e*cot(c + d*x))^(3/2)*((2*b^3)/(3*d) - (2*a^2*b)/d) - (e*cot(c + d*x))^(1/
2)*((2*a^3*e)/d - (6*a*b^2*e)/d) - atan((((16*(e*cot(c + d*x))^(1/2)*(a^6*e
^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 - (8*(4*a^3*d^2*e^5 -
12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 -
a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2))/d^3)*(-
```

(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20
*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*1i + ((16*(e*cot(c + d*x))^(
1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 + (8*(4*a^3
*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a
^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/
2))/d^3)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e
^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*1i)/(((16*(e*cot(
c + d*x))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2
+ (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^
5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(
4*d^2))^(1/2))/d^3)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3
- a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2) - ((16
*(e*cot(c + d*x))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^
6))/d^2 - (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e^3*1i
+ 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^
3*15i)/(4*d^2))^(1/2))/d^3)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^
5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2
) + (16*(3*a^8*b*e^8 - b^9*e^8 + 6*a^4*b^5*e^8 + 8*a^6*b^3*e^8))/d^3))*(-(b
^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a
^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*2i - atan((((16*(e*cot(c + d*x
))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 - (8*(4
*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 +
6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))
^(1/2))/d^3)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b
^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*1i + ((16*(e*
cot(c + d*x))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/
d^2 + (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*
a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15
i)/(4*d^2))^(1/2))/d^3)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*
e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*1i
)/(((16*(e*cot(c + d*x))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4
*b^2*e^6))/d^2 + (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(a^6*e^3*1i - b^6*
e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4
*b^2*e^3*15i)/(4*d^2))^(1/2))/d^3)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3
+ 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2
))^(1/2) - ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6
- 15*a^4*b^2*e^6))/d^2 - (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(a^6*e^3*1
i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e
^3 - a^4*b^2*e^3*15i)/(4*d^2))^(1/2))/d^3)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a
*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i
)/(4*d^2))^(1/2) + (16*(3*a^8*b*e^8 - b^9*e^8 + 6*a^4*b^5*e^8 + 8*a^6*b^3*e
^8))/d^3))*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4
*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*2i - (2*b^3*(e*
cot(c + d*x))^(7/2))/(7*d*e^2) - (6*a*b^2*(e*cot(c + d*x))^(5/2))/(5*d*e)

3.63 $\int \sqrt{e \cot(c+dx)}\,(a+b\cot(c+dx))^3\,dx$

Optimal. Leaf size=342

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{(a+b)\left(a^2-4ab+b^2\right)\sqrt{e}\,\text{ArcTan}\left(1+\right.}{\sqrt{2}\,d}$$

```
[Out] -8/5*a*b^2*(e*cot(d*x+c))^(3/2)/d/e-2/5*b^2*(e*cot(d*x+c))^(3/2)*(a+b*cot(d
*x+c))/d/e+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/
e^(1/2))*e^(1/2)/d*2^(1/2)-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*(e*co
t(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*ln(e^(
1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)+1/4
*(a-b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))
^(1/2))*e^(1/2)/d*2^(1/2)-2*b*(3*a^2-b^2)*(e*cot(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.33, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3647, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt{e}(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}-\frac{\sqrt{e}(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d}-\frac{2b\left(3a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{\sqrt{e}(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}+\frac{\sqrt{e}(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d}-\frac{8ab^2(e\cot(c+dx))^{3/2}}{5de}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^3,x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x
]])/Sqrt[e]])/(Sqrt[2]*d) - ((a + b)*(a^2 - 4*a*b + b^2)*Sqrt[e]*ArcTan[1 +
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d) - (2*b*(3*a^2 - b^2)*
Sqrt[e*Cot[c + d*x]])/d - (8*a*b^2*(e*Cot[c + d*x])^(3/2))/(5*d*e) - (2*b^2
*(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x]))/(5*d*e) - ((a - b)*(a^2 + 4*a
*b + b^2)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c
 + d*x]]])/(2*Sqrt[2]*d) + ((a - b)*(a^2 + 4*a*b + b^2)*Sqrt[e]*Log[Sqrt[e]
 + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^3\,dx = -\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de} - \frac{2\int \sqrt{e\cot(c+dx)}$$

$$= -\frac{8ab^2(e\cot(c+dx))^{3/2}}{5de} - \frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+$$

$$= -\frac{2b(3a^2-b^2)\sqrt{e\cot(c+dx)}}{d} - \frac{8ab^2(e\cot(c+dx))^{3/2}}{5de} - \frac{2b^2}{}$$

$$= -\frac{2b(3a^2-b^2)\sqrt{e\cot(c+dx)}}{d} - \frac{8ab^2(e\cot(c+dx))^{3/2}}{5de} - \frac{2b^2}{}$$

$$= -\frac{2b(3a^2-b^2)\sqrt{e\cot(c+dx)}}{d} - \frac{8ab^2(e\cot(c+dx))^{3/2}}{5de} - \frac{2b^2}{}$$

$$= -\frac{2b(3a^2-b^2)\sqrt{e\cot(c+dx)}}{d} - \frac{8ab^2(e\cot(c+dx))^{3/2}}{5de} - \frac{2b^2}{}$$

$$= -\frac{2b(3a^2-b^2)\sqrt{e\cot(c+dx)}}{d} - \frac{8ab^2(e\cot(c+dx))^{3/2}}{5de} - \frac{2b^2}{}$$

$$= \frac{(a+b)\left(a^2-4ab+b^2\right)\sqrt{e}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.66, size = 247, normalized size = 0.72

$$-\frac{\sqrt{e\cot(c+dx)}\left(2ab^2\cot^{\frac{3}{2}}(c+dx)+\frac{2}{5}b^3\cot^{\frac{5}{2}}(c+dx)+\frac{2}{3}a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)-\frac{1}{4}b(-3a^2+b^2)\left(2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -((Sqrt[e*Cot[c + d*x]]*(2*a*b^2*Cot[c + d*x]^(3/2) + (2*b^3*Cot[c + d*x]^(
5/2))/5 + (2*a*(a^2 - 3*b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7
/4, -Cot[c + d*x]^2])/3 - (b*(-3*a^2 + b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*S
qrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*S
qrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*
x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/4))/(d*S
qrt[Cot[c + d*x]]))
```

Maple [A]
time = 0.58, size = 371, normalized size = 1.08

method	result
derivativedivides	$-\frac{2\left(\frac{b^3(e\cot(dx+c))^{\frac{5}{2}}}{5}+ae\,b^2(e\cot(dx+c))^{\frac{3}{2}}+3a^2b\,e^2\sqrt{e\cot(dx+c)}-b^3e^2\sqrt{e\cot(dx+c)}+e^3\left(\frac{(-3a^2be- \ldots}{\ldots}\right.\right.}{\ldots}$
default	$-\frac{2\left(\frac{b^3(e\cot(dx+c))^{\frac{5}{2}}}{5}+ae\,b^2(e\cot(dx+c))^{\frac{3}{2}}+3a^2b\,e^2\sqrt{e\cot(dx+c)}-b^3e^2\sqrt{e\cot(dx+c)}+e^3\left(\frac{(-3a^2be- \ldots}{\ldots}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e^2*(1/5*b^3*(e*cot(d*x+c))^(5/2)+a*e*b^2*(e*cot(d*x+c))^(3/2)+3*a^2*b
*e^2*(e*cot(d*x+c))^(1/2)-b^3*e^2*(e*cot(d*x+c))^(1/2)+e^3*(1/8*(-3*a^2*b*e
+b^3*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)
-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(a^3-3*a*b^2)/(
e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/
2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2
)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^
(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.51, size = 244, normalized size = 0.71

$$-\frac{\left(10\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+10\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-5\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)+5\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)+\frac{40ab^2}{\tan\left(dx+c\right)^{\frac{3}{2}}}+\frac{8b^3}{\tan\left(dx+c\right)^{\frac{5}{2}}}+\frac{40\left(3a^2b-b^3\right)}{\sqrt{\tan\left(dx+c\right)}}\right)e^{\frac{1}{2}}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/20*(10*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(
2) + 2/sqrt(tan(d*x + c)))) + 10*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - 5*sqrt(2)*(a^3 + 3*a^
2*b - 3*a*b^2 - b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) +
 5*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(-sqrt(2)/sqrt(tan(d*x + c))
+ 1/tan(d*x + c) + 1) + 40*a*b^2/tan(d*x + c)^(3/2) + 8*b^3/tan(d*x + c)^(5
/2) + 40*(3*a^2*b - b^3)/sqrt(tan(d*x + c)))*e^(1/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cot\left(c+dx\right)}\left(a+b\cot\left(c+dx\right)\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)*(a+b*cot(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^3*sqrt(e*cot(d*x + c)), x)
```

Mupad [B]
time = 2.55, size = 2071, normalized size = 6.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))^3,x)
```

```
[Out] (e*cot(c + d*x))^(1/2)*((2*b^3)/d - (6*a^2*b)/d) + atan((((16*(e*cot(c + d*
x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))/d^2 - (8*(
4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 2
0*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*(
(b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b
^5*e + 6*a^5*b*e)/(4*d^2))^(1/2)*1i + ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^4
- b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))/d^2 + (8*(4*b^3*d^2*e^4 - 12*
a^2*b*d^2*e^4)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b
^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((b^6*e*1i - a^6*e*1
i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(
4*d^2))^(1/2)*1i)/(((16*(e*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*
b^4*e^4 - 15*a^4*b^2*e^4))/d^2 - (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4)*((b^
6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*
e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i -
 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2) - ((1
6*(e*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e
^4))/d^2 + (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4)*((b^6*e*1i - a^6*e*1i - a^
2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2)
)^(1/2))/d^3)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^
2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2) + (16*(3*a*b^8*e^5 - a^9*e^
5 + 8*a^3*b^6*e^5 + 6*a^5*b^4*e^5))/d^3))*((b^6*e*1i - a^6*e*1i - a^2*b^4*e
*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2)
*2i + atan((((16*(e*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4
 - 15*a^4*b^2*e^4))/d^2 - (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4)*((a^6*e*1i
- b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a
^5*b*e)/(4*d^2))^(1/2))/d^3)*((a^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3
*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2)*1i + ((16*(e
*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))
/d^2 + (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4)*((a^6*e*1i - b^6*e*1i + a^2*b^
4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1
/2))/d^3)*((a^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*
15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2)*1i)/(((16*(e*cot(c + d*x))^(1/2
)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))/d^2 - (8*(4*b^3*d^
2*e^4 - 12*a^2*b*d^2*e^4)*((a^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^
3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((a^6*e*1
i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6
*a^5*b*e)/(4*d^2))^(1/2) - ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 +
```

```
 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))/d^2 + (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e
^4)*((a^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i +
6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((a^6*e*1i - b^6*e*1i + a^2*b^4
*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/
2) + (16*(3*a*b^8*e^5 - a^9*e^5 + 8*a^3*b^6*e^5 + 6*a^5*b^4*e^5))/d^3))*((a
^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5
*e + 6*a^5*b*e)/(4*d^2))^(1/2)*2i - (2*b^3*(e*cot(c + d*x))^(5/2))/(5*d*e^2
) - (2*a*b^2*(e*cot(c + d*x))^(3/2))/(d*e)
```

3.64 $\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal. Leaf size=313

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}-\frac{(a-b)\left(a^2+4ab+b^2\right)\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}$$

```
[Out] 1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*
2^(1/2)/e^(1/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(
1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*
x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)/e^(1/2)-1/4*(a+b)*(a^2
-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2
^(1/2)/e^(1/2)-16/3*a*b^2*(e*cot(d*x+c))^(1/2)/d/e-2/3*b^2*(a+b*cot(d*x+c))
*(e*cot(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.29, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3647, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}}-\frac{(a-b)\left(a^2+4ab+b^2\right)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}}-\frac{2b^2\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}{3de}-\frac{16ab^2\sqrt{e\cot(c+dx)}}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt
[e]])/(Sqrt[2]*d*Sqrt[e]) - ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[e]) - (16*a*b^2*Sqrt[e*Co
t[c + d*x]])/(3*d*e) - (2*b^2*Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x]))/(3
*d*e) + ((a + b)*(a^2 - 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - S
qrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) - ((a + b)*(a^2 - 4*a*b
 + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])
/(2*Sqrt[2]*d*Sqrt[e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = -\frac{2b^2\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}{3de} - \frac{2\int \frac{-\frac{1}{2}a(3a^2-b^2)e-\frac{3}{2}b(3a^2-b^2)e\cot(c+dx)-4 \cdots}{\sqrt{e\cot(c+dx)}}}{3e}$$

$$= -\frac{16ab^2\sqrt{e\cot(c+dx)}}{3de} - \frac{2b^2\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}{3de} - \frac{2\int \frac{-\frac{3}{2}a(a^2- \cdots}{\cdots}}{\cdots}$$

$$= -\frac{16ab^2\sqrt{e\cot(c+dx)}}{3de} - \frac{2b^2\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}{3de} - \frac{4\text{Subst}\Big(\int \cdots}{\cdots}$$

$$= -\frac{16ab^2\sqrt{e\cot(c+dx)}}{3de} - \frac{2b^2\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}{3de} - \frac{((a+b)\,(a \cdots}{\cdots}$$

$$= -\frac{16ab^2\sqrt{e\cot(c+dx)}}{3de} - \frac{2b^2\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}{3de} - \frac{((a-b)\,(a \cdots}{\cdots}$$

$$= -\frac{16ab^2\sqrt{e\cot(c+dx)}}{3de} - \frac{2b^2\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}{3de} + \frac{(a+b)\,(a^2 \cdots}{\cdots}$$

$$= \frac{(a-b)\,(a^2+4ab+b^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}} - \frac{(a-b)\,(a^2+4ab+ \cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.08, size = 216, normalized size = 0.69

$$-\frac{2\sqrt{\cot(c+dx)}\left(9ab^2\sqrt{\cot(c+dx)}+b^3\cot^{\frac{5}{2}}(c+dx)-b(-3a^2+b^2)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)-\frac{3a(a^2-3b^2)\left(2\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{4\sqrt{2}}\right)}{3d\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cot[c + d*x]]*(9*a*b^2*Sqrt[Cot[c + d*x]] + b^3*Cot[c + d*x]^(3/2)
 - b*(-3*a^2 + b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[
c + d*x]^2] - (3*a*(a^2 - 3*b^2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]
- 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d
*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))
/(4*Sqrt[2])))/(3*d*Sqrt[e*Cot[c + d*x]])
```

Maple [A]
time = 0.51, size = 337, normalized size = 1.08

method	result
derivativedivides	$-\frac{2\left(\frac{b^{3}(e\cot(dx+c))^{\frac{3}{2}}}{3}+3a\,b^{2}e\sqrt{e\cot\left(dx+c\right)}+e^{2}\left(\frac{\left(a^{3}e-3a\,b^{2}e\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot}}\right.\right.}{}\right.\right.}{}$
default	$-\frac{2\left(\frac{b^{3}(e\cot(dx+c))^{\frac{3}{2}}}{3}+3a\,b^{2}e\sqrt{e\cot\left(dx+c\right)}+e^{2}\left(\frac{\left(a^{3}e-3a\,b^{2}e\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e^2*(1/3*b^3*(e*cot(d*x+c))^(3/2)+3*a*b^2*e*(e*cot(d*x+c))^(1/2)+e^2*(
1/8*(a^3*e-3*a*b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(
3*a^2*b-b^3)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.50, size = 222, normalized size = 0.71

$$-\frac{\left(6\sqrt{2}\left(a^{3}+3a^{2}b-3ab^{2}-b^{3}\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+6\sqrt{2}\left(a^{3}+3a^{2}b-3ab^{2}-b^{3}\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+3\sqrt{2}\left(a^{3}-3a^{2}b-3ab^{2}+b^{3}\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-3\sqrt{2}\left(a^{3}-3a^{2}b-3ab^{2}+b^{3}\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\frac{72ab^{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{8b^{3}}{\tan(dx+c)^{\frac{3}{2}}}\right)e^{\left(-\frac{1}{2}\right)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2
) + 2/sqrt(tan(d*x + c)))) + 6*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arct
an(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 3*sqrt(2)*(a^3 - 3*a^2*
b - 3*a*b^2 + b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 3
*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) +
1/tan(d*x + c) + 1) + 72*a*b^2/sqrt(tan(d*x + c)) + 8*b^3/tan(d*x + c)^(3/2
))*e^(-1/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\cot{(c + dx)})^3}{\sqrt{e\cot{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))**3/sqrt(e*cot(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^3/sqrt(e*cot(d*x + c)), x)
```

Mupad [**B**]
time = 1.41, size = 1896, normalized size = 6.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(1/2),x)

[Out] atan((((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 - (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*1i + ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 + (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*1i)/(((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 + (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2) - ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 - (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2) + (16*(3*a^8*b*e^2 - b^9*e^2 + 6*a^4*b^5*e^2 + 8*a^6*b^3*e^2))/d^3))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*2i + atan((((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 - (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*1i + ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 + (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*1i)/(((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 + (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2) - ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 - (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2) + (16*(3*a^8*b*e^2 - b^9*e^2 + 6*a^4*b^5*e^2 + 8*a^6*b^3*e^2))/d^3))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*2i - (2*b^3*(e*cot(c + d*x))^(3/2))/(3*d*e^2) - (6*a*b^2*(e*cot(c + d*x))^(1/2))/(d*e)

3.65 $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=313

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cdots}}{}\right.}{\sqrt{2}\,de^{3/2}}$$

```
[Out] -1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d
/e^(3/2)*2^(1/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^
(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d
*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(3/2)*2^(1/2)-1/4*(a-b)*(a^
2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/
e^(3/2)*2^(1/2)+2*a^2*(a+b*cot(d*x+c))/d/e/(e*cot(d*x+c))^(1/2)-2*b*(a^2+b^
2)*(e*cot(d*x+c))^(1/2)/d/e^2
```

Rubi [A]
time = 0.29, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3646, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}}+\frac{(a+b)\left(a^2-4ab+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{3/2}}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}}-\frac{2b\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}}{de^2}+\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] -(((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sq
rt[e]])/(Sqrt[2]*d*e^(3/2))) + ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + (Sqr
t[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)) - (2*b*(a^2 + b^2)
*Sqrt[e*Cot[c + d*x]])/(d*e^2) + (2*a^2*(a + b*Cot[c + d*x]))/(d*e*Sqrt[e*C
ot[c + d*x]]) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c +
d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - ((a - b)*(a^2
 + 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c +
 d*x]]])/(2*Sqrt[2]*d*e^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
```

```
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} - \frac{2\int \frac{-2a^2be^2+\frac{1}{2}a(a^2-3b^2)e^2\cot(c+dx)-\frac{1}{2}b(a^2+b^2)e^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{e^3}$$

$$= -\frac{2b(a^2+b^2)\sqrt{e\cot(c+dx)}}{de^2} + \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}b(3a^2-b^2)e^2+\frac{1}{2}a(a}{\sqrt{e\cot(c}}}{e^3}$$

$$= -\frac{2b(a^2+b^2)\sqrt{e\cot(c+dx)}}{de^2} + \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} - \frac{4\mathrm{Subst}\left(\int \frac{\frac{1}{2}b(3a^2-b^2)e}{}\right.}{}$$

$$= -\frac{2b(a^2+b^2)\sqrt{e\cot(c+dx)}}{de^2} + \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} + \frac{((a+b)(a^2-4ab+}{}$$

$$= -\frac{2b(a^2+b^2)\sqrt{e\cot(c+dx)}}{de^2} + \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} + \frac{((a-b)(a^2+4ab+}{}$$

$$= -\frac{2b(a^2+b^2)\sqrt{e\cot(c+dx)}}{de^2} + \frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} + \frac{(a-b)(a^2+4ab+b}{}$$

$$= -\frac{(a+b)(a^2-4ab+b^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}} + \frac{(a+b)(a^2-4ab}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 3.55, size = 193, normalized size = 0.62

$$\frac{2\left(3ab^2-b^3\cot(c+dx)+a(a^2-3b^2)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)-\frac{b(-3a^2+b^2)\sqrt{\cot(c+dx)}\left(2\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{4\sqrt{2}}\right)}{de\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (2*(3*a*b^2 - b^3*Cot[c + d*x] + a*(a^2 - 3*b^2)*Hypergeometric2F1[-1/4, 1,
 3/4, -Cot[c + d*x]^2] - (b*(-3*a^2 + b^2)*Sqrt[Cot[c + d*x]]*(2*ArcTan[1 -
 Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + L
og[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Co
t[c + d*x]] + Cot[c + d*x]]))/(4*Sqrt[2])))/(d*e*Sqrt[e*Cot[c + d*x]])
```

Maple [A]
time = 0.49, size = 332, normalized size = 1.06

method	result
derivativedivides	$-\frac{2\left(b^3\sqrt{e\cot(dx+c)}-\frac{a^3e}{\sqrt{e\cot(dx+c)}}-e\left(\frac{\left(-3a^2be+b^3e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot}}\right.\right.}{}\right.\right.}{}$
default	$-\frac{2\left(b^3\sqrt{e\cot(dx+c)}-\frac{a^3e}{\sqrt{e\cot(dx+c)}}-e\left(\frac{\left(-3a^2be+b^3e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot}}\right.\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e^2*(b^3*(e*cot(d*x+c))^(1/2)-a^3*e/(e*cot(d*x+c))^(1/2)-e*(1/8*(-3*a^
2*b*e+b^3*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(a^3-3*a*b
^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*
2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)
+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arcta
n(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.51, size = 220, normalized size = 0.70

$$\frac{\left(8\,a^3\sqrt{\tan(dx+c)}+2\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{8\,b^3}{\sqrt{\tan(dx+c)}}\right)e^{\left(-\frac{3}{2}\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^3*sqrt(tan(d*x + c)) + 2*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*a
rctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^3 - 3*a^
2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
- sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) +
1/tan(d*x + c) + 1) + sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(-sqrt(2)/
sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8*b^3/sqrt(tan(d*x + c)))*e^(-3/
2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\cot (c + dx))^3}{(e\cot (c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))**3/(e*cot(c + d*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^3/(e*cot(d*x + c))^(3/2), x)
```

Mupad [**B**]
time = 1.20, size = 1951, normalized size = 6.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(3/2),x)
```

```
[Out] (2*a^3)/(d*e*(e*cot(c + d*x))^(1/2)) - atan((((e*cot(c + d*x))^(1/2)*(16*a^
6*d^3*e^5 - 16*b^6*d^3*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) + (
32*b^3*d^4*e^7 - 96*a^2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15
*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i +
 a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e
^3))^(1/2)*1i + ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^5 - 16*b^6*d^3*e^5 +
240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) - (32*b^3*d^4*e^7 - 96*a^2*b*d^4
*e^7)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a
^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^
2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2)*1i)/(((e*cot(c + d
*x))^(1/2)*(16*a^6*d^3*e^5 - 16*b^6*d^3*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4
*b^2*d^3*e^5) - (32*b^3*d^4*e^7 - 96*a^2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i
 + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2
))*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*
b^2)*1i)/(4*d^2*e^3))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^5 - 16*
b^6*d^3*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) + (32*b^3*d^4*e^7
- 96*a^2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*
b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i + a^5*b*6i + a^6
 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2) - 16
*a^9*d^2*e^4 + 48*a*b^8*d^2*e^4 + 128*a^3*b^6*d^2*e^4 + 96*a^5*b^4*d^2*e^4)
)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b
^2)*1i)/(4*d^2*e^3))^(1/2)*2i - atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e
^5 - 16*b^6*d^3*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) + (32*b^3*
d^4*e^7 - 96*a^2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^
4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i + a^5*b*
6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1
/2)*1i + ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^5 - 16*b^6*d^3*e^5 + 240*a^2
*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) - (32*b^3*d^4*e^7 - 96*a^2*b*d^4*e^7)*(
-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)
*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 -
 a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2)*1i)/(((e*cot(c + d*x))^(1
/2)*(16*a^6*d^3*e^5 - 16*b^6*d^3*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^
3*e^5) - (32*b^3*d^4*e^7 - 96*a^2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i - a^6
+ b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((
a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i
)/(4*d^2*e^3))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^5 - 16*b^6*d^3
*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) + (32*b^3*d^4*e^7 - 96*a^
2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i
 + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6
- 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2) - 16*a^9*d^
2*e^4 + 48*a*b^8*d^2*e^4 + 128*a^3*b^6*d^2*e^4 + 96*a^5*b^4*d^2*e^4))*(-((a
```

```
*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)
/(4*d^2*e^3))^(1/2)*2i - (2*b^3*(e*cot(c + d*x))^(1/2))/(d*e^2)
```

3.66 $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=313

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}$$

```
[Out] 2/3*a^2*(a+b*cot(d*x+c))/d/e/(e*cot(d*x+c))^(3/2)-1/2*(a-b)*(a^2+4*a*b+b^2)
*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/2*(a-b)
*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2
^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*c
ot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+co
t(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)+16/3*a^2*b
/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3646, 3709, 3615, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}}+\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{5/2}}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}}+\frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}}+\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] -(((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sq
rt[e]])/(Sqrt[2]*d*e^(5/2))) + ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + (Sqr
t[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)) + (16*a^2*b)/(3*d*
e^2*Sqrt[e*Cot[c + d*x]]) + (2*a^2*(a + b*Cot[c + d*x]))/(3*d*e*(e*Cot[c +
d*x])^(3/2)) - ((a + b)*(a^2 - 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d
*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) + ((a + b)*(a^2
- 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c +
d*x]]])/(2*Sqrt[2]*d*e^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
```

```
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} - \frac{2\int \frac{-4a^2be^2+\frac{3}{2}a(a^2-3b^2)e^2\cot(c+dx)+\frac{1}{2}b(a^2-3b^2)e^2\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}}\,d}{3e^3}$$

$$= \frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3}{2}a(a^2-3b^2)e^3+\frac{3}{2}b(3a^2-b^2)e^3\cot}{\sqrt{e\cot(c+dx)}}}{3e^5}$$

$$= \frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} - \frac{4\text{Subst}\left(\int \frac{-\frac{3}{2}a(a^2-3b^2)e^4-\frac{3}{2}b(3a}{e^2+x^4}\right.}{}$$

$$= \frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} + \frac{((a+b)(a^2-4ab+b^2))\,\text{Sub}}{}$$

$$= \frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} - \frac{((a+b)(a^2-4ab+b^2))\,\text{Sub}}{}$$

$$= \frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} - \frac{(a+b)(a^2-4ab+b^2)\log\left(\right.}{}$$

$$= -\frac{(a-b)(a^2+4ab+b^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}} + \frac{(a-b)(a^2+4a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.38, size = 104, normalized size = 0.33

$$\frac{-6b(-3a^2+b^2)\ {}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+2a(a^2-3b^2)\ {}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)\tan(c+dx)+6b^2(b+a\tan(c+dx))}{3de^2\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(5/2),x]
```

```
[Out] (-6*b*(-3*a^2 + b^2)*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + 2*a
*(a^2 - 3*b^2)*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2]*Tan[c + d*x
] + 6*b^2*(b + a*Tan[c + d*x]))/(3*d*e^2*Sqrt[e*Cot[c + d*x]])
```

Maple [A]
time = 0.53, size = 331, normalized size = 1.06

method	result
derivativedivides	$-\frac{2\left(-\frac{a^3e}{3(e\cot(dx+c))^{\frac{3}{2}}}-\frac{3a^2b}{\sqrt{e\cot(dx+c)}}+\frac{(-a^3e+3ab^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c}\right.\right.}{}\right.}{}$
default	$-\frac{2\left(-\frac{a^3e}{3(e\cot(dx+c))^{\frac{3}{2}}}-\frac{3a^2b}{\sqrt{e\cot(dx+c)}}+\frac{(-a^3e+3ab^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e^2*(-1/3*a^3*e/(e*cot(d*x+c))^(3/2)-3*a^2*b/(e*cot(d*x+c))^(1/2)+1/8*
(-a^3*e+3*a*b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-3*
a^2*b+b^3)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*
2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-
2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Maxima [A]
time = 0.53, size = 223, normalized size = 0.71

$$\frac{\left(6\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+3\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-3\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+8\left(a^3+\frac{9a^2b}{\tan(dx+c)}\right)\tan(dx+c)^{\frac{3}{2}}\right)e^{\left(-\frac{5}{2}\right)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(6*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2)
 + 2/sqrt(tan(d*x + c)))) + 6*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 3*sqrt(2)*(a^3 - 3*a^2*b
 - 3*a*b^2 + b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 3*
sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1
/tan(d*x + c) + 1) + 8*(a^3 + 9*a^2*b/tan(d*x + c))*tan(d*x + c)^(3/2))*e^(
-5/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\cot(c + dx))^3}{(e\cot(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))**3/(e*cot(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^3/(e*cot(d*x + c))^(5/2), x)
```

Mupad [B]
time = 1.67, size = 1946, normalized size = 6.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(5/2),x)

[Out] ((2*a^3*e)/3 + 6*a^2*b*e*cot(c + d*x))/(d*e^2*(e*cot(c + d*x))^(3/2)) - ata
n((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d
^3*e^8 - 240*a^4*b^2*d^3*e^8) + (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*
b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/
(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b
^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2)*1i + ((e*cot(c + d*x))^(1/2)*(1
6*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 - 240*a^4*b^2*d^3*e^8)
 - (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6
 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*
6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d
^2*e^5))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^
8 + 240*a^2*b^4*d^3*e^8 - 240*a^4*b^2*d^3*e^8) + (32*a^3*d^4*e^11 - 96*a*b^
2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i -
 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 1
5*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2) - ((e*cot(c +
d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 - 240*a^
4*b^2*d^3*e^8) - (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*
6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1
/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4
*b^2)*1i)/(4*d^2*e^5))^(1/2) - 16*b^9*d^2*e^6 + 48*a^8*b*d^2*e^6 + 96*a^4*b
^5*d^2*e^6 + 128*a^6*b^3*d^2*e^6))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*
a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2)*2i - atan((((e*c
ot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 -
 240*a^4*b^2*d^3*e^8) + (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i +
 a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e
^5))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i +
 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2)*1i + ((e*cot(c + d*x))^(1/2)*(16*a^6*d^
3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 - 240*a^4*b^2*d^3*e^8) - (32*a
^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^
2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5
*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))
^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*
a^2*b^4*d^3*e^8 - 240*a^4*b^2*d^3*e^8) + (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^
11)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*
b^2)*1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^
4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2) - ((e*cot(c + d*x))^(1
/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 - 240*a^4*b^2*d^
3*e^8) - (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i - a^6
 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*(((
a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i
)/(4*d^2*e^5))^(1/2) - 16*b^9*d^2*e^6 + 48*a^8*b*d^2*e^6 + 96*a^4*b^5*d^2*e
^6 + 128*a^6*b^3*d^2*e^6))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4
- a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2)*2i
```

3.67 $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=343

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}}-\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\,c}}{}\right.}{\sqrt{2}\,de^{7/2}}$$

```
[Out] 8/5*a^2*b/d/e^2/(e*cot(d*x+c))^(3/2)+2/5*a^2*(a+b*cot(d*x+c))/d/e/(e*cot(d*
x+c))^(5/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)
/e^(1/2))/d/e^(7/2)*2^(1/2)-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*(e*c
ot(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*ln(e^
(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(7/2)*2^(1/2)+1/
4*(a-b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c)
)^(1/2))/d/e^(7/2)*2^(1/2)-2*a*(a^2-3*b^2)/d/e^3/(e*cot(d*x+c))^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3646, 3709, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}}-\frac{(a+b)\left(a^2-4ab+b^2\right)\operatorname{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{7/2}}-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{7/2}}+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{7/2}}-\frac{2a\left(a^2-3b^2\right)}{de^3\sqrt{e\cot(c+dx)}}+\frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}}+\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt
[e]])/(Sqrt[2]*d*e^(7/2)) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/2)) + (8*a^2*b)/(5*d*e^2*
(e*Cot[c + d*x])^(3/2)) - (2*a*(a^2 - 3*b^2))/(d*e^3*Sqrt[e*Cot[c + d*x]])
+ (2*a^2*(a + b*Cot[c + d*x]))/(5*d*e*(e*Cot[c + d*x])^(5/2)) - ((a - b)*(a
^2 + 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c
 + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[Sqrt[e]
 + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(7/
2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{2\int \frac{-6a^2be^2+\frac{5}{2}a(a^2-3b^2)e^2\cot(c+dx)+\frac{1}{2}b(3a^2-5b^2)e^2\cot^2(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx}{5e^3}$$

$$= \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{2\int \frac{\frac{5}{2}a(a^2-3b^2)e^3+\frac{5}{2}b(3a^2-b^2)e^3\text{co}}{(e\cot(c+dx))^{3/2}}}{5e^5}$$

$$= \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} - \frac{2a(a^2-3b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{2\int \frac{5}{2}}{}$$

$$= \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} - \frac{2a(a^2-3b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{4\text{Subs}}{}$$

$$= \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} - \frac{2a(a^2-3b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{((a+}{}$$

$$= \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} - \frac{2a(a^2-3b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{((a-}{}$$

$$= \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} - \frac{2a(a^2-3b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{(a-}{}$$

$$= \frac{(a+b)\,(a^2-4ab+b^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{7/2}} - \frac{(a+b)\,(a^2-4ab+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.66, size = 108, normalized size = 0.31

$$\frac{2\left(3a(a^2-3b^2)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(c+dx)\right)+b\left(b(9a+5b\cot(c+dx))+5(3a^2-b^2)\cot(c+dx)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)\right)\right)}{15de(e\cot(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(7/2),x]
```

```
[Out] (2*(3*a*(a^2 - 3*b^2)*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2] + b
*(b*(9*a + 5*b*Cot[c + d*x]) + 5*(3*a^2 - b^2)*Cot[c + d*x]*Hypergeometric2
F1[-3/4, 1, 1/4, -Cot[c + d*x]^2])))/(15*d*e*(e*Cot[c + d*x])^(5/2))
```

Maple [A]
time = 0.49, size = 359, normalized size = 1.05

method	result
derivativedivides	$-\frac{2\left(\frac{\left(-3a^2be+b^3e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\,c}}{(e}\right.\right.}{8e^2}\right.}{}$
default	$-\frac{2\left(\frac{\left(-3a^2be+b^3e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot\left(dx+c\right)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\,c}}{(e}\right.\right.}{8e^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e^2*(1/e*(1/8*(-3*a^2*b*e+b^3*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*
x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)+1))+1/8*(a^3-3*a*b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1
/5*a^3*e/(e*cot(d*x+c))^(5/2)-a^2*b/(e*cot(d*x+c))^(3/2)+a/e*(a^2-3*b^2)/(e
*cot(d*x+c))^(1/2))
```

Maxima [A]

time = 0.51, size = 243, normalized size = 0.71

$$\frac{\left(8\left(a^3+\frac{5a^2b}{\tan(dx+c)}-\frac{5\left(a^3-3ab^2\right)}{\tan(dx+c)^2}\right)\tan\left(dx+c\right)^{\frac{5}{2}}-10\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-10\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+5\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-5\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)\right)e^{\left(-\frac{7}{2}\right)}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/20*(8*(a^3 + 5*a^2*b/tan(d*x + c) - 5*(a^3 - 3*a*b^2)/tan(d*x + c)^2)*tan
(d*x + c)^(5/2) - 10*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqr
t(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) - 10*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^
2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 5*sqrt(2)*
(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x
```

```
+ c) + 1) - 5*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(-sqrt(2)/sqrt(tan
(d*x + c)) + 1/tan(d*x + c) + 1))*e^(-7/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \cot{(c + dx)})^3}{(e \cot{(c + dx)})^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(7/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))**3/(e*cot(c + d*x))**(7/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.06, size = 1969, normalized size = 5.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(7/2),x)
```

```
[Out] atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a^2*
b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) + (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e^15
)*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b
^2)*1i)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^
```

4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2)*1i + ((e*cot(c + d*x))
^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*
b^2*d^3*e^11) - (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e^15)*(-((a*b^5*6i + a^5*b*
6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1
/2))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^
4*b^2)*1i)/(4*d^2*e^7))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11
 - 16*b^6*d^3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) - (32*b^3
*d^4*e^15 - 96*a^2*b*d^4*e^15)*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2
*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5
*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))
^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a
^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) + (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e
^15)*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^
4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2
*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2) - 16*a^9*d^2*e^8 +
48*a*b^8*d^2*e^8 + 128*a^3*b^6*d^2*e^8 + 96*a^5*b^4*d^2*e^8))*(-((a*b^5*6i
+ a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*
e^7))^(1/2)*2i + atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^
3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) + (32*b^3*d^4*e^15 -
96*a^2*b*d^4*e^15)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b
^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5*b*6i + a^6
- b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2)*1i +
((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a^2*b^4*d^
3*e^11 - 240*a^4*b^2*d^3*e^11) - (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e^15)*(-((
a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i
)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^
3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)
*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^
3*e^11) - (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e^15)*(-((a*b^5*6i + a^5*b*6i + a
^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(
-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)
*1i)/(4*d^2*e^7))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6
*d^3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) + (32*b^3*d^4*e^15
 - 96*a^2*b*d^4*e^15)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^
3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5*b*6i + a
^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2) -
16*a^9*d^2*e^8 + 48*a*b^8*d^2*e^8 + 128*a^3*b^6*d^2*e^8 + 96*a^5*b^4*d^2*e^
8))*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4
*b^2)*1i)/(4*d^2*e^7))^(1/2)*2i + ((2*a^3*e)/5 + 2*e*cot(c + d*x)^2*(3*a*b^
2 - a^3) + 2*a^2*b*e*cot(c + d*x))/(d*e^2*(e*cot(c + d*x))^(5/2))

3.68 $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=377

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{9/2}}-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{9/2}}$$

```
[Out] 32/35*a^2*b/d/e^2/(e*cot(d*x+c))^(5/2)-2/3*a*(a^2-3*b^2)/d/e^3/(e*cot(d*x+c
))^(3/2)+2/7*a^2*(a+b*cot(d*x+c))/d/e/(e*cot(d*x+c))^(7/2)+1/2*(a-b)*(a^2+4
*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(9/2)*2^(1/2)-
1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/
e^(9/2)*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(
1/2)*(e*cot(d*x+c))^(1/2))/d/e^(9/2)*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e
^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(9/2)*2^(1/2)-2
*b*(3*a^2-b^2)/d/e^4/(e*cot(d*x+c))^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3646, 3709, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{9/2}}-\frac{(a-b)\left(a^2+4ab+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{9/2}}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{9/2}}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{9/2}}-\frac{2b\left(3a^2-b^2\right)}{de^4\sqrt{e\cot(c+dx)}}-\frac{2a\left(a^2-3b^2\right)}{3de^3(e\cot(c+dx))^{3/2}}+\frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(9/2),x]
```

```
[Out] ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt
[e]])/(Sqrt[2]*d*e^(9/2)) - ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(9/2)) + (32*a^2*b)/(35*d*e^
2*(e*Cot[c + d*x])^(5/2)) - (2*a*(a^2 - 3*b^2))/(3*d*e^3*(e*Cot[c + d*x])^(
3/2)) - (2*b*(3*a^2 - b^2))/(d*e^4*Sqrt[e*Cot[c + d*x]]) + (2*a^2*(a + b*Co
t[c + d*x]))/(7*d*e*(e*Cot[c + d*x])^(7/2)) + ((a + b)*(a^2 - 4*a*b + b^2)*
Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt
[2]*d*e^(9/2)) - ((a + b)*(a^2 - 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c +
 d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*d*e^(9/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
```

```
NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{-8a^2be^2+\frac{7}{2}a(a^2-3b^2)e^2\cot(c+dx)+\frac{1}{2}b(5a^2-7b^2)e^2\cot^2(c+dx)}{(e\cot(c+dx))^{7/2}}}{7e^3}$$

$$= \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} + \frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{\frac{7}{2}a(a^2-3b^2)e^3+\frac{7}{2}b(3a^2-b^2)e}{(e\cot(c+dx))^{5/2}}}{7e^5}$$

$$= \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} - \frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}} + \frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} -$$

$$= \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} - \frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}} + \frac{2a^2}{7d}$$

$$= \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} - \frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}} + \frac{2a^2}{7d}$$

$$= \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} - \frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}} + \frac{2a^2}{7d}$$

$$= \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} - \frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}} + \frac{2a^2}{7d}$$

$$= \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} - \frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}} + \frac{2a^2}{7d}$$

$$= \frac{(a-b)(a^2+4ab+b^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{9/2}} - \frac{(a-b)(a^2+4ab}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.74, size = 116, normalized size = 0.31

$$\frac{2\sqrt{e\cot(c+dx)}\left(5a(a^2-3b^2)\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\cot^2(c+dx)\right)+b\left(b(15a+7b\cot(c+dx))+7(3a^2-b^2)\cot(c+dx)\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(c+dx)\right)\right)\right)\tan^4(c+dx)}{35de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(9/2),x]
[Out] (2*Sqrt[e*Cot[c + d*x]]*(5*a*(a^2 - 3*b^2)*Hypergeometric2F1[-7/4, 1, -3/4,
 -Cot[c + d*x]^2] + b*(b*(15*a + 7*b*Cot[c + d*x]) + 7*(3*a^2 - b^2)*Cot[c
+ d*x]*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2]))*Tan[c + d*x]^4)/
(35*d*e^5)
```

Maple [A]
time = 0.46, size = 388, normalized size = 1.03

method	result
derivativedivides	$-\frac{2\left(-\frac{a^{3} e}{7(e\cot(dx+c))^{\frac{7}{2}}}-\frac{3a^{2} b}{5(e\cot(dx+c))^{\frac{5}{2}}}+\frac{a\left(a^{2}-3b^{2}\right)}{3e(e\cot(dx+c))^{\frac{3}{2}}}+\frac{b\left(3a^{2}-b^{2}\right)}{e^{2}\sqrt{e\cot\left(dx+c\right)}}+\frac{\left(a^{3}e-3a\,b^{2}e\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\,c\cdots}{e\,c\cdots}\right.\right.\cdots}{\cdots}\cdots\right)}{\cdots}$
default	$-\frac{2\left(-\frac{a^{3} e}{7(e\cot(dx+c))^{\frac{7}{2}}}-\frac{3a^{2} b}{5(e\cot(dx+c))^{\frac{5}{2}}}+\frac{a\left(a^{2}-3b^{2}\right)}{3e(e\cot(dx+c))^{\frac{3}{2}}}+\frac{b\left(3a^{2}-b^{2}\right)}{e^{2}\sqrt{e\cot\left(dx+c\right)}}+\frac{\left(a^{3}e-3a\,b^{2}e\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\,c\cdots}{e\,c\cdots}\right.\right.\cdots}{\cdots}\cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/e^2*(-1/7*a^3*e/(e*cot(d*x+c))^(7/2)-3/5*a^2*b/(e*cot(d*x+c))^(5/2)+1/
3*a/e*(a^2-3*b^2)/(e*cot(d*x+c))^(3/2)+b*(3*a^2-b^2)/e^2/(e*cot(d*x+c))^(1/
2)+1/e^2*(1/8*(a^3*e-3*a*b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
+1))+1/8*(3*a^2*b-b^3)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.52, size = 267, normalized size = 0.71

$$\frac{\left(8\left(15\,a^{3}+\frac{63\,a^{2}b}{\tan(dx+c)}-\frac{35\,(a^{3}-3\,ab^{2})}{\tan(dx+c)^{2}}-\frac{105\,(3\,a^{2}b-b^{3})}{\tan(dx+c)^{3}}\right)\tan\left(dx+c\right)^{\frac{7}{2}}-210\,\sqrt{2}\,(a^{3}+3\,a^{2}b-3\,ab^{2}-b^{3})\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-210\,\sqrt{2}\,(a^{3}+3\,a^{2}b-3\,ab^{2}-b^{3})\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-105\,\sqrt{2}\,(a^{3}-3\,a^{2}b-3\,ab^{2}+b^{3})\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+105\,\sqrt{2}\,(a^{3}-3\,a^{2}b-3\,ab^{2}+b^{3})\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)\right)e^{\left(-\frac{9}{2}\right)}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] 1/420*(8*(15*a^3 + 63*a^2*b/tan(d*x + c) - 35*(a^3 - 3*a*b^2)/tan(d*x + c)^
2 - 105*(3*a^2*b - b^3)/tan(d*x + c)^3)*tan(d*x + c)^(7/2) - 210*sqrt(2)*(a
^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x
```

```
+ c)))) - 210*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(
sqrt(2) - 2/sqrt(tan(d*x + c)))) - 105*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b
^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 105*sqrt(2)*(a^3
 - 3*a^2*b - 3*a*b^2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c
) + 1))*e^(-9/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \cot (c + dx))^3}{(e \cot (c + dx))^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(9/2),x)
[Out] Integral((a + b*cot(c + d*x))**3/(e*cot(c + d*x))**(9/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**B**]
time = 5.26, size = 1992, normalized size = 5.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(9/2),x)
[Out] atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*
b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) + (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19
```

```
)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^
2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4
- a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2)*1i + ((e*cot(c + d*x))^(
1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*b^4*d^3*e^14 - 240*a^4*b^
2*d^3*e^14) - (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19)*(((a*b^5*6i + a^5*b*6i
- a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2)
)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^
2)*1i)/(4*d^2*e^9))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 1
6*b^6*d^3*e^14 + 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) + (32*a^3*d^4
*e^19 - 96*a*b^2*d^4*e^19)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4
- a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i
- a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2)
 - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*b^4
*d^3*e^14 - 240*a^4*b^2*d^3*e^14) - (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19)*(
((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*
1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a
^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2) - 16*b^9*d^2*e^10 + 48*a^8*
b*d^2*e^10 + 96*a^4*b^5*d^2*e^10 + 128*a^6*b^3*d^2*e^10))*(((a*b^5*6i + a^5
*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))
^(1/2)*2i + atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^1
4 + 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) + (32*a^3*d^4*e^19 - 96*a*
b^2*d^4*e^19)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i
 - 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 +
 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2)*1i + ((e*cot
(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*b^4*d^3*e^14
- 240*a^4*b^2*d^3*e^14) - (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19)*(((a*b^5*6i
 + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2
*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i
 - 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(16*a^6*
d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) +
 (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 +
 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i
 + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2
*e^9))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 +
 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) - (32*a^3*d^4*e^19 - 96*a*b^2
*d^4*e^19)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i -
15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15
*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2) - 16*b^9*d^2*e^
10 + 48*a^8*b*d^2*e^10 + 96*a^4*b^5*d^2*e^10 + 128*a^6*b^3*d^2*e^10))*(((a*
b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/
(4*d^2*e^9))^(1/2)*2i + ((2*a^3*e)/7 + (2*e*cot(c + d*x)^2*(3*a*b^2 - a^3))
/3 - 2*e*cot(c + d*x)^3*(3*a^2*b - b^3) + (6*a^2*b*e*cot(c + d*x))/5)/(d*e^
2*(e*cot(c + d*x))^(7/2))
```

3.69 $\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx$

Optimal. Leaf size=325

$$\frac{2a^{5/2}e^{5/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{b^{3/2}\left(a^2+b^2\right)d}-\frac{(a+b)e^{5/2}\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,\left(a^2+b^2\right)d}+\frac{(a+b)e^{5/2}\operatorname{Arc}}{}$$

```
[Out] 2*a^(5/2)*e^(5/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/b^(3
/2)/(a^2+b^2)/d-1/2*(a+b)*e^(5/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(
1/2))/(a^2+b^2)/d*2^(1/2)+1/2*(a+b)*e^(5/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))
^(1/2)/e^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*e^(5/2)*ln(e^(1/2)+cot(d*x+c)
*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*e^(5/2
)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2
^(1/2)-2*e^2*(e*cot(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.44, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3647, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{e^{5/2}(a+b)\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\left(a^2+b^2\right)}+\frac{e^{5/2}(a+b)\operatorname{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\left(a^2+b^2\right)}+\frac{e^{5/2}(a-b)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)}-\frac{e^{5/2}(a-b)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\left(a^2+b^2\right)}+\frac{2a^{5/2}e^{5/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{b^{3/2}d\left(a^2+b^2\right)}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x]),x]
```

```
[Out] (2*a^(5/2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])]
)/(b^(3/2)*(a^2 + b^2)*d) - ((a + b)*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot
[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d) + ((a + b)*e^(5/2)*ArcTan[1 +
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d) - (2*e^2*
Sqrt[e*Cot[c + d*x]])/(b*d) + ((a - b)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c
+ d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)*d) - ((a - b
)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]
])/(2*Sqrt[2]*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx = -\frac{2e^2\sqrt{e\cot(c+dx)}}{bd} - \frac{2\int \frac{\frac{ae^3}{2}+\frac{1}{2}be^3\cot(c+dx)+\frac{1}{2}ae^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}\,dx}{b}$$

$$= -\frac{2e^2\sqrt{e\cot(c+dx)}}{bd} - \frac{2\int \frac{\frac{b^2e^3}{2}+\frac{1}{2}abe^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{b(a^2+b^2)} - \frac{(a^3e^3)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+b\cdots}}{b(a^2+b^2)}$$

$$= -\frac{2e^2\sqrt{e\cot(c+dx)}}{bd} - \frac{4\text{Subst}\left(\int \frac{-\frac{1}{2}b^2e^4-\frac{1}{2}abe^3x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{b(a^2+b^2)d} - \frac{(a^3e^3\cdots}{\cdots}$$

$$= -\frac{2e^2\sqrt{e\cot(c+dx)}}{bd} + \frac{(2a^3e^2)\,\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{b(a^2+b^2)d} - \frac{((a-\cdots}{\cdots}$$

$$= \frac{2a^{5/2}e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}(a^2+b^2)d} - \frac{2e^2\sqrt{e\cot(c+dx)}}{bd} + \frac{((a-b)e^{5/2})\,\text{Su}\cdots}{\cdots}$$

$$= \frac{2a^{5/2}e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}(a^2+b^2)d} - \frac{2e^2\sqrt{e\cot(c+dx)}}{bd} + \frac{(a-b)e^{5/2}\log\left(\cdots\right.}{\cdots}$$

$$= \frac{2a^{5/2}e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}(a^2+b^2)d} - \frac{(a+b)e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+\cdots}}{\sqrt{e}}\right.}{\sqrt{2}(a^2+b^2)d}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.89, size = 286, normalized size = 0.88

$$\frac{(e\cot(c+dx))^{5/2}\left(8ab^{3/2}\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)-3\left(2\sqrt{2}\,b^{5/2}\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,b^{5/2}\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)-8a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)+8a^2\sqrt{b}\sqrt{\cot(c+dx)}+8b^{5/2}\sqrt{\cot(c+dx)}+\sqrt{2}\,b^{5/2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\,b^{5/2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{12b^{3/2}(a^2+b^2)\,d\cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x]),x]
```

```
[Out] ((e*Cot[c + d*x])^(5/2)*(8*a*b^(3/2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3
/4, 1, 7/4, -Cot[c + d*x]^2] - 3*(2*Sqrt[2]*b^(5/2)*ArcTan[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]]] - 2*Sqrt[2]*b^(5/2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]
- 8*a^(5/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]] + 8*a^2*Sqrt[b]*Sq
rt[Cot[c + d*x]] + 8*b^(5/2)*Sqrt[Cot[c + d*x]] + Sqrt[2]*b^(5/2)*Log[1 - S
qrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*b^(5/2)*Log[1 + Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(12*b^(3/2)*(a^2 + b^2)*d*Cot[c + d*
x]^(5/2))
```

Maple [A]

time = 0.60, size = 347, normalized size = 1.07

method	result
derivativedivides	$-\frac{2e^2\left(\frac{\sqrt{e\cot(dx+c)}}{b}-\frac{a^3 e\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{b(a^2+b^2)\sqrt{aeb}}-\frac{e\left(\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cdots}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cdots}}\cdots\right.\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{2e^2\left(\frac{\sqrt{e\cot(dx+c)}}{b}-\frac{a^3 e\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{b(a^2+b^2)\sqrt{aeb}}-\frac{e\left(\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cdots}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cdots}}\cdots\right.\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^2*(1/b*(e*cot(d*x+c))^(1/2)-1/b*a^3*e/(a^2+b^2)/(a*e*b)^(1/2)*arctan
(b*(e*cot(d*x+c))^(1/2)/(a*e*b)^(1/2))-1/(a^2+b^2)*e*(1/8*b/e*(e^2)^(1/4)*2
^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*
arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*a/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)+1))))
```

Maxima [A]

time = 0.54, size = 191, normalized size = 0.59

$$\frac{\left(\frac{8a^3\arctan\left(\frac{b}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^2b+b^3)\sqrt{ab}}+\frac{2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}(a-b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}(a-b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^2+b^2}-\frac{8}{b\sqrt{\tan(dx+c)}}\right)e^{\frac{5}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^3*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^2*b + b^3)*sqrt(a*b
)) + (2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))
) + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
 - sqrt(2)*(a - b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + s
qrt(2)*(a - b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^2
+ b^2) - 8/(b*sqrt(tan(d*x + c))))*e^(5/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cot(c+dx))^{\frac{5}{2}}}{a+b\cot(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)/(a+b*cot(d*x+c)),x)
```

```
[Out] Integral((e*cot(c + d*x))**(5/2)/(a + b*cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)/(b*cot(d*x + c) + a), x)
```

Mupad [B]
time = 1.97, size = 2500, normalized size = 7.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(5/2)/(a + b*cot(c + d*x)),x)
```

```
[Out] (atan(((((32*(e*cot(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d^4) + (((3
2*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5) + (
((32*(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*
b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4) - (((32*(4*a*b^8*d^4*e^13 + 8*a
^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13))/(b*d^5) + (32*(e*cot(c + d*x))^(1/2)
*(-a^5*b^3*e^5)^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*
d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b^4*d^5*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2
))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^
3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2)*1i)/(b^3*d*(a^2 + b
^2)) + (((32*(e*cot(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d^4) - (((3
2*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5) - (
((32*(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*
b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4) + (((32*(4*a*b^8*d^4*e^13 + 8*a
^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13))/(b*d^5) - (32*(e*cot(c + d*x))^(1/2)
*(-a^5*b^3*e^5)^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*
d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b^4*d^5*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2
))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^
3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2)*1i)/(b^3*d*(a^2 + b
^2)))/((64*(a^5*e^23 - a^3*b^2*e^23))/(b*d^5) + (((32*(e*cot(c + d*x))^(1/2
)*(2*a^6*e^20 - b^6*e^20))/(b*d^4) + (((32*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2
*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5) + (((32*(e*cot(c + d*x))^(1/2)*(16*a^
7*b*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15)
)/(b*d^4) - (((32*(4*a*b^8*d^4*e^13 + 8*a^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^
13))/(b*d^5) + (32*(e*cot(c + d*x))^(1/2)*(-a^5*b^3*e^5)^(1/2)*(16*b^10*d^4
*e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(
b^4*d^5*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*
e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))
*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)) - (((32*(e*cot(c + d*x))^(1/2)*(
2*a^6*e^20 - b^6*e^20))/(b*d^4) - (((32*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^
18 - 15*a^4*b^3*d^2*e^18))/(b*d^5) - (((32*(e*cot(c + d*x))^(1/2)*(16*a^7*b
*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(
b*d^4) + (((32*(4*a*b^8*d^4*e^13 + 8*a^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13)
)/(b*d^5) - (32*(e*cot(c + d*x))^(1/2)*(-a^5*b^3*e^5)^(1/2)*(16*b^10*d^4*e^
10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b^4
*d^5*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5
)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-
a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2))))*(-a^5*b^3*e^5)^(1/2)*2i)/(b^3*d*(
a^2 + b^2)) - atan((((((((32*(4*a*b^8*d^4*e^13 + 8*a^3*b^6*d^4*e^13 + 4*a^5*
b^4*d^4*e^13))/(b*d^5) - (32*(e*cot(c + d*x))^(1/2)*(-e^5/(4*(b^2*d^2*1i -
a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8*d^4*e^10 - 1
6*a^4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i -
a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*
e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4
))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(12*a^6*b*d
^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5))*(-e^5/(4*(b^2*d
```

```
^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(2*a^6
*e^20 - b^6*e^20))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))
)^(1/2)*1i - (((((32*(4*a*b^8*d^4*e^13 + 8*a^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4
*e^13))/(b*d^5) + (32*(e*cot(c + d*x))^(1/2)*(-e^5/(4*(b^2*d^2*1i - a^2*d^2
*1i + 2*a*b*d^2)))^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b
^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2
*1i + 2*a*b*d^2)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*e^15 -
14*a*b^7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4))*(-e^
5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(12*a^6*b*d^2*e^18
 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5))*(-e^5/(4*(b^2*d^2*1i -
 a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(2*a^6*e^20 -
 b^6*e^20))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)
*1i)/((((((32*(4*a*b^8*d^4*e^13 + 8*a^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13))
/(b*d^5) - (32*(e*cot(c + d*x))^(1/2)*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2
*a*b*d^2)))^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*d^4*
e^10 - 16*a^6*b^4*d^4*e^10))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i + 2
*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*e^15 - 14*a*b^
7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4))*(-e^5/(4*(b
^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(12*a^6*b*d^2*e^18 + a^2*
b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5))*(-e^5/(4*(b^2*d^2*1i - a^2*d^
2*1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^
20))/(b*d^4))*(-e^5/(4*(b^2*d^2*1i - a^2*d^2*1i...
```

3.70 $\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx$

Optimal. Leaf size=302

$$-\frac{2a^{3/2}e^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)\,d}-\frac{(a-b)e^{3/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d}+\frac{(a-b)e^{3/2}\mathrm{Ar}}{}$$

```
[Out] -1/2*(a-b)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)
/d*2^(1/2)+1/2*(a-b)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))
/(a^2+b^2)/d*2^(1/2)-1/4*(a+b)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2
)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a+b)*e^(3/2)*ln(e^(1/2)+co
t(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)-2*a^(3/2
)*e^(3/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/(a^2+b^2)/d/
b^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3654, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{e^{3/2}(a-b)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\,(a^2+b^2)}+\frac{e^{3/2}(a-b)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)}-\frac{e^{3/2}(a+b)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)}+\frac{e^{3/2}(a+b)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)}-\frac{2a^{3/2}e^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{b}\,d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x]),x]
```

```
[Out] (-2*a^(3/2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])
])/(Sqrt[b]*(a^2 + b^2)*d) - ((a - b)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Co
t[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d) + ((a - b)*e^(3/2)*ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d) - ((a +
b)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]
]])/(2*Sqrt[2]*(a^2 + b^2)*d) + ((a + b)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[
c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

```
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3654

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2
*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]
], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = \frac{\int \frac{-ae^2+be^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a^2+b^2} + \frac{(a^2e^2)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{2\mathrm{Subst}\left(\int \frac{ae^3-be^2x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d} + \frac{(a^2e^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-ex}\,(a-bx)}\,dx, x\right.}{(a^2+b^2)\,d}$$

$$= -\frac{(2a^2e)\,\mathrm{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d} + \frac{((a-b)e^2)\,\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx\right.}{(a^2+b^2)}$$

$$= -\frac{2a^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)\,d} - \frac{((a+b)e^{3/2})\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\,\sqrt{e}+2}{-e-\sqrt{2}\,\sqrt{e}\,x}\right.}{2\sqrt{2}\,(a^2+}$$

$$= -\frac{2a^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)\,d} - \frac{(a+b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\,\cot(c+\right.}{2\sqrt{2}\,(a^2+}$$

$$= -\frac{2a^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)\,d} - \frac{(a-b)e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(}}{\sqrt{e}}\right.}{\sqrt{2}\,(a^2+b^2)\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 0.54, size = 249, normalized size = 0.82

$$-\frac{(e\cot(c+dx))^{3/2}\left(8b^{3/2}\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)+3a\left(2\sqrt{2}\sqrt{b}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\sqrt{b}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+8\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)+\sqrt{2}\sqrt{b}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\sqrt{b}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{12\sqrt{b}\,(a^2+b^2)\,d\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x]),x]
```

```
[Out] -1/12*((e*Cot[c + d*x])^(3/2)*(8*b^(3/2)*Cot[c + d*x]^(3/2)*Hypergeometric2
F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + 3*a*(2*Sqrt[2]*Sqrt[b]*ArcTan[1 - Sqrt[2
]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*Sqrt[b]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d
*x]]] + 8*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]] + Sqrt[2]*Sq
rt[b]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Sqrt[b]*
Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(Sqrt[b]*(a^2 + b^2)*
d*Cot[c + d*x]^(3/2))
```

Maple [A]
time = 0.61, size = 326, normalized size = 1.08

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2e^2\left(-\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^2*(1/(a^2+b^2)*(-1/8*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))
+1/8*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
```

```
/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*ar
ctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+a^2/(a^2+b^2)/(a*e*b)^(
1/2)*arctan(b*(e*cot(d*x+c))^(1/2)/(a*e*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 177, normalized size = 0.59

$$-\frac{\left(\frac{8\,a^2\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{(a^2+b^2)\sqrt{ab}}-\frac{2\,\sqrt{2}\,(a-b)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a-b)\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\,(a+b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\,(a+b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^2+b^2}\right)e^{\frac{3}{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(8*a^2*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^2 + b^2)*sqrt(a*b)
) - (2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))))
 + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
+ sqrt(2)*(a + b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sq
rt(2)*(a + b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^2 +
 b^2))*e^(3/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}{a+b\cot\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c)),x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)/(a + b*cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="giac")

[Out] integrate((e*cot(d*x + c))^(3/2)/(b*cot(d*x + c) + a), x)

Mupad [B]
time = 1.63, size = 2500, normalized size = 8.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cot(c + d*x))^(3/2)/(a + b*cot(c + d*x)),x)

[Out] atan((((((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 - (32*(e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15))/d^5)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i - (((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 + (32*(e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15))/d^5)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i)/((((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 - (32*(e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15))/d^5)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 + (32*(e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(

```
1/2) + (32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15))/d^5)*
((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot(c + d*x)
)^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a
*b*d^2*2i)))^(1/2) + (64*a^2*b^2*e^18)/d^5))*((e^3*1i)/(4*(b^2*d^2 - a^2*d^
2 + a*b*d^2*2i)))^(1/2)*2i + atan(((((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*
d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 - (32*(e*cot(c + d*x))^(1/2)*(e^3/(4*(b
^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d
^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(e^3/(4*(b^2*d^2
*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(14*a*b^
6*d^2*e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*(e^3/(4*(b^2*d
^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2
*e^15 - 15*a^3*b^3*d^2*e^15))/d^5)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b
*d^2)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*
(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*1i - (((((32*(4*a^2*b
^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 + (32*(e*cot(c
+ d*x))^(1/2)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^9
*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10
))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(e*cot(
c + d*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^
13))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(a*b^
5*d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15))/d^5)*(e^3/(4*(b^2*d^2
*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(b^5*e^1
6 + 2*a^4*b*e^16))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/
2)*1i)/((((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^
12))/d^5 - (32*(e*cot(c + d*x))^(1/2)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*
a*b*d^2)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^
10 - 16*a^6*b^3*d^4*e^10))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^
2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*
e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*
d^2)))^(1/2) + (32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15
))/d^5)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(e*cot(
c + d*x))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d
^2*1i + 2*a*b*d^2)))^(1/2) + (((((32*(4*a^2*b^6...
```

3.71 $\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx$

Optimal. Leaf size=302

$$\frac{2\sqrt{a}\,\sqrt{b}\,\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{(a^2+b^2)\,d}+\frac{(a+b)\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d}-\frac{(a+b)\sqrt{e}}{}$$

```
[Out] 1/2*(a+b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/(a^2+b^2)/
d*2^(1/2)-1/2*(a+b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/
(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(
d*x+c))^(1/2))*e^(1/2)/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*
e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/(a^2+b^2)/d*2^(1/2)+2*arctan(
b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*a^(1/2)*b^(1/2)*e^(1/2)/(a^2+
b^2)/d
```

Rubi [A]

time = 0.24, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3653, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{2\sqrt{a}\,\sqrt{b}\,\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{d\,(a^2+b^2)}+\frac{\sqrt{e}\,(a+b)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\,(a^2+b^2)}-\frac{\sqrt{e}\,(a+b)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)}-\frac{\sqrt{e}\,(a-b)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)}+\frac{\sqrt{e}\,(a-b)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x]),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*S
qrt[e])])/((a^2 + b^2)*d) + ((a + b)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot
[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d) - ((a + b)*Sqrt[e]*ArcTan[1 +
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d) - ((a - b
)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]
])/(2*Sqrt[2]*(a^2 + b^2)*d) + ((a - b)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c
 + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

```
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3653

```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a*c + b*d + (b*c -
a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[d*((b*c - a*d
)/(c^2 + d^2)), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \frac{\int \frac{be+ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a^2+b^2} - \frac{(abe)\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{2\text{Subst}\left(\int \frac{-be^2-aex^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{(abe)\text{Subst}\left(\int \frac{1}{\sqrt{-ex}\,(a-bx)}\,dx, x, -\right.}{(a^2+b^2)\,d}$$

$$= \frac{(2ab)\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d} + \frac{((a-b)e)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e}\right.}{(a^2+b^2)\,d}$$

$$= \frac{2\sqrt{a}\sqrt{b}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{(a^2+b^2)\,d} - \frac{((a-b)\sqrt{e})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+}{-e-\sqrt{2}\sqrt{e}}\right.}{2\sqrt{2}\,(a^2+}$$

$$= \frac{2\sqrt{a}\sqrt{b}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{(a^2+b^2)\,d} - \frac{(a-b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+\right.}{2\sqrt{2}\,(a^2+}$$

$$= \frac{2\sqrt{a}\sqrt{b}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{(a^2+b^2)\,d} + \frac{(a+b)\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cot}}{\sqrt{e}}\right.}{\sqrt{2}\,(a^2+b^2)\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.26, size = 226, normalized size = 0.75

$$\frac{\sqrt{e\cot(c+dx)}\left(6\sqrt{2}\,b\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-6\sqrt{2}\,b\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)+24\sqrt{a}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)-8a\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)+3\sqrt{2}\,b\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-3\sqrt{2}\,b\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{12\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x]),x]
```

```
[Out] (Sqrt[e*Cot[c + d*x]]*(6*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] -
 6*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24*Sqrt[a]*Sqrt[b]*Ar
cTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]] - 8*a*Cot[c + d*x]^(3/2)*Hyperge
ometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + 3*Sqrt[2]*b*Log[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]] + Cot[c + d*x]] - 3*Sqrt[2]*b*Log[1 + Sqrt[2]*Sqrt[Cot[c + d
*x]] + Cot[c + d*x]]))/(12*(a^2 + b^2)*d*Sqrt[Cot[c + d*x]])
```

Maple [A]
time = 0.61, size = 332, normalized size = 1.10

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{ab\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{e\left(a^2+b^2\right)\sqrt{aeb}}+\frac{b\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{}}\right.\right.}{}\right.}{}$
default	$-\frac{2e^2\left(-\frac{ab\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{e\left(a^2+b^2\right)\sqrt{aeb}}+\frac{b\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{}}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^2*(-a/e*b/(a^2+b^2)/(a*e*b)^(1/2)*arctan(b*(e*cot(d*x+c))^(1/2)/(a*e
*b)^(1/2))+1/(a^2+b^2)/e*(1/8*b/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
))+1/8*a/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*
arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.51, size = 176, normalized size = 0.58

$$\frac{\left(\frac{8\,ab\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{\left(a^2+b^2\right)\sqrt{ab}} - \frac{2\,\sqrt{2}\,(a+b)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a+b)\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\sqrt{2}\,(a-b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)+\sqrt{2}\,(a-b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)}{a^2+b^2}\right)e^{\frac{1}{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(8*a*b*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^2 + b^2)*sqrt(a*b))
 - (2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))))
+ 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) -
 sqrt(2)*(a - b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqr
t(2)*(a - b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^2 +
b^2))*e^(1/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cot\left(c+dx\right)}}{a+b\cot\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))/(a + b*cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))/(b*cot(d*x + c) + a), x)
```

Mupad [B]
time = 1.39, size = 2500, normalized size = 8.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)/(a + b*cot(c + d*x)),x)
```

```
[Out] (atan(((((32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4 - (((3
2*(13*a^2*b^4*d^2*e^12 + a^4*b^2*d^2*e^12))/d^5 + (((32*(e*cot(c + d*x))^(1
/2)*(20*a^3*b^4*d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4 + (
((32*(12*a*b^7*d^4*e^11 + 24*a^3*b^5*d^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 -
 (32*(e*cot(c + d*x))^(1/2)*(-a*b*e)^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^
4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/(d^5*(a^2 + b^2)))*(-a
*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/
2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2)*1i)/(d*(a^2 + b^2)) + (((32*(e*cot(c +
d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4 + (((32*(13*a^2*b^4*d^2*e^12 +
 a^4*b^2*d^2*e^12))/d^5 - (((32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^4*d^2*e^11
 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4 - (((32*(12*a*b^7*d^4*e^11
+ 24*a^3*b^5*d^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 + (32*(e*cot(c + d*x))^(1
/2)*(-a*b*e)^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*
e^10 - 16*a^6*b^3*d^4*e^10))/(d^5*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b
^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a
*b*e)^(1/2)*1i)/(d*(a^2 + b^2)))/((64*a*b^3*e^13)/d^5 - (((32*(e*cot(c + d*
x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4 - (((32*(13*a^2*b^4*d^2*e^12 + a
^4*b^2*d^2*e^12))/d^5 + (((32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^4*d^2*e^11 -
 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4 + (((32*(12*a*b^7*d^4*e^11 +
24*a^3*b^5*d^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 - (32*(e*cot(c + d*x))^(1/2
)*(-a*b*e)^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^
10 - 16*a^6*b^3*d^4*e^10))/(d^5*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2
)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b
*e)^(1/2))/(d*(a^2 + b^2)) + (((32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2
*b^3*e^12))/d^4 + (((32*(13*a^2*b^4*d^2*e^12 + a^4*b^2*d^2*e^12))/d^5 - (((
32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^4*d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*
b^2*d^2*e^11))/d^4 - (((32*(12*a*b^7*d^4*e^11 + 24*a^3*b^5*d^4*e^11 + 12*a^
5*b^3*d^4*e^11))/d^5 + (32*(e*cot(c + d*x))^(1/2)*(-a*b*e)^(1/2)*(16*b^9*d^
4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/
(d^5*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2
 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))
)*(-a*b*e)^(1/2)*2i)/(d*(a^2 + b^2)) - atan(((((32*(13*a^2*b^4*d^2*e^12 + a
^4*b^2*d^2*e^12))/d^5 + (((32*(12*a*b^7*d^4*e^11 + 24*a^3*b^5*d^4*e^11 + 12
*a^5*b^3*d^4*e^11))/d^5 - (32*(e*cot(c + d*x))^(1/2)*(-e/(4*(b^2*d^2*1i - a
^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*
a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(-e/(4*(b^2*d^2*1i - a^2*d^2*
1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^4*d^2*e^11 -
```

```
 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4)*(-e/(4*(b^2*d^2*1i - a^2*d^2
*1i + 2*a*b*d^2)))^(1/2))*(-e/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1
/2) - (32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4)*(-e/(4*(
b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*1i - (((32*(13*a^2*b^4*d^2*e^1
2 + a^4*b^2*d^2*e^12))/d^5 + (((32*(12*a*b^7*d^4*e^11 + 24*a^3*b^5*d^4*e^11
 + 12*a^5*b^3*d^4*e^11))/d^5 + (32*(e*cot(c + d*x))^(1/2)*(-e/(4*(b^2*d^2*1
i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10
- 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(-e/(4*(b^2*d^2*1i - a^2
*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^4*d^2*e
^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4)*(-e/(4*(b^2*d^2*1i - a^
2*d^2*1i + 2*a*b*d^2)))^(1/2))*(-e/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)
))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4)*(-e
/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*1i)/((((32*(13*a^2*b^4*d^
2*e^12 + a^4*b^2*d^2*e^12))/d^5 + (((32*(12*a*b^7*d^4*e^11 + 24*a^3*b^5*d^4
*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 - (32*(e*cot(c + d*x))^(1/2)*(-e/(4*(b^2*
d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*
e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(-e/(4*(b^2*d^2*1i
- a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^4*
d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4)*(-e/(4*(b^2*d^2*1i
 - a^2*d^2*1i + 2*a*b*d^2)))^(1/2))*(-e/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b
*d^2)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4
)*(-e/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) + (((32*(13*a^2*b^4*
d^2*e^12 + a^4*b^2*d^2*e^12))/d^5 + (((32*(12*a*b^7*d^4*e^11 + 24*a^3*b^5*d
^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 + (32*(e*cot(c + d*x))^(1/2)*(-e/(4*(b^
2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^
4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(-e/(4*(b^2*d^2*1
i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^
4*d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4)*(-e/(4*(b^2*d^2*
1i - a^2*d^2*1i + 2*a*b*d^2)))^(1/2))*(-e/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a
*b*d^2)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(b...
```

3.72 $\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}\,dx$

Optimal. Leaf size=302

$$-\frac{2b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)\,d\sqrt{e}}+\frac{(a-b)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}}-\frac{(a-b)\text{ArcTan}\left(1+\right.}{\sqrt{2}\,(a^2\cdots}$$

```
[Out] 1/2*(a-b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)/d*2^(1/2
)/e^(1/2)-1/2*(a-b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2
)/d*2^(1/2)/e^(1/2)+1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(
d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)/e^(1/2)-1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*
e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)/e^(1/2)-2*b^(3/2)
*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/(a^2+b^2)/d/a^(1/2)/e
^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3655, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a-b)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}\,(a^2+b^2)}-\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\sqrt{e}\,(a^2+b^2)}-\frac{2b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{a}\,d\sqrt{e}\,(a^2+b^2)}+\frac{(a+b)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}\,(a^2+b^2)}-\frac{(a+b)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])),x]
```

```
[Out] (-2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(Sqrt
[a]*(a^2 + b^2)*d*Sqrt[e]) + ((a - b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*
x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*Sqrt[e]) - ((a - b)*ArcTan[1 + (Sqrt[
2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*Sqrt[e]) + ((a +
b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*S
qrt[2]*(a^2 + b^2)*d*Sqrt[e]) - ((a + b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x]
 + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)*d*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

```
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}\,dx = \frac{\int \frac{a-b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a^2+b^2} + \frac{b^2\int \frac{1+\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{2\text{Subst}\left(\int \frac{-ae+bx^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d} + \frac{b^2\text{Subst}\left(\int \frac{}{\sqrt{-e\cdots}}\right.}{}$$

$$= -\frac{(a-b)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx)}\right)}{(a^2+b^2)\,d} - \frac{(a+b)\text{Subs}\cdots}{}$$

$$= -\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)\,d\sqrt{e}} - \frac{(a-b)\text{Subst}\left(\int \frac{}{e-\sqrt{2}\cdots}\right.}{}$$

$$= -\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)\,d\sqrt{e}} + \frac{(a+b)\log\left(\sqrt{e}+\sqrt{\cdots}\right.}{2\cdots}$$

$$= -\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)\,d\sqrt{e}} + \frac{(a-b)\tan^{-1}\left(1-\frac{\sqrt{\cdots}}{}\right.}{\sqrt{2}\,(a^2+\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 0.24, size = 248, normalized size = 0.82

$$-\frac{\sqrt{\cot(c+dx)}\left(\frac{2b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,(a^2+b^2)}-\frac{2b\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)}{3(a^2+b^2)}-\frac{a\left(4\left(\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\right)+2\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-2\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{8(a^2+b^2)}\right)}{d\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])),x]
```

```
[Out] -((Sqrt[Cot[c + d*x]]*((2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[
a]])/(Sqrt[a]*(a^2 + b^2)) - (2*b*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4,
 1, 7/4, -Cot[c + d*x]^2])/(3*(a^2 + b^2)) - (a*(4*(Sqrt[2]*ArcTan[1 - Sqrt
[2]*Sqrt[Cot[c + d*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) +
 2*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 2*Sqrt[2]*L
og[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(8*(a^2 + b^2))))/(d*Sq
rt[e*Cot[c + d*x]]))
```

Maple [A]
time = 0.60, size = 332, normalized size = 1.10

method	result
derivativedivides	$-\frac{2e^2\left(\frac{\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$
default	$-\frac{2e^2\left(\frac{\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^2*(1/e^2/(a^2+b^2)*(1/8*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1))-1/8*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^
```

```
(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2
^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2
*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+b^2/e^2/(a^2+b^2)/(a
*e*b)^(1/2)*arctan(b*(e*cot(d*x+c))^(1/2)/(a*e*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 176, normalized size = 0.58

$$-\frac{\left(\frac{8\,b^2\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{\left(a^2+b^2\right)\sqrt{ab}}+\frac{2\,\sqrt{2}\,(a-b)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a-b)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\,(a+b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)-\sqrt{2}\,(a+b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)}{a^2+b^2}\right)e^{\left(-\frac{1}{2}\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(8*b^2*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^2 + b^2)*sqrt(a*b)
) + (2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))))
 + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
+ sqrt(2)*(a + b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sq
rt(2)*(a + b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^2 +
 b^2))*e^(-1/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cot\left(c+dx\right)}\,\left(a+b\cot\left(c+dx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c)),x)
```

```
[Out] Integral(1/(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cot(d*x + c) + a)*sqrt(e*cot(d*x + c))), x)
```

Mupad [B]
time = 1.77, size = 2500, normalized size = 8.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))),x)
```

```
[Out] atan((((((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - ((((1/(b^2*d^2*e*1i - a^2*
d^2*e*1i + 2*a*b*d^2*e))^(1/2)*((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10
+ 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 - (16*(e*cot(c + d*x))^(1/2
)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*(16*b^9*d^4*e^10 +
16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4))/2 -
 (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b
^2*d^2*e^9))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2))/2)
*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2))/2 + (96*b^5*e^8*(e*
cot(c + d*x))^(1/2))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(
1/2)*1i)/2 - (((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - ((((1/(b^2*d^2*e*1i
 - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^
2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (16*(e*cot(c + d*x
))^(1/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*(16*b^9*d^4*
e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^
4))/2 + (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 +
2*a^5*b^2*d^2*e^9))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1
/2))/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2))/2 - (96*b^5*
e^8*(e*cot(c + d*x))^(1/2))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^
2*e))^(1/2)*1i)/2)/((((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - ((((1/(b^2*d
^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*((32*(16*b^8*d^2*e^10 + 28*a^2
*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 - (16*(e*cot(
c + d*x))^(1/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*(16*b
^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^
10))/d^4))/2 - (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2
*e^9 + 2*a^5*b^2*d^2*e^9))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2
*e))^(1/2))/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2))/2 + (
96*b^5*e^8*(e*cot(c + d*x))^(1/2))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2
*a*b*d^2*e))^(1/2))/2 + (((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - ((((1/(b
^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*((32*(16*b^8*d^2*e^10 + 28
*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (16*(e*
cot(c + d*x))^(1/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*(
16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^
```

```
4*e^10))/d^4))/2 + (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6
*d^2*e^9 + 2*a^5*b^2*d^2*e^9))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b
*d^2*e))^(1/2))/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2))/2
 - (96*b^5*e^8*(e*cot(c + d*x))^(1/2))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i
 + 2*a*b*d^2*e))^(1/2))/2))*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))
^(1/2)*1i + atan(((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - (((32*(16*b^8*d^
2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^
3 - (32*(e*cot(c + d*x))^(1/2)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i
)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16
*a^6*b^3*d^4*e^10))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1
/2) - (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*
a^5*b^2*d^2*e^9))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2
))*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2) + (96*b^5*e^8*(e*c
ot(c + d*x))^(1/2))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1
/2)*1i - (((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - (((32*(16*b^8*d^2*e^10 +
28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (32*(
e*cot(c + d*x))^(1/2)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2)
*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*
d^4*e^10))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2) + (32
*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d
^2*e^9))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2))*(1i/(4
*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2) - (96*b^5*e^8*(e*cot(c + d*
x))^(1/2))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2)*1i)/(
(((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - (((32*(16*b^8*d^2*e^10 + 28*a^2*b^
6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 - (32*(e*cot(c +
 d*x))^(1/2)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2)*(16*b^9*
d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10)
)/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2) - (32*(e*cot(c
 + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d^2*e^9))/
d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2))*(1i/(4*(b^2*d^2
*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2) + (96*b^5*e^8*(e*cot(c + d*x))^(1/2)
)/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^(1/2) + (((32*(5*a*b
^5*e^9 + a^3*b^3*e^9))/d^3 - (((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 +
 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (32*(e*cot(c + d*x))^(1/2)
*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))...
```

3.73 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx$

Optimal. Leaf size=325

$$\frac{2b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{a^{3/2}\left(a^2+b^2\right)de^{3/2}}-\frac{(a+b)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,\left(a^2+b^2\right)de^{3/2}}+\frac{(a+b)\text{ArcTan}\left(1+\frac{\sqrt{2}}{}\right.}{\sqrt{2}\,\left(a^2+b\right.}$$

```
[Out] 2*b^(5/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/a^(3/2)/(a^2
+b^2)/d/e^(3/2)-1/2*(a+b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a
^2+b^2)/d/e^(3/2)*2^(1/2)+1/2*(a+b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e
^(1/2))/(a^2+b^2)/d/e^(3/2)*2^(1/2)+1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)
-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d/e^(3/2)*2^(1/2)-1/4*(a-b)*ln(e^(
1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d/e^(3/2)*2
^(1/2)+2/a/d/e/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3650, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a+b)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}\left(a^2+b^2\right)}+\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{3/2}\left(a^2+b^2\right)}+\frac{(a-b)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}\left(a^2+b^2\right)}-\frac{(a-b)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}\left(a^2+b^2\right)}+\frac{2b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{a^{3/2}de^{3/2}\left(a^2+b^2\right)}+\frac{2}{ade\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])),x]
```

```
[Out] (2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(a^(3/
2)*(a^2 + b^2)*d*e^(3/2)) - ((a + b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x
]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*e^(3/2)) + ((a + b)*ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*e^(3/2)) + 2/(a*d*
e*Sqrt[e*Cot[c + d*x]]) + ((a - b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqr
t[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)*d*e^(3/2)) - ((a - b)*Lo
g[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2
]*(a^2 + b^2)*d*e^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx &= \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{2\int \frac{-\frac{be^2}{2}-\frac{1}{2}ae^2\cot(c+dx)-\frac{1}{2}be^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}\,dx}{ae^3} \\
&= \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}abe^2-\frac{1}{2}a^2e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{a\left(a^2+b^2\right)e^3} - \frac{b^3\int \ldots}{\sqrt{e\ldots}} \\
&= \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{4\text{Subst}\left(\int \frac{\frac{1}{2}abe^3+\frac{1}{2}a^2e^2x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(\ldots}\right.}{a\left(a^2+b^2\right)de^3} \\
&= \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+\ldots}\right.}{a\left(a^2+b^2\right)de^2} \\
&= \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}\left(a^2+b^2\right)de^{3/2}} + \frac{2}{ade\sqrt{e\cot(c+dx)}} + \ldots \\
&= \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}\left(a^2+b^2\right)de^{3/2}} + \frac{2}{ade\sqrt{e\cot(c+dx)}} + \ldots \\
&= \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}\left(a^2+b^2\right)de^{3/2}} - \frac{(a+b)\tan^{-1}\left(1-\frac{\sqrt{2}\ldots}{}\right.}{\sqrt{2}\left(a^2+\ldots\right.}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.45, size = 198, normalized size = 0.61

$$
\frac{8b^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{b\cot(c+dx)}{a}\right)+a\left(8a\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+\sqrt{2}\,b\sqrt{\cot(c+dx)}\left(-2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)+2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)+\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{4a\left(a^2+b^2\right)de\sqrt{e\cot(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])),x]
```

```
[Out] (8*b^2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*Cot[c + d*x])/a)] + a*(8*a*Hype
rgeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Sqrt[2]*b*Sqrt[Cot[c + d*x]]
*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot
[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] + Log[1 +
Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(4*a*(a^2 + b^2)*d*e*Sqrt[e*C
ot[c + d*x]])
```

Maple [A]
time = 0.55, size = 354, normalized size = 1.09

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2e^2\left(-\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^2*(1/(a^2+b^2)/e^3*(-1/8*b/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
+1))-1/8*a/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*
2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-
2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/a/e^3/(e*cot(d*x+
c))^(1/2)-1/a/e^3*b^3/(a^2+b^2)/(a*e*b)^(1/2)*arctan(b*(e*cot(d*x+c))^(1/2)
/(a*e*b)^(1/2)))
```

Maxima [A]

time = 0.50, size = 191, normalized size = 0.59

$$\frac{\left(\frac{8b^3\arctan\left(\frac{b}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^3+ab^2)\sqrt{ab}}+\frac{2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}(a-b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}(a-b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^2+b^2}+\frac{8\sqrt{\tan(dx+c)}}{a}\right)e^{\left(-\frac{3}{2}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(8*b^3*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^3 + a*b^2)*sqrt(a*b
)) + (2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))
) + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
 - sqrt(2)*(a - b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + s
```

```
qrt(2)*(a - b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^2
+ b^2) + 8*sqrt(tan(d*x + c))/a)*e^(-3/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\,(a+b\cot(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c)),x)
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((b*cot(d*x + c) + a)*(e*cot(d*x + c))^(3/2)), x)
```

Mupad [**B**]
time = 1.86, size = 2500, normalized size = 7.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))),x)
[Out] (log((e*cot(c + d*x))^(1/2)*(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^13) - (
(-1/(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2)*(((((-1/(b^2*d
^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2)*(((e*cot(c + d*x))^(1/2)
*(-1/(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2)*(512*a^9*b^9*
```

d^9*e^19 + 512*a^11*b^7*d^9*e^19 - 512*a^13*b^5*d^9*e^19 - 512*a^15*b^3*d^9
*e^19))/2 - 512*a^8*b^9*d^8*e^18 - 640*a^10*b^7*d^8*e^18 + 256*a^12*b^5*d^8
*e^18 + 384*a^14*b^3*d^8*e^18))/2 - (e*cot(c + d*x))^(1/2)*(512*a^8*b^8*d^7
*e^16 - 448*a^10*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^16 + 64*a^14*b^2*d^7*e^1
6))*(-1/(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2))/2 - 128*a
^7*b^8*d^6*e^15 + 32*a^11*b^4*d^6*e^15 + 32*a^13*b^2*d^6*e^15))/2)*(-1/(b^2
*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2))/2 - atan(((-1i/(4*(b^
2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((e*cot(c + d*x))^(1/2)*(
64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^13) - (-1i/(4*(b^2*d^2*e^3 - a^2*d^2
*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^
2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(
1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*(e*cot(c
 + d*x))^(1/2)*(512*a^9*b^9*d^9*e^19 + 512*a^11*b^7*d^9*e^19 - 512*a^13*b^5
*d^9*e^19 - 512*a^15*b^3*d^9*e^19) - 512*a^8*b^9*d^8*e^18 - 640*a^10*b^7*d^
8*e^18 + 256*a^12*b^5*d^8*e^18 + 384*a^14*b^3*d^8*e^18) - (e*cot(c + d*x))^
(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a^10*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^16
 + 64*a^14*b^2*d^7*e^16)) - 128*a^7*b^8*d^6*e^15 + 32*a^11*b^4*d^6*e^15 + 3
2*a^13*b^2*d^6*e^15))*1i + (-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3
2i)))^(1/2)((e*cot(c + d*x))^(1/2)*(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*
e^13) - (-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/
(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e
^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*
e^3 + a*b*d^2*e^3*2i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^9*b^9*d^9*e^19
+ 512*a^11*b^7*d^9*e^19 - 512*a^13*b^5*d^9*e^19 - 512*a^15*b^3*d^9*e^19) +
512*a^8*b^9*d^8*e^18 + 640*a^10*b^7*d^8*e^18 - 256*a^12*b^5*d^8*e^18 - 384*
a^14*b^3*d^8*e^18) - (e*cot(c + d*x))^(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a^1
0*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^16 + 64*a^14*b^2*d^7*e^16)) + 128*a^7*b
^8*d^6*e^15 - 32*a^11*b^4*d^6*e^15 - 32*a^13*b^2*d^6*e^15))*1i)/((-1i/(4*(b
^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((e*cot(c + d*x))^(1/2)*
(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^13) - (-1i/(4*(b^2*d^2*e^3 - a^2*d^
2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d
^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^
(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*(e*cot(
c + d*x))^(1/2)*(512*a^9*b^9*d^9*e^19 + 512*a^11*b^7*d^9*e^19 - 512*a^13*b^
5*d^9*e^19 - 512*a^15*b^3*d^9*e^19) - 512*a^8*b^9*d^8*e^18 - 640*a^10*b^7*d
^8*e^18 + 256*a^12*b^5*d^8*e^18 + 384*a^14*b^3*d^8*e^18) - (e*cot(c + d*x))
^(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a^10*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^1
6 + 64*a^14*b^2*d^7*e^16)) - 128*a^7*b^8*d^6*e^15 + 32*a^11*b^4*d^6*e^15 +
32*a^13*b^2*d^6*e^15)) - (-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2
i)))^(1/2)*((e*cot(c + d*x))^(1/2)*(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^
13) - (-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4
*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3
 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^
3 + a*b*d^2*e^3*2i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^9*b^9*d^9*e^19 +
512*a^11*b^7*d^9*e^19 - 512*a^13*b^5*d^9*e^19 - 512*a^15*b^3*d^9*e^19) + 51

2*a^8*b^9*d^8*e^18 + 640*a^10*b^7*d^8*e^18 - 256*a^12*b^5*d^8*e^18 - 384*a^14*b^3*d^8*e^18) - (e*cot(c + d*x))^(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a^10*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^16 + 64*a^14*b^2*d^7*e^16)) + 128*a^7*b^8*d^6*e^15 - 32*a^11*b^4*d^6*e^15 - 32*a^13*b^2*d^6*e^15))))*(-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*2i - log((e*cot(c + d*x))^(1/2)*(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^13) - (-1/(4*(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3)))^(1/2)*((-1/(4*(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3)))^(1/2)*((-1/(4*(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3)))^(1/2)*((-1/(4*(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3)))^(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^9*b^9*d^9*e^19 + 512*a^11*b^7*d^9*e^19 - 512*a^13*b^5*d^9*e^19 - 512*a^15*b^3*d^9*e^19) + 512*a^8*b^9*d^8*e^18 + 640*a^10*b^7*d^8*e^18 - 256*a^12*b^5*d^8*e^18 - 384*a^14*b^3*d^8*e^18) - (e*cot(c + d*x))^(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a^10*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^16 + 64*a^14*b^2*d^7*e^16)) + 128*a^7*b^8*d^6*e^15 - 32*a^11*b^4*d^6*e^15 - 32*a^13*b^2*d^6*e^15))*(-1/(4*(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3)))^(1/2) - (atan(((((e*cot(c + d*x))^(1/2)*(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^13) + ((-a^3*b^5*e^3)^(1/2)*((((e*cot(c + d*x))^(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a^10*b^6*...

3.74 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx$

Optimal. Leaf size=351

$$-\frac{2b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{a^{5/2}\left(a^2+b^2\right)de^{5/2}}-\frac{(a-b)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,\left(a^2+b^2\right)de^{5/2}}+\frac{(a-b)\text{ArcTan}\left(1+\right.}{\sqrt{2}\,\left(a^2+\right.}$$

```
[Out] -2*b^(7/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/a^(5/2)/(a^
2+b^2)/d/e^(5/2)+2/3/a/d/e/(e*cot(d*x+c))^(3/2)-1/2*(a-b)*arctan(1-2^(1/2)*
(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)/d/e^(5/2)*2^(1/2)+1/2*(a-b)*arctan(
1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)/d/e^(5/2)*2^(1/2)-1/4*(a+
b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d/
e^(5/2)*2^(1/2)+1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+
c))^(1/2))/(a^2+b^2)/d/e^(5/2)*2^(1/2)-2*b/a^2/d/e^2/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3650, 3730, 3735, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a-b)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{5/2}\left(a^2+b^2\right)}+\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{5/2}\left(a^2+b^2\right)}-\frac{(a+b)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}\left(a^2+b^2\right)}+\frac{(a+b)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{5/2}\left(a^2+b^2\right)}-\frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}}-\frac{2b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{a^{5/2}de^{5/2}\left(a^2+b^2\right)}+\frac{2}{3ade(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + b*Cot[c + d*x])),x]
```

```
[Out] (-2*b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(a^(5
/2)*(a^2 + b^2)*d*e^(5/2)) - ((a - b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*
x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*e^(5/2)) + ((a - b)*ArcTan[1 + (Sqrt[
2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*e^(5/2)) + 2/(3*a
*d*e*(e*Cot[c + d*x])^(3/2)) - (2*b)/(a^2*d*e^2*Sqrt[e*Cot[c + d*x]]) - ((a
 + b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(
2*Sqrt[2]*(a^2 + b^2)*d*e^(5/2)) + ((a + b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d
*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)*d*e^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3735

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
```

```
eQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = \frac{2}{3ade(e\cot(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3be^2}{2}-\frac{3}{2}ae^2\cot(c+dx)-\frac{3}{2}be^2\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))} \cdots}{3ae^3}$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}} + \frac{4\int \frac{-\frac{3}{4}(a^2 \cdots}{\sqrt{e\cot \cdots}}}{\cdots}$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}} + \frac{4\int \frac{-\frac{3}{4}a^3e^4+ \cdots}{\sqrt{e \cdots}}}{3a^2 (\cdots}$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}} + \frac{8\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}} - \frac{(2b^4)\,\text{Subst} \cdots}{\cdots}$$

$$= -\frac{2b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}(a^2+b^2)de^{5/2}} + \frac{2}{3ade(e\cot(c+dx))^{3/2}} \cdots$$

$$= -\frac{2b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}(a^2+b^2)de^{5/2}} + \frac{2}{3ade(e\cot(c+dx))^{3/2}} \cdots$$

$$= -\frac{2b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}(a^2+b^2)de^{5/2}} - \frac{(a-b)\tan^{-1}\left(1-\cdots\right.}{\sqrt{2}\,(a^2 \cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.26, size = 109, normalized size = 0.31

$$\frac{2\left(b^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{b\cot(c+dx)}{a}\right)+a\left(a\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(c+dx)\right)-3b\cot(c+dx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)\right)\right)}{3a\left(a^2+b^2\right)de(e\cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + b*Cot[c + d*x])),x]
```

```
[Out] (2*(b^2*Hypergeometric2F1[-3/2, 1, -1/2, -((b*Cot[c + d*x])/a)] + a*(a*Hype
rgeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2] - 3*b*Cot[c + d*x]*Hypergeomet
ric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2])))/(3*a*(a^2 + b^2)*d*e*(e*Cot[c + d*
x])^(3/2))
```

Maple [A]
time = 0.58, size = 371, normalized size = 1.06

method	result
derivativedivides	$-\frac{2e^2\left(\frac{b^4\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{a^2e^4(a^2+b^2)\sqrt{aeb}}-\frac{1}{3a\,e^3(e\cot(dx+c))^{\frac{3}{2}}}+\frac{b}{a^2e^4\sqrt{e\cot(dx+c)}}+\frac{-\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\,\mathrm{co}}{e\,\mathrm{co}}\right.\right.}{}}{}\right.}{}$
default	$-\frac{2e^2\left(\frac{b^4\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{a^2e^4(a^2+b^2)\sqrt{aeb}}-\frac{1}{3a\,e^3(e\cot(dx+c))^{\frac{3}{2}}}+\frac{b}{a^2e^4\sqrt{e\cot(dx+c)}}+\frac{-\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\,\mathrm{co}}{e\,\mathrm{co}}\right.\right.}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^2*(1/a^2/e^4*b^4/(a^2+b^2)/(a*e*b)^(1/2)*arctan(b*(e*cot(d*x+c))^(1/
2)/(a*e*b)^(1/2))-1/3/a/e^3/(e*cot(d*x+c))^(3/2)+1/a^2/e^4*b/(e*cot(d*x+c))
^(1/2)+1/(a^2+b^2)/e^4*(-1/8*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)
)+1/8*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.52, size = 207, normalized size = 0.59

$$-\frac{\left(\frac{24\,b^4\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{\tan(dx+c)}}\right)}{(a^4+a^2b^2)\sqrt{ab}}-\frac{8\left(a-\frac{3\,b}{\tan(dx+c)}\right)\tan(dx+c)^{\frac{3}{2}}}{a^2}-\frac{3\left(2\sqrt{2}\,(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\,(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\,(a+b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\,(a+b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)}{a^2+b^2}\right)e^{\left(-\frac{5}{2}\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(24*b^4*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^4 + a^2*b^2)*sqr
t(a*b)) - 8*(a - 3*b/tan(d*x + c))*tan(d*x + c)^(3/2)/a^2 - 3*(2*sqrt(2)*(a
 - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a -
 b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(a + b)
*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(a + b)*log
(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^2 + b^2))*e^(-5/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\,(a+b\cot(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+b*cot(d*x+c)),x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(5/2)*(a + b*cot(c + d*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cot(d*x + c) + a)*(e*cot(d*x + c))^(5/2)), x)
```

Mupad [**B**]
time = 2.81, size = 2500, normalized size = 7.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + b*cot(c + d*x))),x)
```

```
[Out] (2/(3*a*e) - (2*b*cot(c + d*x))/(a^2*e))/(d*(e*cot(c + d*x))^(3/2)) - atan(
((((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18))/2
+ ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((((1/(b^2*
d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((e*cot(c + d*x))^(1/2
)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(512*a^18*b^9
*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 512*a^24*b^3*d^
9*e^28))/4 - 256*a^16*b^10*d^8*e^26 - 256*a^18*b^8*d^8*e^26 + 192*a^20*b^6*
d^8*e^26 + 128*a^22*b^4*d^8*e^26 - 64*a^24*b^2*d^8*e^26))/2 - ((e*cot(c + d
*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 128*a^21*b^4*d
^7*e^23 - 64*a^23*b^2*d^7*e^23))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2
*a*b*d^2*e^5))^(1/2))/2 + 192*a^15*b^9*d^6*e^21 - 16*a^19*b^5*d^6*e^21 - 16
*a^21*b^3*d^6*e^21))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5
))^(1/2)*1i + (((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 + 32*a^18*b^5*
d^5*e^18))/2 + ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)
*(((((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((e*cot(c
 + d*x))^(1/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*
(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 51
2*a^24*b^3*d^9*e^28))/4 + 256*a^16*b^10*d^8*e^26 + 256*a^18*b^8*d^8*e^26 -
192*a^20*b^6*d^8*e^26 - 128*a^22*b^4*d^8*e^26 + 64*a^24*b^2*d^8*e^26))/2 -
((e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 1
28*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23))/2)*(1/(b^2*d^2*e^5*1i - a^2*d
^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2))/2 - 192*a^15*b^9*d^6*e^21 + 16*a^19*b^5*
d^6*e^21 + 16*a^21*b^3*d^6*e^21))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i +
2*a*b*d^2*e^5))^(1/2)*1i)/((((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 +
 32*a^18*b^5*d^5*e^18))/2 + ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^
2*e^5))^(1/2)*(((((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/
2)*(((e*cot(c + d*x))^(1/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2
*e^5))^(1/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*
d^9*e^28 - 512*a^24*b^3*d^9*e^28))/4 + 256*a^16*b^10*d^8*e^26 + 256*a^18*b^
8*d^8*e^26 - 192*a^20*b^6*d^8*e^26 - 128*a^22*b^4*d^8*e^26 + 64*a^24*b^2*d^
8*e^26))/2 - ((e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6
*d^7*e^23 - 128*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23))/2)*(1/(b^2*d^2*e
^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2))/2 - 192*a^15*b^9*d^6*e^21 +
 16*a^19*b^5*d^6*e^21 + 16*a^21*b^3*d^6*e^21))/2)*(1/(b^2*d^2*e^5*1i - a^2*
d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2) - (((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*
d^5*e^18 + 32*a^18*b^5*d^5*e^18))/2 + ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i
+ 2*a*b*d^2*e^5))^(1/2)*(((((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2
*e^5))^(1/2)*(((e*cot(c + d*x))^(1/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i +
 2*a*b*d^2*e^5))^(1/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512
*a^22*b^5*d^9*e^28 - 512*a^24*b^3*d^9*e^28))/4 - 256*a^16*b^10*d^8*e^26 - 2
56*a^18*b^8*d^8*e^26 + 192*a^20*b^6*d^8*e^26 + 128*a^22*b^4*d^8*e^26 - 64*a
^24*b^2*d^8*e^26))/2 - ((e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 44
8*a^19*b^6*d^7*e^23 - 128*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23))/2)*(1/
(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2))/2 + 192*a^15*b^9*
d^6*e^21 - 16*a^19*b^5*d^6*e^21 - 16*a^21*b^3*d^6*e^21))/2)*(1/(b^2*d^2*e^5
```

*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2) + 64*a^14*b^8*d^4*e^16))*(1/(b
^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*1i - atan(((1i/(4*(b
^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((e*cot(c + d*x))^(1/2)*
(64*a^14*b^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18) + (1i/(4*(b^2*d^2*e^5 - a^2*d
^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d
^2*e^5*2i)))^(1/2)*((1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(
1/2)*((1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*(e*cot(c
+ d*x))^(1/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5
*d^9*e^28 - 512*a^24*b^3*d^9*e^28) - 512*a^16*b^10*d^8*e^26 - 512*a^18*b^8*
d^8*e^26 + 384*a^20*b^6*d^8*e^26 + 256*a^22*b^4*d^8*e^26 - 128*a^24*b^2*d^8
*e^26) - (e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*
e^23 - 128*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23)) + 384*a^15*b^9*d^6*e^
21 - 32*a^19*b^5*d^6*e^21 - 32*a^21*b^3*d^6*e^21))*1i + (1i/(4*(b^2*d^2*e^5
 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((e*cot(c + d*x))^(1/2)*(64*a^14*b
^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18) + (1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a
*b*d^2*e^5*2i)))^(1/2)*((1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)
))^(1/2)*((1i/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*((1i/
(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^(1/2)*(e*cot(c + d*x))^(1
/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28
- 512*a^24*b^3*d^9*e^28) + 512*a^16*b^10*d^8*e^26 + 512*a^18*b^8*d^8*e^26 -
 384*a^20*b^6*d^8*e^26 - 256*a^22*b^4*d^8*e^26 + 128*a^24*b^2*d^8*e^26) - (
e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 +...

3.75 $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx$

Optimal. Leaf size=437

$$\frac{a^{5/2}(3a^2+7b^2)\,e^{7/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{b^{5/2}\,(a^2+b^2)^2\,d}+\frac{(a^2-2ab-b^2)\,e^{7/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}$$

```
[Out] a^(5/2)*(3*a^2+7*b^2)*e^(7/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e
^(1/2))/b^(5/2)/(a^2+b^2)^2/d+a^2*e^2*(e*cot(d*x+c))^(3/2)/b/(a^2+b^2)/d/(a
+b*cot(d*x+c))+1/2*(a^2-2*a*b-b^2)*e^(7/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^
(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)-1/2*(a^2-2*a*b-b^2)*e^(7/2)*arctan(1+2
^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)+1/4*(a^2+2*a*b-b
^2)*e^(7/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^
2+b^2)^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*e^(7/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2
)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)-(3*a^2+2*b^2)*e^3*(e*
cot(d*x+c))^(1/2)/b^2/(a^2+b^2)/d
```

Rubi [A]

time = 0.74, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3646, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{e^{7/2}(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{e^{7/2}(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{e^{7/2}(a^2+2ab-b^2)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{e^{7/2}(a^2+2ab-b^2)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{e^3(3a^2+2b^2)\,\sqrt{e\cot(c+dx)}}{b^2d\,(a^2+b^2)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\,(a^2+b^2)\,(a+b\cot(c+dx))}+\frac{a^{5/2}e^{7/2}(3a^2+7b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{b^{5/2}d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(7/2)/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] (a^(5/2)*(3*a^2 + 7*b^2)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqr
t[a]*Sqrt[e])])/(b^(5/2)*(a^2 + b^2)^2*d) + ((a^2 - 2*a*b - b^2)*e^(7/2)*Ar
cTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d)
 - ((a^2 - 2*a*b - b^2)*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/S
qrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d) - ((3*a^2 + 2*b^2)*e^3*Sqrt[e*Cot[c + d*
x]])/(b^2*(a^2 + b^2)*d) + (a^2*e^2*(e*Cot[c + d*x])^(3/2))/(b*(a^2 + b^2)*
d*(a + b*Cot[c + d*x])) + ((a^2 + 2*a*b - b^2)*e^(7/2)*Log[Sqrt[e] + Sqrt[e
]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d)
 - ((a^2 + 2*a*b - b^2)*e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2
]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx = \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b\,(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{\int \frac{\sqrt{e\cot(c+dx)}\,\left(-\frac{3}{2}a^2e^3+abe^3\cot(c+dx)-\frac{1}{2}(3a^2 \cdots\right.}{a+b\cot(c+dx)}}{b\,(a^2+b^2)}$$

$$= -\frac{(3a^2+2b^2)\,e^3\sqrt{e\cot(c+dx)}}{b^2\,(a^2+b^2)\,d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b\,(a^2+b^2)\,d(a+b\cot(c+dx))} + \frac{2\int \frac{-\frac{1}{4}a(3 \cdots}{\cdots}}{\cdots}$$

$$= -\frac{(3a^2+2b^2)\,e^3\sqrt{e\cot(c+dx)}}{b^2\,(a^2+b^2)\,d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b\,(a^2+b^2)\,d(a+b\cot(c+dx))} + \frac{2\int \frac{\frac{1}{2}b^2(a \cdots}{\cdots}}{\cdots}$$

$$= -\frac{(3a^2+2b^2)\,e^3\sqrt{e\cot(c+dx)}}{b^2\,(a^2+b^2)\,d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b\,(a^2+b^2)\,d(a+b\cot(c+dx))} + \frac{4\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{(3a^2+2b^2)\,e^3\sqrt{e\cot(c+dx)}}{b^2\,(a^2+b^2)\,d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b\,(a^2+b^2)\,d(a+b\cot(c+dx))} + \frac{(a^3(3a^2 \cdots}{\cdots}$$

$$= \frac{a^{5/2}(3a^2+7b^2)\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{b^{5/2}\,(a^2+b^2)^2\,d} - \frac{(3a^2+2b^2)\,e^3\sqrt{e\cot(c+ \cdots}}{b^2\,(a^2+b^2)\,d}$$

$$= \frac{a^{5/2}(3a^2+7b^2)\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{b^{5/2}\,(a^2+b^2)^2\,d} - \frac{(3a^2+2b^2)\,e^3\sqrt{e\cot(c+ \cdots}}{b^2\,(a^2+b^2)\,d}$$

$$= \frac{a^{5/2}(3a^2+7b^2)\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{b^{5/2}\,(a^2+b^2)^2\,d} + \frac{(a^2-2ab-b^2)\,e^{7/2}\tan^{-\cdots}}{\sqrt{2}\,(\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 6.16, size = 445, normalized size = 1.02

$$-\frac{(e\cot(c+dx))^{7/2}\left(\frac{4ab\cot^{\frac{7}{2}}(c+dx)}{7(a^2+b^2)^2}-\frac{4a^2\left(3\cot^{\frac{5}{2}}(c+dx)-5a\left(-\frac{3a\left(-\frac{\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}}+\frac{\sqrt{\cot(c+dx)}}{b}\right)}{b}+\frac{\cot^{\frac{3}{2}}(c+dx)}{b}\right)\right)}{15(a^2+b^2)^2}+\frac{4ab\left(7\cot^{\frac{3}{2}}(c+dx)-3\cot^{\frac{7}{2}}(c+dx)-7\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)}{21(a^2+b^2)^2}+\frac{2b^2\cot^{\frac{9}{2}}(c+dx)\,{}_2F_1\left(2,\frac{9}{2};\frac{11}{2};-\frac{b\cot(c+dx)}{a}\right)}{9a^2(a^2+b^2)}-\frac{(a-b)(a+b)\left(40\sqrt{\cot(c+dx)}-8\cot^{\frac{5}{2}}(c+dx)+\frac{5}{2}\left(4\left(\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)\right)+2\sqrt{2}\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-2\sqrt{2}\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{20(a^2+b^2)^2}\right)}{d\cot^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(7/2)/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -(((e*Cot[c + d*x])^(7/2)*((4*a*b*Cot[c + d*x]^(7/2))/(7*(a^2 + b^2)^2) - (
4*a^2*(3*Cot[c + d*x]^(5/2) - 5*a*((-3*a*(-((Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[C
ot[c + d*x]])/Sqrt[a]])/b^(3/2)) + Sqrt[Cot[c + d*x]]/b))/b + Cot[c + d*x]^
(3/2)/b)))/(15*(a^2 + b^2)^2) + (4*a*b*(7*Cot[c + d*x]^(3/2) - 3*Cot[c + d*
x]^(7/2) - 7*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x
]^2]))/(21*(a^2 + b^2)^2) + (2*b^2*Cot[c + d*x]^(9/2)*Hypergeometric2F1[2,
9/2, 11/2, -((b*Cot[c + d*x])/a)])/(9*a^2*(a^2 + b^2)) - ((a - b)*(a + b)*(
40*Sqrt[Cot[c + d*x]] - 8*Cot[c + d*x]^(5/2) + (5*(4*(Sqrt[2]*ArcTan[1 - Sq
rt[2]*Sqrt[Cot[c + d*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])
 + 2*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 2*Sqrt[2]
*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/2))/(20*(a^2 + b^2)^2
)))/(d*Cot[c + d*x]^(7/2)))
```

Maple [A]
time = 0.65, size = 409, normalized size = 0.94

method	result
derivativedivides	$-\frac{2e^3\left(\frac{\sqrt{e\cot(dx+c)}}{b^2}-\frac{a^3e\left(\frac{\left(-\frac{a^2}{2}-\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{(3a^2+7b^2)\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right.}{b^2(a^2+b^2)^2}\right.}{}$

default	$-\frac{2e^3\left(\frac{\sqrt{e\cot(dx+c)}}{b^2}-\frac{a^3e\left(\frac{\left(-\frac{a^2}{2}-\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{(3a^2+7b^2)\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{b^2(a^2+b^2)^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^3*(1/b^2*(e*cot(d*x+c))^(1/2)-a^3*e/b^2/(a^2+b^2)^2*((-1/2*a^2-1/2*b
^2)*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)*b+a*e)+1/2*(3*a^2+7*b^2)/(a*e*b)^(1/
2)*arctan(b*(e*cot(d*x+c))^(1/2)/(a*e*b)^(1/2)))+e/(a^2+b^2)^2*(1/8*(a^2*e-
b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*
2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-
2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4*a*b/(e^2)^(1/4)*
2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1
/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2
*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.52, size = 304, normalized size = 0.70

$$-\frac{\left(\frac{4a^3}{\left(a^3b^2+ab^4+\frac{a^2b^3+b^5}{\tan(dx+c)}\right)\sqrt{\tan(dx+c)}}-\frac{4(3a^5+7a^3b^2)\arctan\left(\frac{b}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^4b^2+2a^2b^4+b^6)\sqrt{ab}}+\frac{2\sqrt{2}(a^2-2ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a^2-2ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}(a^2+2ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}(a^2+2ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^4+2a^2b^2+b^4}+\frac{8}{b^2\sqrt{\tan(dx+c)}}\right)e^{\frac{7}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^3/((a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)/tan(d*x + c))*sqrt(tan(d*x
+ c))) - 4*(3*a^5 + 7*a^3*b^2)*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a
^4*b^2 + 2*a^2*b^4 + b^6)*sqrt(a*b)) + (2*sqrt(2)*(a^2 - 2*a*b - b^2)*arcta
n(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 - 2*a*b -
b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(a^2 +
 2*a*b - b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2
)*(a^2 + 2*a*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1)
)/(a^4 + 2*a^2*b^2 + b^4) + 8/(b^2*sqrt(tan(d*x + c))))*e^(7/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(7/2)/(a+b*cot(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(7/2)/(b*cot(d*x + c) + a)^2, x)
```

Mupad [**B**]
time = 3.97, size = 2500, normalized size = 5.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(7/2)/(a + b*cot(c + d*x))^2,x)
```

```
[Out] (atan(((((16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10
*e^24 + 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^2
4))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4
) + (((16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^21 +
 600*a^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^1
1*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (((1
6*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*
b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*
d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a
^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4) + (((16*(8*a*b^17
*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a^7*b^11*d^4
*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14
))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5)
```

- (8*(e*cot(c + d*x))^(1/2)*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*d^4*e^10 + 160*a^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^4*e^10 - 160*a^12*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/((b^9*d + 2*a^2*b^7*d + a^4*b^5*d)*(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2)*1i)/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)) + (((16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10*e^24 + 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^24))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4) - (((16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^21 + 600*a^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + (((16*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4) - (((16*(8*a*b^17*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a^7*b^11*d^4*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + (8*(e*cot(c + d*x))^(1/2)*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*d^4*e^10 + 160*a^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^4*e^10 - 160*a^12*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/((b^9*d + 2*a^2*b^7*d + a^4*b^5*d)*(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2)*1i)/(2*(b^9*d + 2*a^2*b^7*d + a^4*b^5*d)))/((32*(7*a^3*b^7*e^28 + 3*a^5*b^5*e^28))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (((16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^10*e^24 + 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e^24))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4) + (((16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^21 + 600*a^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (((16*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 - 52*a^3*b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*a^9*b^7*d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4) + (((16*(8*a*b^17*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a^7*b^11*d^4*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*b^5*d^4*e^14))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4

```
*a^6*b^5*d^5 + a^8*b^3*d^5) - (8*(e*cot(c + d*x))^(1/2)*(3*a^2 + 7*b^2)*(-a
^5*b^5*e^7)^(1/2)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*
d^4*e^10 + 160*a^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^
4*e^10 - 160*a^12*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/((b^9*d + 2*a^2*b^7
*d + a^4*b^5*d)*(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 +
 a^8*b^3*d^4)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7
*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b
^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2
*b^7*d + a^4*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5...
```

3.76 $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx$

Optimal. Leaf size=393

$$-\frac{a^{3/2}(a^2+5b^2)\,e^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{b^{3/2}\,(a^2+b^2)^2\,d}-\frac{(a^2+2ab-b^2)\,e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right.}{\sqrt{2}\,\left(a^2+b^2\right)^2d}$$

```
[Out] -a^(3/2)*(a^2+5*b^2)*e^(5/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^
(1/2))/b^(3/2)/(a^2+b^2)^2/d-1/2*(a^2+2*a*b-b^2)*e^(5/2)*arctan(1-2^(1/2)*(
e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)+1/2*(a^2+2*a*b-b^2)*e^(5
/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)+1/
4*(a^2-2*a*b-b^2)*e^(5/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+
c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*e^(5/2)*ln(e^(1/2)+cot
(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)+a^2*e^2
*(e*cot(d*x+c))^(1/2)/b/(a^2+b^2)/d/(a+b*cot(d*x+c))
```

Rubi [A]

time = 0.50, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3646, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{e^{5/2}(a^2+2ab-b^2)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{e^{5/2}(a^2+2ab-b^2)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{e^{5/2}(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{e^{5/2}(a^2-2ab-b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd\,(a^2+b^2)\,(a+b\cot(c+dx))}-\frac{a^{3/2}e^{5/2}(a^2+5b^2)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -((a^(3/2)*(a^2 + 5*b^2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqr
t[a]*Sqrt[e])])/(b^(3/2)*(a^2 + b^2)^2*d)) - ((a^2 + 2*a*b - b^2)*e^(5/2)*A
rcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d
) + ((a^2 + 2*a*b - b^2)*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/
Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d) + (a^2*e^2*Sqrt[e*Cot[c + d*x]])/(b*(a^
2 + b^2)*d*(a + b*Cot[c + d*x])) + ((a^2 - 2*a*b - b^2)*e^(5/2)*Log[Sqrt[e]
 + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 +
b^2)^2*d) - ((a^2 - 2*a*b - b^2)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x]
 + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx = \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b\left(a^2+b^2\right)d(a+b\cot(c+dx))} - \frac{\int \frac{-\frac{1}{2}a^2e^3+abe^3\cot(c+dx)-\frac{1}{2}\left(a^2+2b^2\right)e^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}\,dx}{b\left(a^2+b^2\right)}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b\left(a^2+b^2\right)d(a+b\cot(c+dx))} - \frac{\int \frac{2ab^2e^3+b\left(a^2-b^2\right)e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{b\left(a^2+b^2\right)^2} + \frac{\left(a^2\left(a^2+5b^2\right)\right.}{}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b\left(a^2+b^2\right)d(a+b\cot(c+dx))} - \frac{2\text{Subst}\left(\int \frac{-2ab^2e^4-b\left(a^2-b^2\right)e^3x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c\right.}{b\left(a^2+b^2\right)^2 d}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b\left(a^2+b^2\right)d(a+b\cot(c+dx))} - \frac{\left(a^2\left(a^2+5b^2\right)e^2\right)\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot\right.}{b\left(a^2+b^2\right)^2 d}$$

$$= -\frac{a^{3/2}\left(a^2+5b^2\right)e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}\left(a^2+b^2\right)^2 d} + \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b\left(a^2+b^2\right)d(a+b\cot(c+}$$

$$= -\frac{a^{3/2}\left(a^2+5b^2\right)e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}\left(a^2+b^2\right)^2 d} + \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b\left(a^2+b^2\right)d(a+b\cot(c+}$$

$$= -\frac{a^{3/2}\left(a^2+5b^2\right)e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}\left(a^2+b^2\right)^2 d} - \frac{\left(a^2+2ab-b^2\right)e^{5/2}\tan^{-1}}{\sqrt{2}\,(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.81, size = 390, normalized size = 0.99

$$-\frac{(e\cot(c+dx))^{5/2}\left(-28a^2b^{3/2}\left(a^2-b^2\right)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)+12b^{7/2}\left(a^2+b^2\right)\cot^{\frac{7}{2}}(c+dx)\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{b\cot(c+dx)}{a}\right)-7a^2\left(-6\sqrt{2}ab^{5/2}\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)+6\sqrt{2}ab^{5/2}\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+24a^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)-24a^3\sqrt{b}\sqrt{\cot(c+dx)}-24ab^{5/2}\sqrt{\cot(c+dx)}+4a^2b^{3/2}\cot^{\frac{3}{2}}(c+dx)+4b^{7/2}\cot^{\frac{3}{2}}(c+dx)-3\sqrt{2}ab^{5/2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)+3\sqrt{2}ab^{5/2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{42a^2b^{3/2}\left(a^2+b^2\right)^2 d\cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -1/42*((e*Cot[c + d*x])^(5/2)*(-28*a^2*b^(3/2)*(a^2 - b^2)*Cot[c + d*x]^(3/
2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + 12*b^(7/2)*(a^2 + b^2)
*Cot[c + d*x]^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*Cot[c + d*x])/a)] -
 7*a^2*(-6*Sqrt[2]*a*b^(5/2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] + 6*Sqr
t[2]*a*b^(5/2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24*a^(7/2)*ArcTan[(
Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]] - 24*a^3*Sqrt[b]*Sqrt[Cot[c + d*x]] -
24*a*b^(5/2)*Sqrt[Cot[c + d*x]] + 4*a^2*b^(3/2)*Cot[c + d*x]^(3/2) + 4*b^(7
/2)*Cot[c + d*x]^(3/2) - 3*Sqrt[2]*a*b^(5/2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d
```

```
*x]] + Cot[c + d*x]] + 3*Sqrt[2]*a*b^(5/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x
]] + Cot[c + d*x]]))/(a^2*b^(3/2)*(a^2 + b^2)^2*d*Cot[c + d*x]^(5/2))
```

Maple [A]
time = 0.62, size = 387, normalized size = 0.98

method	result
derivativedivides	$-\frac{2e^3\left(\frac{a^2\left(-\frac{(a^2+b^2)\sqrt{e\cot(dx+c)}}{2b(e\cot(dx+c)b+ae)}+\frac{(a^2+5b^2)\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{2b\sqrt{aeb}}\right)}{(a^2+b^2)^2}+\frac{-\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\,c\cdots}{e\,c\cdots}\right.\right.\cdots}{\cdots}\cdots}{\cdots}\right.\cdots}{\cdots}$
default	$-\frac{2e^3\left(\frac{a^2\left(-\frac{(a^2+b^2)\sqrt{e\cot(dx+c)}}{2b(e\cot(dx+c)b+ae)}+\frac{(a^2+5b^2)\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{2b\sqrt{aeb}}\right)}{(a^2+b^2)^2}+\frac{-\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\,c\cdots}{e\,c\cdots}\right.\right.\cdots}{\cdots}\cdots}{\cdots}\right.\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^3*(a^2/(a^2+b^2)^2*(-1/2*(a^2+b^2)/b*(e*cot(d*x+c))^(1/2)/(e*cot(d*x
+c)*b+a*e)+1/2*(a^2+5*b^2)/b/(a*e*b)^(1/2)*arctan(b*(e*cot(d*x+c))^(1/2)/(a
*e*b)^(1/2)))+1/(a^2+b^2)^2*(-1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+
c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1))+1/8*(-a^2+b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.50, size = 286, normalized size = 0.73

$$-\frac{\left(\frac{4\,(a^4+5\,a^2b^2)\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{\tan(dx+c)}}\right)}{(a^4b+2\,a^2b^3+b^5)\sqrt{ab}}-\frac{4\,a^2}{\left(a^3b+ab^3+\frac{a^2b^2+b^4}{\tan(dx+c)}\right)\sqrt{\tan(dx+c)}}-\frac{2\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^4+2\,a^2b^2+b^4}\right)e^{\frac{5}{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(a^4 + 5*a^2*b^2)*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^4*b
+ 2*a^2*b^3 + b^5)*sqrt(a*b)) - 4*a^2/((a^3*b + a*b^3 + (a^2*b^2 + b^4)/tan
(d*x + c))*sqrt(tan(d*x + c))) - (2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*
sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 + 2*a*b - b^2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^2 - 2*a*b
 - b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^2
 - 2*a*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4
 + 2*a^2*b^2 + b^4))*e^(5/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cot(c+dx))^{\frac{5}{2}}}{(a+b\cot(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)/(a+b*cot(d*x+c))**2,x)
```

```
[Out] Integral((e*cot(c + d*x))**(5/2)/(a + b*cot(c + d*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)/(b*cot(d*x + c) + a)^2, x)
```

Mupad [**B**]
time = 3.12, size = 2500, normalized size = 6.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cot(c + d*x))^(5/2)/(a + b*cot(c + d*x))^2,x)

[Out] atan(((((((8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12*d^4*e^13 + 960*a^6*b^10*d^4*e^13 + 960*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4*e^13 + 96*a^12*b^4*d^4*e^13))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) - (16*(e*cot(c + d*x))^(1/2)*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^18*d^4*e^10 + 160*a^2*b^16*d^4*e^10 + 288*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4*e^10 - 160*a^8*b^10*d^4*e^10 - 288*a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^10 - 32*a^14*b^4*d^4*e^10))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(60*a*b^13*d^2*e^15 + 8*a^13*b*d^2*e^15 + 52*a^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15 + 424*a^7*b^7*d^2*e^15 + 380*a^9*b^5*d^2*e^15 + 100*a^11*b^3*d^2*e^15))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (8*(4*a*b^11*d^2*e^18 + 16*a^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 - 120*a^5*b^7*d^2*e^18 + 320*a^7*b^5*d^2*e^18 + 148*a^9*b^3*d^2*e^18))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(a^10*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20 - 27*a^4*b^6*e^20 + 15*a^6*b^4*e^20 + 9*a^8*b^2*e^20))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*1i - (((((8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12*d^4*e^13 + 960*a^6*b^10*d^4*e^13 + 960*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4*e^13 + 96*a^12*b^4*d^4*e^13))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) + (16*(e*cot(c + d*x))^(1/2)*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^18*d^4*e^10 + 160*a^2*b^16*d^4*e^10 + 288*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4*e^10 - 160*a^8*b^10*d^4*e^10 - 288*a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^10 - 32*a^14*b^4*d^4*e^10))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(60*a*b^13*d^2*e^15 + 8*a^13*b*d^2*e^15 + 52*a^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15 + 424*a^7*b^7*d^2*e^15 + 380*a^9*b^5*d^2*e^15 + 100*a^11*b^3*d^2*e^15))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (8*(4*a*b^11*d^2*e^18 + 16*a^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 - 120*a^5*b^7*d^2*e^18 + 320*a^7*b^5*d^2*e^18 + 148*a^9*b^3*d^2*e^18))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(a^10*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20 - 27*a^4*b^6*e^20 + 15*a^6*b^4*e^20 + 9*a^8*b^2*e^20))/(b^9*d^4 +

```
a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(
a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*1i
)/((16*(a^8*e^23 + 10*a^2*b^6*e^23 + 27*a^4*b^4*e^23 + 10*a^6*b^2*e^23))/(b
^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) + (((((
8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12*d^4*e^13 + 960*a^6*b^10*d^4*e^13 + 9
60*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4*e^13 + 96*a^12*b^4*d^4*e^13))/(b^9*d
^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) - (16*(e*co
t(c + d*x))^(1/2)*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^
2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^18*d^4*e^10 + 160*a^2*b^16*d^4*e^10 + 2
88*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4*e^10 - 160*a^8*b^10*d^4*e^10 - 288*
a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^10 - 32*a^14*b^4*d^4*e^10))/(b^9*d^4
 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4
*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)
+ (16*(e*cot(c + d*x))^(1/2)*(60*a*b^13*d^2*e^15 + 8*a^13*b*d^2*e^15 + 52*a
^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15 + 424*a^7*b^7*d^2*e^15 + 380*a^9*b^
5*d^2*e^15 + 100*a^11*b^3*d^2*e^15))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 +
 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^
2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (8*(4*a*b^11*d^2*e^18 + 16*a
^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 - 120*a^5*b^7*d^2*e^18 + 320*a^7*b^5*
d^2*e^18 + 148*a^9*b^3*d^2*e^18))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*
a^4*b^5*d^5 + 4*a^6*b^3*d^5))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4
i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(a^1
0*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20 - 27*a^4*b^6*e^20 + 15*a^6*b^4*e^20 +
 9*a^8*b^2*e^20))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*
a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i
 - 6*a^2*b^2*d^2)))^(1/2) + (((((8*(96*a^2*b^14...
```

3.77 $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx$

Optimal. Leaf size=387

$$-\frac{\sqrt{a}\,(a^2-3b^2)\,e^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,d}-\frac{(a^2-2ab-b^2)\,e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right.}{\sqrt{2}\,(a^2+b^2)^2\,d}$$

```
[Out] -1/2*(a^2-2*a*b-b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))
/(a^2+b^2)^2/d*2^(1/2)+1/2*(a^2-2*a*b-b^2)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(
d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*e^(3/2)*ln
(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(
1/2)+1/4*(a^2+2*a*b-b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*c
ot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)-(a^2-3*b^2)*e^(3/2)*arctan(b^(1/2)*
(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*a^(1/2)/(a^2+b^2)^2/d/b^(1/2)-a*e*(e*
cot(d*x+c))^(1/2)/(a^2+b^2)/d/(a+b*cot(d*x+c))
```

Rubi [A]

time = 0.45, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3648, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{\sqrt{a}\,e^{3/2}(a^2-3b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{b}\,d\,(a^2+b^2)^2}-\frac{e^{3/2}(a^2-2ab-b^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{e^{3/2}(a^2-2ab-b^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{e^{3/2}(a^2+2ab-b^2)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{e^{3/2}(a^2+2ab-b^2)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{ae\sqrt{e\cot(c+dx)}}{d\,(a^2+b^2)\,(a+b\cot(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -((Sqrt[a]*(a^2 - 3*b^2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqr
t[a]*Sqrt[e])])/(Sqrt[b]*(a^2 + b^2)^2*d)) - ((a^2 - 2*a*b - b^2)*e^(3/2)*A
rcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d
) + ((a^2 - 2*a*b - b^2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/
Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d) - (a*e*Sqrt[e*Cot[c + d*x]])/((a^2 + b^
2)*d*(a + b*Cot[c + d*x])) - ((a^2 + 2*a*b - b^2)*e^(3/2)*Log[Sqrt[e] + Sqr
t[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2
*d) + ((a^2 + 2*a*b - b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqr
t[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = -\frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{\int \frac{\frac{ae^2}{2}-be^2\cot(c+dx)-\frac{1}{2}ae^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}\,dx}{a^2+b^2}$$

$$= -\frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{\int \frac{(a^2-b^2)e^2-2abe^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{(a^2+b^2)^2} + \frac{(a(a^2-3b^2)\,e^2}{}$$

$$= -\frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{2\text{Subst}\left(\int \frac{-(a^2-b^2)e^3+2abe^2x^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+}\right.}{(a^2+b^2)^2\,d}$$

$$= -\frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{(a(a^2-3b^2)\,e)\,\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(}\right.}{(a^2+b^2)^2\,d}$$

$$= -\frac{\sqrt{a}\,(a^2-3b^2)\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,d} - \frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+}$$

$$= -\frac{\sqrt{a}\,(a^2-3b^2)\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,d} - \frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+}$$

$$= -\frac{\sqrt{a}\,(a^2-3b^2)\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,d} - \frac{(a^2-2ab-b^2)\,e^{3/2}\tan^{-}}{\sqrt{2}\,(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 3.39, size = 322, normalized size = 0.83

$$-\frac{(e\cot(c+dx))^{3/2}\left(-240a^2\left(-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}}+\sqrt{\cot(c+dx)}\right)+80ab\cot^{\frac{3}{2}}(c+dx)+80ab\cot^{\frac{3}{2}}(c+dx)\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)+\frac{24b^2(a^2+b^2)\cot^{\frac{5}{2}}(c+dx)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{b\cot(c+dx)}{a}\right)}{a^2}+15(a-b)(a+b)\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{60\,(a^2+b^2)^2\,d\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -1/60*((e*Cot[c + d*x])^(3/2)*(-240*a^2*(-((Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[Co
t[c + d*x]])/Sqrt[a]])/Sqrt[b]) + Sqrt[Cot[c + d*x]]) + 80*a*b*Cot[c + d*x]
^(3/2) + 80*a*b*Cot[c + d*x]^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Co
t[c + d*x]^2]) + (24*b^2*(a^2 + b^2)*Cot[c + d*x]^(5/2)*Hypergeometric2F1[2
, 5/2, 7/2, -((b*Cot[c + d*x])/a)])/a^2 + 15*(a - b)*(a + b)*(2*Sqrt[2]*Arc
Tan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot
[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d
```

```
*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c +
 d*x]]]))/((a^2 + b^2)^2*d*Cot[c + d*x]^(3/2))
```

Maple [A]
time = 0.66, size = 391, normalized size = 1.01

method	result
derivativedivides	$-\frac{2e^3\left(\frac{\left(-a^2e+b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e c}}{(}\right.\right.}{8e^2}\right.}{}$
default	$-\frac{2e^3\left(\frac{\left(-a^2e+b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e c}}{(}\right.\right.}{8e^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^3*(1/(a^2+b^2)^2/e*(1/8*(-a^2*e+b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((
e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d
*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/
2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1))+1/4*a*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+a/(
a^2+b^2)^2/e*((1/2*a^2+1/2*b^2)*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)*b+a*e)+1
/2*(a^2-3*b^2)/(a*e*b)^(1/2)*arctan(b*(e*cot(d*x+c))^(1/2)/(a*e*b)^(1/2))))
```

Maxima [A]
time = 0.51, size = 276, normalized size = 0.71

$$-\frac{\left(\frac{4\left(a^3-3ab^2\right)\arctan\left(\frac{b}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{\left(a^4+2a^2b^2+b^4\right)\sqrt{ab}}-\frac{2\sqrt{2}\left(a^2-2ab-b^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\left(a^2-2ab-b^2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\left(a^2+2ab-b^2\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\left(a^2+2ab-b^2\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^4+2a^2b^2+b^4}+\frac{4a}{\left(a^3+ab^2+\frac{a^2b+b^3}{\tan(dx+c)}\right)\sqrt{\tan(dx+c)}}\right)e^{\frac{3}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(a^3 - 3*a*b^2)*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^4 + 2*
a^2*b^2 + b^4)*sqrt(a*b)) - (2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(
2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(a^2 + 2*a*b - b^
2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(a^2 + 2*
a*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4 + 2*
a^2*b^2 + b^4) + 4*a/((a^3 + a*b^2 + (a^2*b + b^3)/tan(d*x + c))*sqrt(tan(d
*x + c))))*e^(3/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cot(c+dx))^{\frac{3}{2}}}{(a+b\cot(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c))**2,x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)/(a + b*cot(c + d*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)/(b*cot(d*x + c) + a)^2, x)
```

Mupad [**B**]
time = 3.37, size = 2500, normalized size = 6.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)/(a + b*cot(c + d*x))^2,x)
```

```
[Out] (atan((((a^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*(2*b^9*e^16 + a^8*b*e^16
- 5*a^2*b^7*e^16 + 17*a^4*b^5*e^16 - 7*a^6*b^3*e^16))/(a^8*d^4 + b^8*d^4 +
4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + ((a^2 - 3*b^2)*((16*(2*a^1
0*b*d^2*e^15 - 78*a^2*b^9*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*b^5*d^2*e^
15 - 24*a^8*b^3*d^2*e^15))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d
^5 + 4*a^6*b^2*d^5) - ((a^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*(20*a^3*b^
10*d^2*e^13 - 60*a*b^12*d^2*e^13 + 168*a^5*b^8*d^2*e^13 + 40*a^7*b^6*d^2*e^
13 - 44*a^9*b^4*d^2*e^13 + 4*a^11*b^2*d^2*e^13))/(a^8*d^4 + b^8*d^4 + 4*a^2
*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + ((a^2 - 3*b^2)*((16*(40*a*b^14*
d^4*e^12 + 192*a^3*b^12*d^4*e^12 + 360*a^5*b^10*d^4*e^12 + 320*a^7*b^8*d^4*
e^12 + 120*a^9*b^6*d^4*e^12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5 + b^8*d^5 + 4*
a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (8*(e*cot(c + d*x))^(1/2)*(a
^2 - 3*b^2)*(-a*b*e^3)^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 28
8*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^
10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/((b^5*d +
2*a^2*b^3*d + a^4*b*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 +
 4*a^6*b^2*d^4)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-
a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(1/2))/(2*(
b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(1/2)*1i)/(2*(b^5*d + 2*a^2*b^3
*d + a^4*b*d)) + ((a^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*(2*b^9*e^16 + a
^8*b*e^16 - 5*a^2*b^7*e^16 + 17*a^4*b^5*e^16 - 7*a^6*b^3*e^16))/(a^8*d^4 +
b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) - ((a^2 - 3*b^2)*(
(16*(2*a^10*b*d^2*e^15 - 78*a^2*b^9*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*
b^5*d^2*e^15 - 24*a^8*b^3*d^2*e^15))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6
*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + ((a^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*
(20*a^3*b^10*d^2*e^13 - 60*a*b^12*d^2*e^13 + 168*a^5*b^8*d^2*e^13 + 40*a^7*
b^6*d^2*e^13 - 44*a^9*b^4*d^2*e^13 + 4*a^11*b^2*d^2*e^13))/(a^8*d^4 + b^8*d
^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) - ((a^2 - 3*b^2)*((16*(
40*a*b^14*d^4*e^12 + 192*a^3*b^12*d^4*e^12 + 360*a^5*b^10*d^4*e^12 + 320*a^
7*b^8*d^4*e^12 + 120*a^9*b^6*d^4*e^12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5 + b^
8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (8*(e*cot(c + d*x)
)^(1/2)*(a^2 - 3*b^2)*(-a*b*e^3)^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4
*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^1
0 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/
((b^5*d + 2*a^2*b^3*d + a^4*b*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4
*b^4*d^4 + 4*a^6*b^2*d^4)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4
*b*d)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(
1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(1/2)*1i)/(2*(b^5*d +
 2*a^2*b^3*d + a^4*b*d)))/((32*(3*a*b^6*e^18 - a^3*b^4*e^18))/(a^8*d^5 + b^
8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - ((a^2 - 3*b^2)*((1
6*(e*cot(c + d*x))^(1/2)*(2*b^9*e^16 + a^8*b*e^16 - 5*a^2*b^7*e^16 + 17*a^4
```

```
*b^5*e^16 - 7*a^6*b^3*e^16))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4
*d^4 + 4*a^6*b^2*d^4) + ((a^2 - 3*b^2)*((16*(2*a^10*b*d^2*e^15 - 78*a^2*b^9
*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*b^5*d^2*e^15 - 24*a^8*b^3*d^2*e^15)
)/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - ((a
^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*(20*a^3*b^10*d^2*e^13 - 60*a*b^12*d
^2*e^13 + 168*a^5*b^8*d^2*e^13 + 40*a^7*b^6*d^2*e^13 - 44*a^9*b^4*d^2*e^13
+ 4*a^11*b^2*d^2*e^13))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4
+ 4*a^6*b^2*d^4) + ((a^2 - 3*b^2)*((16*(40*a*b^14*d^4*e^12 + 192*a^3*b^12*d
^4*e^12 + 360*a^5*b^10*d^4*e^12 + 320*a^7*b^8*d^4*e^12 + 120*a^9*b^6*d^4*e^
12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d
^5 + 4*a^6*b^2*d^5) - (8*(e*cot(c + d*x))^(1/2)*(a^2 - 3*b^2)*(-a*b*e^3)^(1
/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160
*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^1
2*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/((b^5*d + 2*a^2*b^3*d + a^4*b*d)*(a
^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)))*(-a*b*e
^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(1/2))/(2*(b^5*d
 + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4
*b*d)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)) + ((a^2 - 3*b
^2)*((16*(e*cot(c + d*x))^(1/2)*(2*b^9*e^16 + a^8*b*e^16 - 5*a^2*b^7*e^16 +
 17*a^4*b^5*e^16 - 7*a^6*b^3*e^16))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*
a^4*b^4*d^4 + 4*a^6*b^2*d^4) - ((a^2 - 3*b^2)*((16*(2*a^10*b*d^2*e^15 - 78*
a^2*b^9*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*b^5*d^2*e^15 - 24*a^8*b^3*d^
2*e^15))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5
) + ((a^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*(20*a^3*b^10*d^2*e^13 - 60*a
*b^12*d^2*e^13 + 168*a^5*b^8*d^2*e^13 + 40*a^7*b^6*d^2*e^13 - 44*a^9*b^4*d^
2*e^13 + 4*a^11*b^2*d^2*e^13))/(a^8*d^4 + b^8*d...
```

3.78 $\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx$

Optimal. Leaf size=386

$$\frac{\sqrt{b}\,(3a^2-b^2)\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)^2 d}+\frac{(a^2+2ab-b^2)\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right.}{\sqrt{2}\,(a^2+b^2)^2 d}$$

```
[Out] 1/2*(a^2+2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/
(a^2+b^2)^2/d*2^(1/2)-1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(
1/2)/e^(1/2))*e^(1/2)/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+
cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/(a^2+b^2)^2/d*2^(1
/2)+1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c)
)^(1/2))*e^(1/2)/(a^2+b^2)^2/d*2^(1/2)+(3*a^2-b^2)*arctan(b^(1/2)*(e*cot(d*
x+c))^(1/2)/a^(1/2)/e^(1/2))*b^(1/2)*e^(1/2)/(a^2+b^2)^2/d/a^(1/2)+b*(e*cot
(d*x+c))^(1/2)/(a^2+b^2)/d/(a+b*cot(d*x+c))
```

Rubi [A]
time = 0.43, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3649, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{\sqrt{b}\,\sqrt{e}\,(3a^2-b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{a}\,d\,(a^2+b^2)^2}+\frac{\sqrt{e}\,(a^2+2ab-b^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}-\frac{\sqrt{e}\,(a^2+2ab-b^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{b\sqrt{e\cot(c+dx)}}{d\,(a^2+b^2)\,(a+b\cot(c+dx))}-\frac{\sqrt{e}\,(a^2-2ab-b^2)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}+\frac{\sqrt{e}\,(a^2-2ab-b^2)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] (Sqrt[b]*(3*a^2 - b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[
a]*Sqrt[e])])/(Sqrt[a]*(a^2 + b^2)^2*d) + ((a^2 + 2*a*b - b^2)*Sqrt[e]*ArcT
an[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d) -
 ((a^2 + 2*a*b - b^2)*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqr
t[e]])/(Sqrt[2]*(a^2 + b^2)^2*d) + (b*Sqrt[e*Cot[c + d*x]])/((a^2 + b^2)*d*
(a + b*Cot[c + d*x])) - ((a^2 - 2*a*b - b^2)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*
Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) +
 ((a^2 - 2*a*b - b^2)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*
Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx = \frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{\int \frac{-\frac{be}{2}-ae\cot(c+dx)+\frac{1}{2}be\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}\,dx}{a^2+b^2}$$

$$= \frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{\int \frac{-2abe-(a^2-b^2)e\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx}{(a^2+b^2)^2} - \frac{(b(3a^2-b^2)\,e)\int}{}$$

$$= \frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{2\text{Subst}\left(\int \frac{2abe^2+(a^2-b^2)ex^2}{e^2+x^4}\,dx, x, \sqrt{e\cot(c+dx}\right.}{(a^2+b^2)^2\,d}$$

$$= \frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} + \frac{(b(3a^2-b^2))\,\text{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}}\,dx, x, \sqrt{e\cot(c+}\right.}{(a^2+b^2)^2\,d}$$

$$= \frac{\sqrt{b}\,(3a^2-b^2)\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)^2\,d} + \frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+d}$$

$$= \frac{\sqrt{b}\,(3a^2-b^2)\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)^2\,d} + \frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+d}$$

$$= \frac{\sqrt{b}\,(3a^2-b^2)\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)^2\,d} + \frac{(a^2+2ab-b^2)\sqrt{e}\,\tan^{-1}}{\sqrt{2}\,(a^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.10, size = 401, normalized size = 1.04

$$-\frac{\sqrt{e\cot(c+dx)}\left(-\frac{4a^{3/2}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{(a^2+b^2)^2}+\frac{4ab\sqrt{\cot(c+dx)}}{(a^2+b^2)^2}-\frac{\sqrt{b}\left(-a\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)+\sqrt{a}\sqrt{b}\sqrt{\cot(c+dx)}-b\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)\cot(c+dx)\right)}{\sqrt{a}\,(a^2+b^2)(a+b\cot(c+dx))}+\frac{2(a-b)(a+b)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)}{3(a^2+b^2)^2}-\frac{ab\left(2\left(\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{2(a^2+b^2)^2}\right)}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] -((Sqrt[e*Cot[c + d*x]]*((-4*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d
*x]])/Sqrt[a]])/(a^2 + b^2)^2 + (4*a*b*Sqrt[Cot[c + d*x]])/(a^2 + b^2)^2 -
(Sqrt[b]*(-(a*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]) + Sqrt[a]*Sqrt[
b]*Sqrt[Cot[c + d*x]] - b*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]*Cot[
c + d*x]))/(Sqrt[a]*(a^2 + b^2)*(a + b*Cot[c + d*x])) + (2*(a - b)*(a + b)*
Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])/(3*(a^2
 + b^2)^2) - (a*b*(2*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - Sqrt
[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) + 8*Sqrt[Cot[c + d*x]] + Sqrt[2
```

```
]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(2*(a^2 + b^2)^2)))/(d*Sqrt[Cot[c
+ d*x]]))
```

Maple [A]

time = 0.63, size = 391, normalized size = 1.01

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{b\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{(3a^2-b^2)\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{e^2(a^2+b^2)^2}+\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\,\mathrm{co}\cdots}{e\,\mathrm{co}\cdots}\right.\right.\cdots}{\cdots}\cdots\right)}{\cdots}$
default	$-\frac{2e^3\left(-\frac{b\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{(3a^2-b^2)\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{e^2(a^2+b^2)^2}+\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\,\mathrm{co}\cdots}{e\,\mathrm{co}\cdots}\right.\right.\cdots}{\cdots}\cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^3*(-b/e^2/(a^2+b^2)^2*((1/2*a^2+1/2*b^2)*(e*cot(d*x+c))^(1/2)/(e*cot
(d*x+c)*b+a*e)+1/2*(3*a^2-b^2)/(a*e*b)^(1/2)*arctan(b*(e*cot(d*x+c))^(1/2)/
(a*e*b)^(1/2)))+1/(a^2+b^2)^2/e^2*(1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot
(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)
-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1))+1/8*(a^2-b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

Maxima [A]

time = 0.53, size = 278, normalized size = 0.72

$$\frac{\left(\frac{4\,(3\,a^2b-b^3)\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{\tan(dx+c)}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{ab}}-\frac{2\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{4\,b}{\left(a^3+ab^2+\frac{a^2b+b^3}{\tan(dx+c)}\right)\sqrt{\tan(dx+c)}}\right)e^{\frac{1}{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(3*a^2*b - b^3)*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^4 + 2*a
^2*b^2 + b^4)*sqrt(a*b)) - (2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/2*sqrt(2
)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^2 - 2*a*b - b^2
)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^2 - 2*a
*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4 + 2*a
^2*b^2 + b^4) + 4*b/((a^3 + a*b^2 + (a^2*b + b^3)/tan(d*x + c))*sqrt(tan(d*
x + c))))*e^(1/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))/(a + b*cot(c + d*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))/(b*cot(d*x + c) + a)^2, x)
```

Mupad [**B**]
time = 3.08, size = 2500, normalized size = 6.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)/(a + b*cot(c + d*x))^2,x)
```

```
[Out] (b*e*(e*cot(c + d*x))^(1/2))/((a*d*e + b*d*e*cot(c + d*x))*(a^2 + b^2)) - a
tan((((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^15*d^4*e^11
 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4*e^11))/(a
^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (16*(e*
cot(c + d*x))^(1/2)*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*
d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 2
88*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a
^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/(a^8*d^4
+ b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4*(a^4*d^2*
1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (16
*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^11 - 88*a^5
*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^11*b^2*d^2*
e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))
*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6
i)))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24*a^5*b^6*d^2
*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^8*d^5 + 4*
a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(e/(4*(a^4*d^2*1i + b^4*d^2*1
i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(e*cot(c + d*
x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*b^3*e^12))
/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4
*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(
1/2)*1i - (((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^15*d^4
*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4*e^11
))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (1
6*(e*cot(c + d*x))^(1/2)*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a
^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^1
0 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 -
288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/(a^8
*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4*(a^4
*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)
- (16*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^11 - 8
8*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^11*b^2
*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*
d^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*
d^2*6i)))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24*a^5*b^
6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^8*d^5
 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(e/(4*(a^4*d^2*1i + b^4*
d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) + (16*(e*cot(c
 + d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*b^3*e
^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*
(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i
```

```
)))^(1/2)*1i)/((((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^1
5*d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4
*e^11))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5)
 - (16*(e*cot(c + d*x))^(1/2)*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2
- 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^
4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^
10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))
/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4
*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(
1/2) + (16*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^1
1 - 88*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^1
1*b^2*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6
*b^2*d^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2
*b^2*d^2*6i)))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24*a
^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^
8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(e/(4*(a^4*d^2*1i +
 b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) - (16*(e*
cot(c + d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*
b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d
^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d
^2*6i)))^(1/2) + (((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b
^15*d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d
^4*e^11))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^
5) + (16*(e*cot(c + d*x))^(1/2)*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^
2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*
d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11...
```

3.79 $\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^2}\,dx$

Optimal. Leaf size=394

$$-\frac{b^{3/2}(5a^2+b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{a^{3/2}\left(a^2+b^2\right)^2 d\sqrt{e}}+\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,\left(a^2+b^2\right)^2 d\sqrt{e}}-\frac{(a^2}{}$$

```
[Out] -b^(3/2)*(5*a^2+b^2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/a
^(3/2)/(a^2+b^2)^2/d/e^(1/2)+1/2*(a^2-2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*
x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)/e^(1/2)-1/2*(a^2-2*a*b-b^2)*arct
an(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)/e^(1/2)+1/
4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2
))/(a^2+b^2)^2/d*2^(1/2)/e^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*
e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)/e^(1/2)-b^2*(e*
cot(d*x+c))^(1/2)/a/(a^2+b^2)/d/e/(a+b*cot(d*x+c))
```

Rubi [A]
time = 0.48, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3650, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}\,(a^2+b^2)^2}-\frac{(a^2-2ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\sqrt{e}\,(a^2+b^2)^2}-\frac{b^2\sqrt{e\cot(c+dx)}}{ade\,(a^2+b^2)\,(a+b\cot(c+dx))}+\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}\,(a^2+b^2)^2}-\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}\,(a^2+b^2)^2}-\frac{b^{3/2}(5a^2+b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{a^{3/2}d\sqrt{e}\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2),x]
```

```
[Out] -((b^(3/2)*(5*a^2 + b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqr
t[e])])/(a^(3/2)*(a^2 + b^2)^2*d*Sqrt[e])) + ((a^2 - 2*a*b - b^2)*ArcTan[1
- (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d*Sqrt[e]
) - ((a^2 - 2*a*b - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]
)/(Sqrt[2]*(a^2 + b^2)^2*d*Sqrt[e]) - (b^2*Sqrt[e*Cot[c + d*x]])/(a*(a^2 +
b^2)*d*e*(a + b*Cot[c + d*x])) + ((a^2 + 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]
*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d*S
qrt[e]) - ((a^2 + 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]
*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^2}\,dx = -\frac{b^2\sqrt{e\cot(c+dx)}}{a\left(a^2+b^2\right)de(a+b\cot(c+dx))} - \frac{\int \frac{-\frac{1}{2}\left(2a^2+b^2\right)e+abe\cot(c+dx)-}{\sqrt{e\cot(c+dx)}\,(a+b}}{a\left(a^2+b^2\right)}$$

$$= -\frac{b^2\sqrt{e\cot(c+dx)}}{a\left(a^2+b^2\right)de(a+b\cot(c+dx))} + \frac{\left(b^2\left(5a^2+b^2\right)\right)\int \frac{}{\sqrt{e\cot(c}}}{2a\left(a^2+\right.}$$

$$= -\frac{b^2\sqrt{e\cot(c+dx)}}{a\left(a^2+b^2\right)de(a+b\cot(c+dx))} + \frac{\left(b^2\left(5a^2+b^2\right)\right)\text{Subst}\left(\int -\right.}{2a\left(\right.}$$

$$= -\frac{b^2\sqrt{e\cot(c+dx)}}{a\left(a^2+b^2\right)de(a+b\cot(c+dx))} - \frac{\left(a^2-2ab-b^2\right)\text{Subst}\left(\int \right.}{\left(a^2-\right.}$$

$$= -\frac{b^{3/2}\left(5a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}\left(a^2+b^2\right)^2d\sqrt{e}} - \frac{b^2\sqrt{e}}{a\left(a^2+b^2\right)de}$$

$$= -\frac{b^{3/2}\left(5a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}\left(a^2+b^2\right)^2d\sqrt{e}} - \frac{b^2\sqrt{e}}{a\left(a^2+b^2\right)de}$$

$$= -\frac{b^{3/2}\left(5a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}\left(a^2+b^2\right)^2d\sqrt{e}} + \frac{\left(a^2-2ab-b\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.93, size = 300, normalized size = 0.76

$$-\frac{\sqrt{\cot(c+dx)}\left(96\sqrt{a}\,b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)+\frac{24b^2\left(a^2+b^2\right)\sqrt{\cot(c+dx)}\left(\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\cot(c+dx)}}+\frac{a}{a+b\cot(c+dx)}\right)}{a^2}-32ab\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)-6\sqrt{2}(a-b)(a+b)\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{24\left(a^2+b^2\right)^2d\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2),x]
```

```
[Out] -1/24*(Sqrt[Cot[c + d*x]]*(96*Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c +
d*x]])/Sqrt[a]] + (24*b^2*(a^2 + b^2)*Sqrt[Cot[c + d*x]]*((Sqrt[a]*ArcTan[(
Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]])/(Sqrt[b]*Sqrt[Cot[c + d*x]]) + a/(a +
 b*Cot[c + d*x])))/a^2 - 32*a*b*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1
, 7/4, -Cot[c + d*x]^2] - 6*Sqrt[2]*(a - b)*(a + b)*(2*ArcTan[1 - Sqrt[2]*S
qrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqr
```

```
t[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]
] + Cot[c + d*x]])))/((a^2 + b^2)^2*d*Sqrt[e*Cot[c + d*x]])
```

Maple [A]
time = 0.65, size = 396, normalized size = 1.01

method	result
derivativedivides	$-\frac{2e^3\left(\frac{b^2\left(\frac{(a^2+b^2)\sqrt{e\cot(dx+c)}}{2a(e\cot(dx+c)b+ae)}+\frac{(5a^2+b^2)\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{2a\sqrt{aeb}}\right)}{e^3(a^2+b^2)^2}+\frac{(a^2e-b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\,(\ln \cdots}{\cdots}\right)}{\cdots}$
default	$-\frac{2e^3\left(\frac{b^2\left(\frac{(a^2+b^2)\sqrt{e\cot(dx+c)}}{2a(e\cot(dx+c)b+ae)}+\frac{(5a^2+b^2)\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{2a\sqrt{aeb}}\right)}{e^3(a^2+b^2)^2}+\frac{(a^2e-b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\,(\ln \cdots}{\cdots}\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^3*(b^2/e^3/(a^2+b^2)^2*(1/2*(a^2+b^2)/a*(e*cot(d*x+c))^(1/2)/(e*cot(
d*x+c)*b+a*e)+1/2*(5*a^2+b^2)/a/(a*e*b)^(1/2)*arctan(b*(e*cot(d*x+c))^(1/2)
/(a*e*b)^(1/2)))+1/e^3/(a^2+b^2)^2*(1/8*(a^2*e-b^2*e)*(e^2)^(1/4)/e^2*2^(1/
2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/
(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arct
an(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)+1))-1/4*a*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1))))
```

Maxima [A]
time = 0.53, size = 285, normalized size = 0.72

$$-\frac{\left(\frac{4\,(5\,a^2b^2+b^4)\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{\tan(dx+c)}}\right)}{(a^5+2\,a^3b^2+ab^4)\sqrt{ab}}+\frac{4\,b^2}{\left(a^4+a^2b^2+\frac{a^3b+ab^3}{\tan(dx+c)}\right)\sqrt{\tan(dx+c)}}+\frac{2\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\,(a^2-2\,ab-b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\,(a^2+2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^4+2\,a^2b^2+b^4}\right)e^{\left(-\frac{1}{2}\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(5*a^2*b^2 + b^4)*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^5 +
2*a^3*b^2 + a*b^4)*sqrt(a*b)) + 4*b^2/((a^4 + a^2*b^2 + (a^3*b + a*b^3)/tan
(d*x + c))*sqrt(tan(d*x + c))) + (2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*
sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 - 2*a*b - b^2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(a^2 + 2*a*b
 - b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(a^2
 + 2*a*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4
 + 2*a^2*b^2 + b^4))*e^(-1/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c))**2,x)
```

```
[Out] Integral(1/(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cot(d*x + c) + a)^2*sqrt(e*cot(d*x + c))), x)
```

Mupad [B]
time = 8.16, size = 2500, normalized size = 6.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))^2),x)
```

```
[Out] (log(- (((((((((128*b^2*e^10*(2*b^6 - a^6 + 9*a^2*b^4 + 6*a^4*b^2))/(a*d) -
 256*b^3*e^10*(e*cot(c + d*x))^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*e*(
a*1i - b)^4))^(1/2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 - (64*b^2*e^9*(e*co
t(c + d*x))^(1/2)*(2*b^8 - a^8 + 5*a^2*b^6 + 67*a^4*b^4 - a^6*b^2))/(a*d^2*
(a^2 + b^2)^2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 - (32*b^5*e^9*(25*a^6 +
b^6 - 13*a^2*b^4 - 85*a^4*b^2))/(a^2*d^3*(a^2 + b^2)^3))*(1i/(d^2*e*(a*1i -
 b)^4))^(1/2))/2 - (16*b^5*e^8*(e*cot(c + d*x))^(1/2)*(b^6 - 27*a^6 + 7*a^2
*b^4 + 11*a^4*b^2))/(a^2*d^4*(a^2 + b^2)^4))*(1i/(d^2*e*(a*1i - b)^4))^(1/2
))/2 - (16*b^6*e^8*(5*a^2 + b^2))/(a*d^5*(a^2 + b^2)^4))*(-1/(a^4*d^2*e*1i
+ b^4*d^2*e*1i - a^2*b^2*d^2*e*6i + 4*a*b^3*d^2*e - 4*a^3*b*d^2*e))^(1/2))/
2 - log(- (((((((((128*b^2*e^10*(2*b^6 - a^6 + 9*a^2*b^4 + 6*a^4*b^2))/(a*d
) + 256*b^3*e^10*(e*cot(c + d*x))^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*
e*(a*1i - b)^4))^(1/2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 + (64*b^2*e^9*(e
*cot(c + d*x))^(1/2)*(2*b^8 - a^8 + 5*a^2*b^6 + 67*a^4*b^4 - a^6*b^2))/(a*d
^2*(a^2 + b^2)^2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 - (32*b^5*e^9*(25*a^6
 + b^6 - 13*a^2*b^4 - 85*a^4*b^2))/(a^2*d^3*(a^2 + b^2)^3))*(1i/(d^2*e*(a*1
i - b)^4))^(1/2))/2 + (16*b^5*e^8*(e*cot(c + d*x))^(1/2)*(b^6 - 27*a^6 + 7*
a^2*b^4 + 11*a^4*b^2))/(a^2*d^4*(a^2 + b^2)^4))*(1i/(d^2*e*(a*1i - b)^4))^(
1/2))/2 - (16*b^6*e^8*(5*a^2 + b^2))/(a*d^5*(a^2 + b^2)^4))*(-1/(4*(a^4*d^2
*e*1i + b^4*d^2*e*1i - a^2*b^2*d^2*e*6i + 4*a*b^3*d^2*e - 4*a^3*b*d^2*e)))^
(1/2) + atan(((-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*
e*4i - a^3*b*d^2*e*4i)))^(1/2)*(((16*(24*a^2*b^11*d^2*e^9 - 2*b^13*d^2*e^9
+ 196*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9))/(a^10*d^
5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (-1i/(4*
(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)
))^(1/2)*((-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i
 - a^3*b*d^2*e*4i)))^(1/2)*((16*(16*a*b^16*d^4*e^10 + 136*a^3*b^14*d^4*e^10
 + 432*a^5*b^12*d^4*e^10 + 680*a^7*b^10*d^4*e^10 + 560*a^9*b^8*d^4*e^10 + 2
16*a^11*b^6*d^4*e^10 + 16*a^13*b^4*d^4*e^10 - 8*a^15*b^2*d^4*e^10))/(a^10*d
^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) - (16*(-1
i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*
e*4i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(32*a^2*b^17*d^4*e^10 + 160*a^4*b^15*d
^4*e^10 + 288*a^6*b^13*d^4*e^10 + 160*a^8*b^11*d^4*e^10 - 160*a^10*b^9*d^4*
e^10 - 288*a^12*b^7*d^4*e^10 - 160*a^14*b^5*d^4*e^10 - 32*a^16*b^3*d^4*e^10
))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)
) + (16*(e*cot(c + d*x))^(1/2)*(8*a*b^14*d^2*e^9 + 36*a^3*b^12*d^2*e^9 + 31
6*a^5*b^10*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 - 12*a^11*b^
4*d^2*e^9 - 4*a^13*b^2*d^2*e^9))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 +
6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)))*(-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2
*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2) + (16*(e*cot(c + d*x))^(1
/2)*(b^11*e^8 + 7*a^2*b^9*e^8 + 11*a^4*b^7*e^8 - 27*a^6*b^5*e^8))/(a^10*d^4
```

 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4))*1i - (-1i/
(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*
4i)))^(1/2)*(((16*(24*a^2*b^11*d^2*e^9 - 2*b^13*d^2*e^9 + 196*a^4*b^9*d^2*e
^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9))/(a^10*d^5 + a^2*b^8*d^5 + 4
*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (-1i/(4*(a^4*d^2*e + b^4*d^
2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*((-1i/(4*(
a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i))
)^(1/2)*((16*(16*a*b^16*d^4*e^10 + 136*a^3*b^14*d^4*e^10 + 432*a^5*b^12*d^4
*e^10 + 680*a^7*b^10*d^4*e^10 + 560*a^9*b^8*d^4*e^10 + 216*a^11*b^6*d^4*e^1
0 + 16*a^13*b^4*d^4*e^10 - 8*a^15*b^2*d^4*e^10))/(a^10*d^5 + a^2*b^8*d^5 +
4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (16*(-1i/(4*(a^4*d^2*e + b
^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*(e*co
t(c + d*x))^(1/2)*(32*a^2*b^17*d^4*e^10 + 160*a^4*b^15*d^4*e^10 + 288*a^6*b
^13*d^4*e^10 + 160*a^8*b^11*d^4*e^10 - 160*a^10*b^9*d^4*e^10 - 288*a^12*b^7
*d^4*e^10 - 160*a^14*b^5*d^4*e^10 - 32*a^16*b^3*d^4*e^10))/(a^10*d^4 + a^2*
b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)) - (16*(e*cot(c +
d*x))^(1/2)*(8*a*b^14*d^2*e^9 + 36*a^3*b^12*d^2*e^9 + 316*a^5*b^10*d^2*e^9
+ 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 - 12*a^11*b^4*d^2*e^9 - 4*a^13*
b^2*d^2*e^9))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a
^8*b^2*d^4)))*(-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*
e*4i - a^3*b*d^2*e*4i)))^(1/2) - (16*(e*cot(c + d*x))^(1/2)*(b^11*e^8 + 7*a
^2*b^9*e^8 + 11*a^4*b^7*e^8 - 27*a^6*b^5*e^8))/(a^10*d^4 + a^2*b^8*d^4 + 4*
a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4))*1i)/((-1i/(4*(a^4*d^2*e + b^4
*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2)*(((16*(
24*a^2*b^11*d^2*e^9 - 2*b^13*d^2*e^9 + 196*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^
2*e^9 - 50*a^8*b^5*d^2*e^9))/(a^10*d^5 + a^2*b^...

3.80 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx$

Optimal. Leaf size=437

$$\frac{b^{5/2}(7a^2+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{a^{5/2}\,(a^2+b^2)^2\,de^{3/2}}-\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,de^{3/2}}+\frac{(a^2}{}$$

```
[Out] b^(5/2)*(7*a^2+3*b^2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/
a^(5/2)/(a^2+b^2)^2/d/e^(3/2)-1/2*(a^2+2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d
*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d/e^(3/2)*2^(1/2)+1/2*(a^2+2*a*b-b^2)*arc
tan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d/e^(3/2)*2^(1/2)+1
/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/
2))/(a^2+b^2)^2/d/e^(3/2)*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)
*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d/e^(3/2)*2^(1/2)+(2*a^2
+3*b^2)/a^2/(a^2+b^2)/d/e/(e*cot(d*x+c))^(1/2)-b^2/a/(a^2+b^2)/d/e/(a+b*cot
(d*x+c))/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3650, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}\,(a^2+b^2)^2}+\frac{(a^2+2ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{3/2}\,(a^2+b^2)^2}+\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}\,(a^2+b^2)^2}-\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}\,(a^2+b^2)^2}-\frac{b^2}{ade\,(a^2+b^2)\,\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}+\frac{2a^2+3b^2}{a^2de\,(a^2+b^2)\,\sqrt{e\cot(c+dx)}}+\frac{b^{5/2}(7a^2+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{a^{5/2}de^{3/2}\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^2),x]
```

```
[Out] (b^(5/2)*(7*a^2 + 3*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqr
t[e])])/(a^(5/2)*(a^2 + b^2)^2*d*e^(3/2)) - ((a^2 + 2*a*b - b^2)*ArcTan[1 -
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^2*d*e^(3/2))
 + ((a^2 + 2*a*b - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])
/(Sqrt[2]*(a^2 + b^2)^2*d*e^(3/2)) + (2*a^2 + 3*b^2)/(a^2*(a^2 + b^2)*d*e*S
qrt[e*Cot[c + d*x]]) - b^2/(a*(a^2 + b^2)*d*e*Sqrt[e*Cot[c + d*x]]*(a + b*C
ot[c + d*x])) + ((a^2 - 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - S
qrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d*e^(3/2)) - ((a^2 -
 2*a*b - b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d
*x]]])/(2*Sqrt[2]*(a^2 + b^2)^2*d*e^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = -\frac{b^2}{a\left(a^2+b^2\right)de\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))} - \frac{\int \frac{-\frac{1}{2}(2a^2}{}}{}$$

$$= \frac{2a^2+3b^2}{a^2\left(a^2+b^2\right)de\sqrt{e\cot(c+dx)}} - \frac{b^2}{a\left(a^2+b^2\right)de\sqrt{e\cot(c+dx}}$$

$$= \frac{2a^2+3b^2}{a^2\left(a^2+b^2\right)de\sqrt{e\cot(c+dx)}} - \frac{b^2}{a\left(a^2+b^2\right)de\sqrt{e\cot(c+dx}}$$

$$= \frac{2a^2+3b^2}{a^2\left(a^2+b^2\right)de\sqrt{e\cot(c+dx)}} - \frac{b^2}{a\left(a^2+b^2\right)de\sqrt{e\cot(c+dx}}$$

$$= \frac{2a^2+3b^2}{a^2\left(a^2+b^2\right)de\sqrt{e\cot(c+dx)}} - \frac{b^2}{a\left(a^2+b^2\right)de\sqrt{e\cot(c+dx}}$$

$$= \frac{b^{5/2}(7a^2+3b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}\left(a^2+b^2\right)^2de^{3/2}} + \frac{2a}{a^2\left(a^2+b^2\right)d}$$

$$= \frac{b^{5/2}(7a^2+3b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}\left(a^2+b^2\right)^2de^{3/2}} + \frac{2a}{a^2\left(a^2+b^2\right)d}$$

$$= \frac{b^{5/2}(7a^2+3b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}\left(a^2+b^2\right)^2de^{3/2}} - \frac{(a^2+2ab-b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.62, size = 244, normalized size = 0.56

$$\frac{8a^2b^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{b\cot(c+dx)}{a}\right)+4b^2(a^2+b^2)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{b\cot(c+dx)}{a}\right)+a^2\left(4(a^2-b^2)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+\sqrt{2}\,ab\sqrt{\cot(c+dx)}\left(-2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)+2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)+\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{2a^2\left(a^2+b^2\right)^2de\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^2),x]
```

```
[Out] (8*a^2*b^2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*Cot[c + d*x])/a)] + 4*b^2*(
a^2 + b^2)*Hypergeometric2F1[-1/2, 2, 1/2, -((b*Cot[c + d*x])/a)] + a^2*(4*
(a^2 - b^2)*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Sqrt[2]*a*b*
Sqrt[Cot[c + d*x]]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] + 2*ArcTan[1
+ Sqrt[2]*Sqrt[Cot[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
+ d*x]] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(2*a^2*(a^2
 + b^2)^2*d*e*Sqrt[e*Cot[c + d*x]])
```

Maple [A]
time = 0.63, size = 414, normalized size = 0.95

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{b^3\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{(7a^2+3b^2)\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{a^2e^4(a^2+b^2)^2}+\frac{-\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\cdots\right.\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{2e^3\left(-\frac{b^3\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{(7a^2+3b^2)\arctan\left(\frac{b\sqrt{e\cot(dx+c)}}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{a^2e^4(a^2+b^2)^2}+\frac{-\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\cdots\right.\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^3*(-b^3/a^2/e^4/(a^2+b^2)^2*((1/2*a^2+1/2*b^2)*(e*cot(d*x+c))^(1/2)/
(e*cot(d*x+c)*b+a*e)+1/2*(7*a^2+3*b^2)/(a*e*b)^(1/2)*arctan(b*(e*cot(d*x+c)
)^(1/2)/(a*e*b)^(1/2)))+1/(a^2+b^2)^2/e^4*(-1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(
ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*c
ot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2
^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)+1))+1/8*(-a^2+b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)
-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^
```

```
(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)+1)))-1/a^2/e^4/(e*cot(d*x+c))^(1/2))
```

Maxima [A]
time = 0.51, size = 323, normalized size = 0.74

$$\frac{\left(\frac{4\,(7\,a^2b^3+3\,b^5)\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{(a^6+2\,a^4b^2+a^2b^4)\sqrt{ab}}+\frac{2\,\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a^2+2\,ab-b^2)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\,(a^2-2\,ab-b^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{4\left(2\,a^3+2\,ab^2+\frac{2\,a^2b+3\,b^3}{\tan(dx+c)}\right)}{\frac{a^5+a^3b^2}{\sqrt{\tan\left(dx+c\right)}}+\frac{a^4b+a^2b^3}{\tan(dx+c)^{\frac{3}{2}}}}\right)e^{\left(-\frac{3}{2}\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(7*a^2*b^3 + 3*b^5)*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^6 +
 2*a^4*b^2 + a^2*b^4)*sqrt(a*b)) + (2*sqrt(2)*(a^2 + 2*a*b - b^2)*arctan(1/
2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(a^2 + 2*a*b - b^2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^2 - 2*a
*b - b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a
^2 - 2*a*b - b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a
^4 + 2*a^2*b^2 + b^4) + 4*(2*a^3 + 2*a*b^2 + (2*a^2*b + 3*b^3)/tan(d*x + c)
)/((a^5 + a^3*b^2)/sqrt(tan(d*x + c)) + (a^4*b + a^2*b^3)/tan(d*x + c)^(3/2
)))*e^(-3/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}\left(a+b\cot\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c))**2,x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")

[Out] integrate(1/((b*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(3/2)), x)

Mupad [B]
time = 4.34, size = 2500, normalized size = 5.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))^2),x)

[Out] (2/a + (b*cot(c + d*x)*(2*a^2 + 3*b^2))/(a^2*(a^2 + b^2)))/(b*d*(e*cot(c + d*x))^(3/2) + a*d*e*(e*cot(c + d*x))^(1/2)) - atan((((e*cot(c + d*x))^(1/2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^21*d^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^22*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a^28*b^9*d^5*e^13 - 560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13) + (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(26496*a^25*b^14*d^6*e^15 - 1152*a^15*b^24*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776*a^19*b^20*d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 - ((e*cot(c + d*x))^(1/2)*(1152*a^15*b^26*d^7*e^16 + 13440*a^17*b^24*d^7*e^16 + 69056*a^19*b^22*d^7*e^16 + 202752*a^21*b^20*d^7*e^16 + 372800*a^23*b^18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16 + 337792*a^27*b^14*d^7*e^16 + 156160*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*d^7*e^16 + 3200*a^33*b^8*d^7*e^16 + 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^7*e^16 + 64*a^39*b^2*d^7*e^16) + (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(768*a^16*b^27*d^8*e^18 - (e*cot(c + d*x))^(1/2)*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^25*d^9*e^19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a^26*b^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 84480*a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19 - 22528*a^38*b^7*d^9*e^19 - 5120*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19) + 8704*a^18*b^25*d^8*e^18 + 44288*a^20*b^23*d^8*e^18 + 133120*a^22*b^21*d^8*e^18 + 261120*a^24*b^19*d^8*e^18 + 347136*a^26*b^17*d^8*e^18 + 311808*a^28*b^15*d^8*e^18 + 178176*a^30*b^13*d^8*e^18 + 49920*a^32*b^11*d^8*e^18 - 7680*a^34*b^9*d^8*e^18 - 12032*a^36*b^7*d^8*e^18 - 4096*a^38*b^5*d^8*e^18 - 512*a^40*b^3*d^8*e^18))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2) + 33984*a^27*b^12*d^6*e^15 + 18624*a^29*b^10*d^6*e^15 + 5376*a^31*b^8*d^6*e^15 + 1152*a^33*b^6*d^6*e^15 + 288*a^35*b^4*d^6*e^15 + 32*a^37*b^2*d^6*e^15))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*1i + ((e*cot(c + d*x))^(1/2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^21*d^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17* d^5*e^13 + 3872*a^22*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a^28*b^9*d^5*e^13 - 560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13) - (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(((e*cot(c + d*x))^(1/2)*(1152*a^15*b^26*d^7*e^16 + 13440*a^17*b^24*d^7*e^16 + 69056*a^19*b^22*d^7*e^16 + 202752*a^21*b^20*d^7*e^16 + 372800*a^23*b^18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16 + 337792*a^27*b^14*d^7*e^16 + 156160*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*d^7*e^16 + 3200*a^33*b^8*d^7*e^16 + 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^7*e^16 + 64*a^39*b^2*d^7*e^16) - (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*((e*cot(c + d*x))^(1/2)*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^25*d^9*e^19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a^26*b^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 84480*a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19 - 22528*a^38*b^7*d^9*e^19 - 5120*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19) + 768*a^16*b^27*d^8*e^18 + 8704*a^18*b^25*d^8*e^18 + 44288*a^20*b^23*d^8*e^18 + 133120*a^22*b^21*d^8*e^18 + 261120*a^24*b^19*d^8*e^18 + 347136*a^26*b^17*d^8*e^18 + 311808*a^28*b^15*d^8*e^18 + 178176*a^30*b^13*d^8*e^18 + 49920*a^32*b^11*d^8*e^18 - 7680*a^34*b^9*d^8*e^18 - 12032*a^36*b^7*d^8*e^18 - 4096*a^38*b^5*d^8*e^18 - 512*a^40*b^3*d^8*e^18))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2) - 1152*a^15*b^24*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776*a^19*b^20*d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 + 26496*a^25*b^14*d^6*e^15 + 33984*a^27*b^12*d^6*e^15 + 18624*a^29*b^10*d^6*e^15 + 5376*a^31*b^8*d^6*e^15 + 1152*a^33*b^6*d^6*e^15 + 288*a^35*b^4*d^6*e^15 + 32*a^37*b^2*d^6*e^15))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^21*d^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^22*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a^28*b^9*d^5*e^13 - 560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13) + (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(26496*a^25*b^14*d^6*e^15 - 1152*a^15*b^24*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776*a^19*b^20*d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 - ((e*cot(c + d*x))^...

3.81 $\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx$

Optimal. Leaf size=529

$$\frac{a^{5/2}(15a^4+46a^2b^2+63b^4)\,e^{9/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4b^{7/2}\,(a^2+b^2)^3\,d}+\frac{(a-b)\,(a^2+4ab+b^2)\,e^{9/2}\operatorname{ArcTan}\left(1-\right.}{\sqrt{2}\,(a^2+b^2)^3\,d}$$

```
[Out] 1/4*a^(5/2)*(15*a^4+46*a^2*b^2+63*b^4)*e^(9/2)*arctan(b^(1/2)*(e*cot(d*x+c)
)^(1/2)/a^(1/2)/e^(1/2))/b^(7/2)/(a^2+b^2)^3/d+1/2*a^2*e^2*(e*cot(d*x+c))^(
5/2)/b/(a^2+b^2)/d/(a+b*cot(d*x+c))^2+1/4*a^2*(5*a^2+13*b^2)*e^3*(e*cot(d*x
+c))^(3/2)/b^2/(a^2+b^2)^2/d/(a+b*cot(d*x+c))+1/2*(a-b)*(a^2+4*a*b+b^2)*e^(
9/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1
/2*(a-b)*(a^2+4*a*b+b^2)*e^(9/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1
/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*e^(9/2)*ln(e^(1/2)+cot
(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a+
b)*(a^2-4*a*b+b^2)*e^(9/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x
+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(15*a^4+31*a^2*b^2+8*b^4)*e^4*(e*cot(
d*x+c))^(1/2)/b^3/(a^2+b^2)^2/d
```

Rubi [A]
time = 1.09, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3646, 3726, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{e^{9/2}(a-b)(a^2+4ab+b^2)\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d(a^2+b^2)^3}-\frac{e^{9/2}(a-b)(a^2+4ab+b^2)\operatorname{ArcTan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d(a^2+b^2)^3}-\frac{e^{9/2}(a+b)(a^2-4ab+b^2)\log\left(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d(a^2+b^2)^3}+\frac{e^{9/2}(a+b)(a^2-4ab+b^2)\log\left(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d(a^2+b^2)^3}+\frac{a^2e^3(5a^2+13b^2)(e\cot(c+dx))^{3/2}}{4b^2d(a^2+b^2)^2(a+b\cot(c+dx))}+\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd(a^2+b^2)(a+b\cot(c+dx))^2}-\frac{e^4(15a^4+31a^2b^2+8b^4)\sqrt{e\cot(c+dx)}}{4b^3d(a^2+b^2)^2}+\frac{a^{5/2}e^{9/2}(15a^4+46a^2b^2+63b^4)\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{7/2}d(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(9/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] (a^(5/2)*(15*a^4 + 46*a^2*b^2 + 63*b^4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[
c + d*x]])/(Sqrt[a]*Sqrt[e])])/(4*b^(7/2)*(a^2 + b^2)^3*d) + ((a - b)*(a^2
+ 4*a*b + b^2)*e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
(Sqrt[2]*(a^2 + b^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)*e^(9/2)*ArcTan[1 +
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d) - ((15*
a^4 + 31*a^2*b^2 + 8*b^4)*e^4*Sqrt[e*Cot[c + d*x]])/(4*b^3*(a^2 + b^2)^2*d)
 + (a^2*e^2*(e*Cot[c + d*x])^(5/2))/(2*b*(a^2 + b^2)*d*(a + b*Cot[c + d*x])
^2) + (a^2*(5*a^2 + 13*b^2)*e^3*(e*Cot[c + d*x])^(3/2))/(4*b^2*(a^2 + b^2)^
2*d*(a + b*Cot[c + d*x])) - ((a + b)*(a^2 - 4*a*b + b^2)*e^(9/2)*Log[Sqrt[e
] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 +
 b^2)^3*d) + ((a + b)*(a^2 - 4*a*b + b^2)*e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Cot
[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx = \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\,(a^2+b^2)\,d(a+b\cot(c+dx))^2} - \frac{\int \frac{(e\cot(c+dx))^{3/2}\left(-\frac{5}{2}a^2e^3+2abe^3\cot(c+dx)-\frac{1}{2}(5a^2+}{(a+b\cot(c+dx))^2}}{2b\,(a^2+b^2)}$$

$$= \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\,(a^2+b^2)\,d(a+b\cot(c+dx))^2} + \frac{a^2(5a^2+13b^2)\,e^3(e\cot(c+dx))^{3/2}}{4b^2\,(a^2+b^2)^2\,d(a+b\cot(c+dx))} + \frac{\int}{}$$

$$= -\frac{(15a^4+31a^2b^2+8b^4)\,e^4\sqrt{e\cot(c+dx)}}{4b^3\,(a^2+b^2)^2\,d} + \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\,(a^2+b^2)\,d(a+b\cot(c+dx))}$$

$$= -\frac{(15a^4+31a^2b^2+8b^4)\,e^4\sqrt{e\cot(c+dx)}}{4b^3\,(a^2+b^2)^2\,d} + \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\,(a^2+b^2)\,d(a+b\cot(c+dx))}$$

$$= -\frac{(15a^4+31a^2b^2+8b^4)\,e^4\sqrt{e\cot(c+dx)}}{4b^3\,(a^2+b^2)^2\,d} + \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\,(a^2+b^2)\,d(a+b\cot(c+dx))}$$

$$= -\frac{(15a^4+31a^2b^2+8b^4)\,e^4\sqrt{e\cot(c+dx)}}{4b^3\,(a^2+b^2)^2\,d} + \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\,(a^2+b^2)\,d(a+b\cot(c+dx))}$$

$$= \frac{a^{5/2}(15a^4+46a^2b^2+63b^4)\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4b^{7/2}\,(a^2+b^2)^3\,d} - \frac{(15a^4+31a^2}{}$$

$$= \frac{a^{5/2}(15a^4+46a^2b^2+63b^4)\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4b^{7/2}\,(a^2+b^2)^3\,d} - \frac{(15a^4+31a^2}{}$$

$$= \frac{a^{5/2}(15a^4+46a^2b^2+63b^4)\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4b^{7/2}\,(a^2+b^2)^3\,d} + \frac{(a-b)\,(a^2+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.26, size = 556, normalized size = 1.05

$$-\frac{(e\cot(c+dx))^{9/2}\left(\frac{2b(3a^2-b^2)\cot^{\frac{9}{2}}(c+dx)}{9(a^2+b^2)^3} - \frac{2a(3a^2-b^2)\left(15\cot^{\frac{7}{2}}(c+dx)-7a\left(\frac{3\cot^{\frac{5}{2}}(c+dx)}{b}-\frac{5a\left(-\frac{3a\left(-\frac{\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}}+\frac{\sqrt{\cot(c+dx)}}{b}\right)}{b}+\frac{\cot^{\frac{3}{2}}(c+dx)}{3b}\right)}{b}\right)\right)}{105(a^2+b^2)^3} - \frac{2a(a^2-3b^2)\left(7\cot^{\frac{3}{2}}(c+dx)-3\cot^{\frac{7}{2}}(c+dx)-7\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)}{21(a^2+b^2)^3} + \frac{4b^2\cot^{\frac{11}{2}}(c+dx)\,{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};-\frac{b\cot(c+dx)}{a}\right)}{11a(a^2+b^2)^2} + \frac{2b^2\cot^{\frac{11}{2}}(c+dx)\,{}_2F_1\left(3,\frac{11}{2};\frac{13}{2};-\frac{b\cot(c+dx)}{a}\right)}{11a^3(a^2+b^2)} - \frac{b(3a^2-b^2)\left(360\sqrt{\cot(c+dx)}-72\cot^{\frac{5}{2}}(c+dx)+40\cot^{\frac{9}{2}}(c+dx)+45\left(2\left(\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\right)+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{180(a^2+b^2)^3}\right)}{d\cot^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(9/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -(((e*Cot[c + d*x])^(9/2)*((2*b*(3*a^2 - b^2)*Cot[c + d*x]^(9/2))/(9*(a^2 +
 b^2)^3) - (2*a*(3*a^2 - b^2)*(15*Cot[c + d*x]^(7/2) - 7*a*((3*Cot[c + d*x]
^(5/2))/b - (5*a*((-3*a*(-((Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqr
t[a]])/b^(3/2)) + Sqrt[Cot[c + d*x]]/b))/b + Cot[c + d*x]^(3/2)/b))/b)))/(1
05*(a^2 + b^2)^3) - (2*a*(a^2 - 3*b^2)*(7*Cot[c + d*x]^(3/2) - 3*Cot[c + d*
x]^(7/2) - 7*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x
]^2]))/(21*(a^2 + b^2)^3) + (4*b^2*Cot[c + d*x]^(11/2)*Hypergeometric2F1[2,
 11/2, 13/2, -((b*Cot[c + d*x])/a)])/(11*a*(a^2 + b^2)^2) + (2*b^2*Cot[c +
d*x]^(11/2)*Hypergeometric2F1[3, 11/2, 13/2, -((b*Cot[c + d*x])/a)])/(11*a^
3*(a^2 + b^2)) - (b*(3*a^2 - b^2)*(360*Sqrt[Cot[c + d*x]] - 72*Cot[c + d*x]
^(5/2) + 40*Cot[c + d*x]^(9/2) + 45*(2*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot
[c + d*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) + Sqrt[2]*Log
[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*S
qrt[Cot[c + d*x]] + Cot[c + d*x]])))/(180*(a^2 + b^2)^3)))/(d*Cot[c + d*x]^
(9/2)))
```

Maple [A]
time = 0.71, size = 471, normalized size = 0.89

method	result
derivativedivides	$-\frac{2e^4\left(\frac{\sqrt{e\cot(dx+c)}}{b^3}+\frac{e\left(\frac{(3a^2be-b^3e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right.\right.}{}\right.}{}\right.}{}$
default	$-\frac{2e^4\left(\frac{\sqrt{e\cot(dx+c)}}{b^3}+\frac{e\left(\frac{(3a^2be-b^3e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right.\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(9/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^4*(1/b^3*(e*cot(d*x+c))^(1/2)+e/(a^2+b^2)^3*(1/8*(3*a^2*b*e-b^3*e)*(
e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2
^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan
(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(a^3-3*a*b^2)/(e^2)^(1/4
)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^
(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))
+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-a^3*e/b^3/(a^2+b^2)^3*(((-9/8*a^4*b-13/4
*a^2*b^3-17/8*b^5)*(e*cot(d*x+c))^(3/2)-1/8*a*e*(7*a^4+22*a^2*b^2+15*b^4)*(
e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)*b+a*e)^2+1/8*(15*a^4+46*a^2*b^2+63*b^4)/
(a*e*b)^(1/2)*arctan(b*(e*cot(d*x+c))^(1/2)/(a*e*b)^(1/2))))
```

Maxima [A]
time = 0.52, size = 442, normalized size = 0.84

$$\frac{\left(\frac{(15a^7+46a^5b^2+63a^3b^4)\arctan\left(\frac{b}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^6b^3+3a^4b^5+3a^2b^7+b^9)\sqrt{ab}} - \frac{2\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{\frac{7a^6+15a^4b^2}{\sqrt{\tan(dx+c)}}+\frac{9a^5b+17a^3b^3}{\tan(dx+c)^{\frac{3}{2}}}}{a^6b^3+2a^4b^5+a^2b^7+\frac{2(a^5b^4+2a^3b^6+ab^8)}{\tan(dx+c)}+\frac{a^4b^5+2a^2b^7+b^9}{\tan(dx+c)^2}} - \frac{8}{b^3\sqrt{\tan(dx+c)}}\right)e^{\frac{9}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(9/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((15*a^7 + 46*a^5*b^2 + 63*a^3*b^4)*arctan(b/(sqrt(a*b)*sqrt(tan(d*x +
c))))/((a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*sqrt(a*b)) - (2*sqrt(2)*(a^3
 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x +
c)))) + 2*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(sqrt
(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(
sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^3 - 3*a^2*b -
 3*a*b^2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((7*a^6 + 15*a^4*b^2)/sqrt(tan(d*x + c))
+ (9*a^5*b + 17*a^3*b^3)/tan(d*x + c)^(3/2))/(a^6*b^3 + 2*a^4*b^5 + a^2*b^7
 + 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)/tan(d*x + c) + (a^4*b^5 + 2*a^2*b^7 + b^
9)/tan(d*x + c)^2) - 8/(b^3*sqrt(tan(d*x + c))))*e^(9/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(9/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(9/2)/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(9/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(9/2)/(b*cot(d*x + c) + a)^3, x)
```

Mupad [B]
time = 10.40, size = 2500, normalized size = 4.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(9/2)/(a + b*cot(c + d*x))^3,x)
```

```
[Out] atan(((((((128*a*b^26*d^4*e^15 + 3648*a^3*b^24*d^4*e^15 + 25536*a^5*b^22*d^
4*e^15 + 88320*a^7*b^20*d^4*e^15 + 182784*a^9*b^18*d^4*e^15 + 244608*a^11*b
^16*d^4*e^15 + 217728*a^13*b^14*d^4*e^15 + 128256*a^15*b^12*d^4*e^15 + 4800
0*a^17*b^10*d^4*e^15 + 10304*a^19*b^8*d^4*e^15 + 960*a^21*b^6*d^4*e^15)/(b^
21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d
^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) +
((e*cot(c + d*x))^(1/2)*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a
^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(5
12*b^30*d^4*e^10 + 4608*a^2*b^28*d^4*e^10 + 17920*a^4*b^26*d^4*e^10 + 38400
*a^6*b^24*d^4*e^10 + 46080*a^8*b^22*d^4*e^10 + 21504*a^10*b^20*d^4*e^10 - 2
1504*a^12*b^18*d^4*e^10 - 46080*a^14*b^16*d^4*e^10 - 38400*a^16*b^14*d^4*e^
10 - 17920*a^18*b^12*d^4*e^10 - 4608*a^20*b^10*d^4*e^10 - 512*a^22*b^8*d^4*
e^10))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*
a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b
^5*d^4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 1
5*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x
))^(1/2)*(1800*a^23*b*d^2*e^19 - 1472*a*b^23*d^2*e^19 - 1024*a^3*b^21*d^2*e
^19 + 8448*a^5*b^19*d^2*e^19 + 46088*a^7*b^17*d^2*e^19 + 177344*a^9*b^15*d^
2*e^19 + 402912*a^11*b^13*d^2*e^19 + 541632*a^13*b^11*d^2*e^19 + 455472*a^1
5*b^9*d^2*e^19 + 248064*a^17*b^7*d^2*e^19 + 87008*a^19*b^5*d^2*e^19 + 18240
*a^21*b^3*d^2*e^19))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*
b^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^
```

7*d^4 + a^16*b^5*d^4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^
5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + (
2250*a^20*b*d^2*e^24 + 32*a^2*b^19*d^2*e^24 + 12288*a^4*b^17*d^2*e^24 - 109
74*a^6*b^15*d^2*e^24 - 105162*a^8*b^13*d^2*e^24 - 150758*a^10*b^11*d^2*e^24
 - 85314*a^12*b^9*d^2*e^24 - 3578*a^14*b^7*d^2*e^24 + 22210*a^16*b^5*d^2*e^
24 + 11550*a^18*b^3*d^2*e^24)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5
+ 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 +
8*a^14*b^7*d^5 + a^16*b^5*d^5))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^
2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^
(1/2) + ((e*cot(c + d*x))^(1/2)*(32*b^18*e^28 - 225*a^18*e^28 + 128*a^2*b^1
6*e^28 + 192*a^4*b^14*e^28 - 3841*a^6*b^12*e^28 + 18050*a^8*b^10*e^28 + 268
01*a^10*b^8*e^28 + 16860*a^12*b^6*e^28 + 4049*a^14*b^4*e^28 - 30*a^16*b^2*e
^28))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a
^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^
5*d^4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15
*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*1i - (((((128*a*b^
26*d^4*e^15 + 3648*a^3*b^24*d^4*e^15 + 25536*a^5*b^22*d^4*e^15 + 88320*a^7*
b^20*d^4*e^15 + 182784*a^9*b^18*d^4*e^15 + 244608*a^11*b^16*d^4*e^15 + 2177
28*a^13*b^14*d^4*e^15 + 128256*a^15*b^12*d^4*e^15 + 48000*a^17*b^10*d^4*e^1
5 + 10304*a^19*b^8*d^4*e^15 + 960*a^21*b^6*d^4*e^15)/(b^21*d^5 + 8*a^2*b^19
*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b^13*d^5 + 56*a^10*b^11*d
^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d^5) - ((e*cot(c + d*x))^(
1/2)*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^
2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^30*d^4*e^10 +
4608*a^2*b^28*d^4*e^10 + 17920*a^4*b^26*d^4*e^10 + 38400*a^6*b^24*d^4*e^10
+ 46080*a^8*b^22*d^4*e^10 + 21504*a^10*b^20*d^4*e^10 - 21504*a^12*b^18*d^4*
e^10 - 46080*a^14*b^16*d^4*e^10 - 38400*a^16*b^14*d^4*e^10 - 17920*a^18*b^1
2*d^4*e^10 - 4608*a^20*b^10*d^4*e^10 - 512*a^22*b^8*d^4*e^10))/(b^21*d^4 +
8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b^13*d^4 + 56*a
^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^4))*(-(e^9*1i)
/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3
*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(1800*a^23
*b*d^2*e^19 - 1472*a*b^23*d^2*e^19 - 1024*a^3*b^21*d^2*e^19 + 8448*a^5*b^19
*d^2*e^19 + 46088*a^7*b^17*d^2*e^19 + 177344*a^9*b^15*d^2*e^19 + 402912*a^1
1*b^13*d^2*e^19 + 541632*a^13*b^11*d^2*e^19 + 455472*a^15*b^9*d^2*e^19 + 24
8064*a^17*b^7*d^2*e^19 + 87008*a^19*b^5*d^2*e^19 + 18240*a^21*b^3*d^2*e^19)
)/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b^15*d^4 + 70*a^8*b
^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^7*d^4 + a^16*b^5*d^
4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2
*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + (2250*a^20*b*d^2*e^2
4 + 32*a^2*b^19*d^2*e^24 + 12288*a^4*b^17*d^2*e^24 - 10974*a^6*b^15*d^2*e^2
4 - 105162*a^8*b^13*d^2*e^24 - 150758*a^10*b^11*d^2*e^24 - 85314*a^12*b^9*d
^2*e^24 - 3578*a^14*b^7*d^2*e^24 + 22210*a^16*b^5*d^2*e^24 + 11550*a^18*b^3
*d^2*e^24)/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 +
 70*a^8*b^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^...

3.82 $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx$

Optimal. Leaf size=476

$$-\frac{a^{3/2}(3a^4+6a^2b^2+35b^4)\,e^{7/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4b^{5/2}\,(a^2+b^2)^3\,d}+\frac{(a+b)\,(a^2-4ab+b^2)\,e^{7/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}\,(a^2+b^2)^3\,d}$$

```
[Out] -1/4*a^(3/2)*(3*a^4+6*a^2*b^2+35*b^4)*e^(7/2)*arctan(b^(1/2)*(e*cot(d*x+c))
^(1/2)/a^(1/2)/e^(1/2))/b^(5/2)/(a^2+b^2)^3/d+1/2*a^2*e^2*(e*cot(d*x+c))^(3
/2)/b/(a^2+b^2)/d/(a+b*cot(d*x+c))^2+1/2*(a+b)*(a^2-4*a*b+b^2)*e^(7/2)*arct
an(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/2*(a+b)*
(a^2-4*a*b+b^2)*e^(7/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2
+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*e^(7/2)*ln(e^(1/2)+cot(d*x+c)*e
^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+4
*a*b+b^2)*e^(7/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2
))/(a^2+b^2)^3/d*2^(1/2)+1/4*a^2*(3*a^2+11*b^2)*e^3*(e*cot(d*x+c))^(1/2)/b^
2/(a^2+b^2)^2/d/(a+b*cot(d*x+c))
```

Rubi [A]
time = 0.81, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3646, 3726, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{e^{7/2}(a+b)\,(a^2-4ab+b^2)\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\,(a^2+b^2)^3}-\frac{e^{7/2}(a+b)\,(a^2-4ab+b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^3}+\frac{e^{7/2}(a-b)\,(a^2+4ab+b^2)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^3}-\frac{e^{7/2}(a-b)\,(a^2+4ab+b^2)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^3}+\frac{a^2e^3(3a^2+11b^2)\,\sqrt{e\cot(c+dx)}}{4b^2d\,(a^2+b^2)^2\,(a+b\cot(c+dx))}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\,(a^2+b^2)\,(a+b\cot(c+dx))^2}-\frac{a^{3/2}e^{7/2}(3a^4+6a^2b^2+35b^4)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4b^{5/2}d\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(7/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -1/4*(a^(3/2)*(3*a^4 + 6*a^2*b^2 + 35*b^4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*C
ot[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(b^(5/2)*(a^2 + b^2)^3*d) + ((a + b)*(a^2
 - 4*a*b + b^2)*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])
/(Sqrt[2]*(a^2 + b^2)^3*d) - ((a + b)*(a^2 - 4*a*b + b^2)*e^(7/2)*ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (a^2
*e^2*(e*Cot[c + d*x])^(3/2))/(2*b*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2) + (
a^2*(3*a^2 + 11*b^2)*e^3*Sqrt[e*Cot[c + d*x]])/(4*b^2*(a^2 + b^2)^2*d*(a +
b*Cot[c + d*x])) + ((a - b)*(a^2 + 4*a*b + b^2)*e^(7/2)*Log[Sqrt[e] + Sqrt[
e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d
) - ((a - b)*(a^2 + 4*a*b + b^2)*e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x]
 + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx = \frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} - \frac{\int \frac{\sqrt{e\cot(c+dx)}\left(-\frac{3}{2}a^2e^3+2abe^3\cot(c+dx)-\frac{1}{2}(\right.}{(a+b\cot(c+dx))^2}}{2b\left(a^2+b^2\right)}$$

$$= \frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2\left(3a^2+11b^2\right)e^3\sqrt{e\cot(c+dx)}}{4b^2\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} + \frac{\int \frac{1}{4}a}{}$$

$$= \frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2\left(3a^2+11b^2\right)e^3\sqrt{e\cot(c+dx)}}{4b^2\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} + \frac{\int \frac{2ab}{}}{}$$

$$= \frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2\left(3a^2+11b^2\right)e^3\sqrt{e\cot(c+dx)}}{4b^2\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} + \frac{\text{Subs}}{}$$

$$= \frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{a^2\left(3a^2+11b^2\right)e^3\sqrt{e\cot(c+dx)}}{4b^2\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} - \frac{(a^2(}{}$$

$$= -\frac{a^{3/2}\left(3a^4+6a^2b^2+35b^4\right)e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{5/2}\left(a^2+b^2\right)^3d} + \frac{a^2e^2(e\,c}{2b\left(a^2+b^2\right)d}$$

$$= -\frac{a^{3/2}\left(3a^4+6a^2b^2+35b^4\right)e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{5/2}\left(a^2+b^2\right)^3d} + \frac{a^2e^2(e\,c}{2b\left(a^2+b^2\right)d}$$

$$= -\frac{a^{3/2}\left(3a^4+6a^2b^2+35b^4\right)e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{5/2}\left(a^2+b^2\right)^3d} + \frac{(a+b)\left(a^2-\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 6.19, size = 525, normalized size = 1.10

$$-\frac{(e\cot(c+dx))^{7/2}\left(\frac{2b(3a^2-b^2)\cot^{\frac{7}{2}}(c+dx)}{7(a^2+b^2)^3}-\frac{2a(3a^2-b^2)\left(3\cot^{\frac{5}{2}}(c+dx)-5a\left(-\frac{3a\left(-\frac{\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}}+\frac{\sqrt{\cot(c+dx)}}{b}\right)}{b}+\frac{\cot^{\frac{3}{2}}(c+dx)}{b}\right)\right)}{15(a^2+b^2)^3}+\frac{2b(3a^2-b^2)\left(7\cot^{\frac{3}{2}}(c+dx)-3\cot^{\frac{7}{2}}(c+dx)-7\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)}{21(a^2+b^2)^3}+\frac{4b^2\cot^{\frac{9}{2}}(c+dx)\,{}_2F_1\left(2,\frac{9}{2};\frac{11}{2};-\frac{b\cot(c+dx)}{a}\right)}{9a(a^2+b^2)^2}+\frac{2b^2\cot^{\frac{9}{2}}(c+dx)\,{}_2F_1\left(3,\frac{9}{2};\frac{11}{2};-\frac{b\cot(c+dx)}{a}\right)}{9a^3(a^2+b^2)}-\frac{a(a^2-3b^2)\left(40\sqrt{\cot(c+dx)}-8\cot^{\frac{5}{2}}(c+dx)+\frac{5}{2}\left(4\left(\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\right)+2\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-2\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{20(a^2+b^2)^3}\right)}{d\cot^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(7/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -(((e*Cot[c + d*x])^(7/2)*((2*b*(3*a^2 - b^2)*Cot[c + d*x]^(7/2))/(7*(a^2 +
 b^2)^3) - (2*a*(3*a^2 - b^2)*(3*Cot[c + d*x]^(5/2) - 5*a*((-3*a*(-((Sqrt[a
]*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]])/b^(3/2)) + Sqrt[Cot[c + d*x
]]/b))/b + Cot[c + d*x]^(3/2)/b)))/(15*(a^2 + b^2)^3) + (2*b*(3*a^2 - b^2)*
(7*Cot[c + d*x]^(3/2) - 3*Cot[c + d*x]^(7/2) - 7*Cot[c + d*x]^(3/2)*Hyperge
ometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(21*(a^2 + b^2)^3) + (4*b^2*Cot[
c + d*x]^(9/2)*Hypergeometric2F1[2, 9/2, 11/2, -((b*Cot[c + d*x])/a)])/(9*a
*(a^2 + b^2)^2) + (2*b^2*Cot[c + d*x]^(9/2)*Hypergeometric2F1[3, 9/2, 11/2,
 -((b*Cot[c + d*x])/a)])/(9*a^3*(a^2 + b^2)) - (a*(a^2 - 3*b^2)*(40*Sqrt[Co
t[c + d*x]] - 8*Cot[c + d*x]^(5/2) + (5*(4*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) + 2*Sqrt[
2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 2*Sqrt[2]*Log[1 + S
qrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/2))/(20*(a^2 + b^2)^3)))/(d*Cot
[c + d*x]^(7/2)))
```

Maple [A]

time = 0.66, size = 460, normalized size = 0.97

method	result
derivativedivides	$-2e^4\left(\frac{\frac{(a^3e-3ab^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)\ldots}\right.\ldots}{8e^2}\ldots}{\ldots}\right.$

default	$-\frac{2e^4\left(\frac{\frac{\left(a^3e-3a\,b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cdots}}{(\cdots}\right.\right.}{8e^2}}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^4*(1/(a^2+b^2)^3*(1/8*(a^3*e-3*a*b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln(
(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(
d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1
/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)+1))+1/8*(-3*a^2*b+b^3)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c
)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)+1)))-a^2/(a^2+b^2)^3*((1/8*(5*a^4+18*a^2*b^2+13*b^4)/b*(e*cot(d*x+c))^(
3/2)+1/8*a*e*(3*a^4+14*a^2*b^2+11*b^4)/b^2*(e*cot(d*x+c))^(1/2))/(e*cot(d*x
+c)*b+a*e)^2-1/8*(3*a^4+6*a^2*b^2+35*b^4)/b^2/(a*e*b)^(1/2)*arctan(b*(e*cot
(d*x+c))^(1/2)/(a*e*b)^(1/2))))
```

Maxima [A]
time = 0.52, size = 428, normalized size = 0.90

$$-\frac{\left(\frac{\left(3a^6+6a^4b^2+35a^2b^4\right)\arctan\left(\frac{b}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{\left(a^6b^2+3a^4b^4+3a^2b^6+b^8\right)\sqrt{ab}}+\frac{2\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\left(a^3-3a^2b-3ab^2+b^3\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}\left(a^3+3a^2b-3ab^2-b^3\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\frac{3a^5+11a^3b^2}{\sqrt{\tan(dx+c)}}+\frac{5a^4b+13a^2b^3}{\tan(dx+c)^{\frac{3}{2}}}}{a^6b^2+2a^4b^4+a^2b^6+\frac{2\left(a^5b^3+2a^3b^5+ab^7\right)}{\tan(dx+c)}+\frac{a^4b^4+2a^2b^6+b^8}{\tan(dx+c)^2}}\right)e^{\frac{7}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((3*a^6 + 6*a^4*b^2 + 35*a^2*b^4)*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c
))))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*sqrt(a*b)) + (2*sqrt(2)*(a^3
- 3*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c
)))) + 2*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(
2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(a^3 + 3*a^2*b -
3*a*b^2 - b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((3*a^5 + 11*a^3*b^2)/sqrt(tan(d*x + c)) +
 (5*a^4*b + 13*a^2*b^3)/tan(d*x + c)^(3/2))/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6
+ 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)/tan(d*x + c) + (a^4*b^4 + 2*a^2*b^6 + b^8
)/tan(d*x + c)^2))*e^(7/2)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(7/2)/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(7/2)/(b*cot(d*x + c) + a)^3, x)
```

Mupad [**B**]
time = 7.26, size = 2500, normalized size = 5.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(7/2)/(a + b*cot(c + d*x))^3,x)
```

```
[Out] (((e*cot(c + d*x))^(1/2)*(3*a^5*e^5 + 11*a^3*b^2*e^5))/(4*b^2*(a^4 + b^4 +
2*a^2*b^2)) + (e^4*(e*cot(c + d*x))^(3/2)*(5*a^4 + 13*a^2*b^2))/(4*b*(a^4 +
 b^4 + 2*a^2*b^2)))/(a^2*d*e^2 + b^2*d*e^2*cot(c + d*x)^2 + 2*a*b*d*e^2*cot
(c + d*x)) - atan(((((32*a*b^18*d^2*e^21 - 18*a^19*d^2*e^21 - 6528*a^3*b^16
*d^2*e^21 + 2758*a^5*b^14*d^2*e^21 + 26482*a^7*b^12*d^2*e^21 + 21582*a^9*b^
10*d^2*e^21 + 7594*a^11*b^8*d^2*e^21 + 3314*a^13*b^6*d^2*e^21 + 246*a^15*b^
4*d^2*e^21 + 90*a^17*b^2*d^2*e^21)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15
*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^
```

5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) + (((1600*a^2*b^23*d^4*e^14 + 12864*a^4*
b^21*d^4*e^14 + 45312*a^6*b^19*d^4*e^14 + 91392*a^8*b^17*d^4*e^14 + 115584*
a^10*b^15*d^4*e^14 + 94080*a^12*b^13*d^4*e^14 + 48384*a^14*b^11*d^4*e^14 +
14592*a^16*b^9*d^4*e^14 + 2112*a^18*b^7*d^4*e^14 + 64*a^20*b^5*d^4*e^14)/(b
^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*
d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) +
((e*cot(c + d*x))^(1/2)*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^
5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(51
2*b^28*d^4*e^10 + 4608*a^2*b^26*d^4*e^10 + 17920*a^4*b^24*d^4*e^10 + 38400*
a^6*b^22*d^4*e^10 + 46080*a^8*b^20*d^4*e^10 + 21504*a^10*b^18*d^4*e^10 - 21
504*a^12*b^16*d^4*e^10 - 46080*a^14*b^14*d^4*e^10 - 38400*a^16*b^12*d^4*e^1
0 - 17920*a^18*b^10*d^4*e^10 - 4608*a^20*b^8*d^4*e^10 - 512*a^22*b^6*d^4*e^
10))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^
8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*
d^4))*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^
2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(
1/2)*(1472*a*b^21*d^2*e^17 + 72*a^21*b*d^2*e^17 + 1024*a^3*b^19*d^2*e^17 +
1352*a^5*b^17*d^2*e^17 + 28224*a^7*b^15*d^2*e^17 + 70240*a^9*b^13*d^2*e^17
+ 72640*a^11*b^11*d^2*e^17 + 39088*a^13*b^9*d^2*e^17 + 13248*a^15*b^7*d^2*e
^17 + 3488*a^17*b^5*d^2*e^17 + 576*a^19*b^3*d^2*e^17))/(b^19*d^4 + 8*a^2*b^
17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*
d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((e^7*1i)/(4*(b^6*d
^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*2
0i + 15*a^4*b^2*d^2)))^(1/2))*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6
i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/
2) + ((e*cot(c + d*x))^(1/2)*(9*a^16*e^24 + 32*b^16*e^24 + 128*a^2*b^14*e^2
4 + 1417*a^4*b^12*e^24 - 6802*a^6*b^10*e^24 - 1017*a^8*b^8*e^24 - 1020*a^10
*b^6*e^24 + 39*a^12*b^4*e^24 - 18*a^14*b^2*e^24))/(b^19*d^4 + 8*a^2*b^17*d^
4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 +
28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((e^7*1i)/(4*(b^6*d^2 -
a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i +
15*a^4*b^2*d^2)))^(1/2)*1i - (((32*a*b^18*d^2*e^21 - 18*a^19*d^2*e^21 - 652
8*a^3*b^16*d^2*e^21 + 2758*a^5*b^14*d^2*e^21 + 26482*a^7*b^12*d^2*e^21 + 21
582*a^9*b^10*d^2*e^21 + 7594*a^11*b^8*d^2*e^21 + 3314*a^13*b^6*d^2*e^21 + 2
46*a^15*b^4*d^2*e^21 + 90*a^17*b^2*d^2*e^21)/(b^19*d^5 + 8*a^2*b^17*d^5 + 2
8*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a
^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) + (((1600*a^2*b^23*d^4*e^14 +
12864*a^4*b^21*d^4*e^14 + 45312*a^6*b^19*d^4*e^14 + 91392*a^8*b^17*d^4*e^14
+ 115584*a^10*b^15*d^4*e^14 + 94080*a^12*b^13*d^4*e^14 + 48384*a^14*b^11*d
^4*e^14 + 14592*a^16*b^9*d^4*e^14 + 2112*a^18*b^7*d^4*e^14 + 64*a^20*b^5*d^
4*e^14)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70
*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b
^3*d^5) - ((e*cot(c + d*x))^(1/2)*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d
^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))
^(1/2)*(512*b^28*d^4*e^10 + 4608*a^2*b^26*d^4*e^10 + 17920*a^4*b^24*d^4*e^1

```
0 + 38400*a^6*b^22*d^4*e^10 + 46080*a^8*b^20*d^4*e^10 + 21504*a^10*b^18*d^4
*e^10 - 21504*a^12*b^16*d^4*e^10 - 46080*a^14*b^14*d^4*e^10 - 38400*a^16*b^
12*d^4*e^10 - 17920*a^18*b^10*d^4*e^10 - 4608*a^20*b^8*d^4*e^10 - 512*a^22*
b^6*d^4*e^10))/(b^19*d^4 + 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d
^4 + 70*a^8*b^11*d^4 + 56*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 +
 a^16*b^3*d^4))*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*
6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c
 + d*x))^(1/2)*(1472*a*b^21*d^2*e^17 + 72*a^21*b*d^2*e^17 + 1024*a^3*b^19*d
^2*e^17 + 1352*a^5*b^17*d^2*e^17 + 28224*a^7*b^15*d^2*e^17 + 70240*a^9*b^13
*d^2*e^17 + 72640*a^11*b^11*d^2*e^17 + 39088*a^13*b^9*d^2*e^17 + 13248*a^15
*b^7*d^2*e^17 + 3488*a^17*b^5*d^2*e^17 + 576*a^19*b^3*d^2*e^17))/(b^19*d^4
+ 8*a^2*b^17*d^4 + 28*a^4*b^15*d^4 + 56*a^6*b^13*d^4 + 70*a^8*b^11*d^4 + 56
*a^10*b^9*d^4 + 28*a^12*b^7*d^4 + 8*a^14*b^5*d^4 + a^16*b^3*d^4))*((e^7*1i)
/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d...
```

3.83 $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx$

Optimal. Leaf size=470

$$-\frac{\sqrt{a}\,(a^4+18a^2b^2-15b^4)\,e^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4b^{3/2}\,(a^2+b^2)^3\,d}-\frac{(a-b)\,(a^2+4ab+b^2)\,e^{5/2}\text{ArcTan}\left(1-\right.}{\sqrt{2}\,(a^2+b^2)^3\,d}$$

```
[Out] -1/2*(a-b)*(a^2+4*a*b+b^2)*e^(5/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^
(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/2*(a-b)*(a^2+4*a*b+b^2)*e^(5/2)*arctan(1+2^(
1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a+b)*(a^2-4*a
*b+b^2)*e^(5/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))
/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*e^(5/2)*ln(e^(1/2)+cot(d*x
+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a^4+18
*a^2*b^2-15*b^4)*e^(5/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2
))*a^(1/2)/b^(3/2)/(a^2+b^2)^3/d+1/2*a^2*e^2*(e*cot(d*x+c))^(1/2)/b/(a^2+b^
2)/d/(a+b*cot(d*x+c))^2-1/4*a*(a^2+9*b^2)*e^2*(e*cot(d*x+c))^(1/2)/b/(a^2+b
^2)^2/d/(a+b*cot(d*x+c))
```

Rubi [A]
time = 0.83, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3646, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{e^{5/2}(a-b)\,(a^2+4ab+b^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\,(a^2+b^2)^3}+\frac{e^{5/2}(a-b)\,(a^2+4ab+b^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^3}+\frac{e^{5/2}(a+b)\,(a^2-4ab+b^2)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^3}-\frac{e^{5/2}(a+b)\,(a^2-4ab+b^2)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^3}-\frac{ae^2(a^2+9b^2)\,\sqrt{e\cot(c+dx)}}{4bd\,(a^2+b^2)^2\,(a+b\cot(c+dx))}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\,(a^2+b^2)\,(a+b\cot(c+dx))^2}-\frac{\sqrt{a}\,e^{5/2}(a^4+18a^2b^2-15b^4)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4b^{3/2}d\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -1/4*(Sqrt[a]*(a^4 + 18*a^2*b^2 - 15*b^4)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Co
t[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(b^(3/2)*(a^2 + b^2)^3*d) - ((a - b)*(a^2
+ 4*a*b + b^2)*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/
(Sqrt[2]*(a^2 + b^2)^3*d) + ((a - b)*(a^2 + 4*a*b + b^2)*e^(5/2)*ArcTan[1 +
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (a^2*
e^2*Sqrt[e*Cot[c + d*x]])/(2*b*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2) - (a*(
a^2 + 9*b^2)*e^2*Sqrt[e*Cot[c + d*x]])/(4*b*(a^2 + b^2)^2*d*(a + b*Cot[c +
d*x])) + ((a + b)*(a^2 - 4*a*b + b^2)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c +
 d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((a +
b)*(a^2 - 4*a*b + b^2)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]
*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3646

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx = \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}a^2e^3+2abe^3\cot(c+dx)-\frac{1}{2}\left(a^2+4b^2\right)e^3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^2}\,d}{2b\left(a^2+b^2\right)}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} - \frac{a\left(a^2+9b^2\right)e^2\sqrt{e\cot(c+dx)}}{4b\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} + \frac{\int \frac{1}{4}a^2(a}{}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} - \frac{a\left(a^2+9b^2\right)e^2\sqrt{e\cot(c+dx)}}{4b\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} + \frac{\int \frac{-2ab^2}{}}{}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} - \frac{a\left(a^2+9b^2\right)e^2\sqrt{e\cot(c+dx)}}{4b\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} + \frac{\text{Subst}\Big(}{}$$

$$= \frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} - \frac{a\left(a^2+9b^2\right)e^2\sqrt{e\cot(c+dx)}}{4b\left(a^2+b^2\right)^2d(a+b\cot(c+dx))} - \frac{(a(a^4 +}{}$$

$$= -\frac{\sqrt{a}\left(a^4+18a^2b^2-15b^4\right)e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{3/2}\left(a^2+b^2\right)^3d} + \frac{a^2e^2\sqrt{e}}{2b\left(a^2+b^2\right)d(}$$

$$= -\frac{\sqrt{a}\left(a^4+18a^2b^2-15b^4\right)e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{3/2}\left(a^2+b^2\right)^3d} + \frac{a^2e^2\sqrt{e}}{2b\left(a^2+b^2\right)d(}$$

$$= -\frac{\sqrt{a}\left(a^4+18a^2b^2-15b^4\right)e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{3/2}\left(a^2+b^2\right)^3d} - \frac{(a-b)\left(a^2+4\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 6.17, size = 488, normalized size = 1.04

$$-\frac{(e\cot(c+dx))^{5/2}\left(\frac{2b(3a^2-b^2)\cot^{\frac{5}{2}}(c+dx)}{5(a^2+b^2)^3}-\frac{2a(3a^2-b^2)\left(-3a\left(-\frac{\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}}+\frac{\sqrt{\cot(c+dx)}}{b}\right)+\cot^{\frac{3}{2}}(c+dx)\right)}{3(a^2+b^2)^3}+\frac{2a(a^2-3b^2)\left(\cot^{\frac{3}{2}}(c+dx)-\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)}{3(a^2+b^2)^3}+\frac{4b^2\cot^{\frac{7}{2}}(c+dx)\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{b\cot(c+dx)}{a}\right)}{7a(a^2+b^2)^2}+\frac{2b^2\cot^{\frac{7}{2}}(c+dx)\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};-\frac{b\cot(c+dx)}{a}\right)}{7a^3(a^2+b^2)}+\frac{b(3a^2-b^2)\left(40\sqrt{\cot(c+dx)}-8\cot^{\frac{5}{2}}(c+dx)+\frac{5}{2}\left(4\left(\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}\right)\right)+2\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-2\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)\right)}{20(a^2+b^2)^3}\right)}{d\cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -(((e*Cot[c + d*x])^(5/2)*((2*b*(3*a^2 - b^2)*Cot[c + d*x]^(5/2))/(5*(a^2 +
 b^2)^3) - (2*a*(3*a^2 - b^2)*(-3*a*(-((Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[Cot[c
+ d*x]])/Sqrt[a]])/b^(3/2)) + Sqrt[Cot[c + d*x]]/b) + Cot[c + d*x]^(3/2)))/
(3*(a^2 + b^2)^3) + (2*a*(a^2 - 3*b^2)*(Cot[c + d*x]^(3/2) - Cot[c + d*x]^(
3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(3*(a^2 + b^2)^3) +
(4*b^2*Cot[c + d*x]^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*Cot[c + d*x])
/a)])/(7*a*(a^2 + b^2)^2) + (2*b^2*Cot[c + d*x]^(7/2)*Hypergeometric2F1[3,
7/2, 9/2, -((b*Cot[c + d*x])/a)])/(7*a^3*(a^2 + b^2)) + (b*(3*a^2 - b^2)*(4
0*Sqrt[Cot[c + d*x]] - 8*Cot[c + d*x]^(5/2) + (5*(4*(Sqrt[2]*ArcTan[1 - Sqr
t[2]*Sqrt[Cot[c + d*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]])
+ 2*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 2*Sqrt[2]*
Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/2))/(20*(a^2 + b^2)^3)
))/(d*Cot[c + d*x]^(5/2)))
```

Maple [A]

time = 0.70, size = 457, normalized size = 0.97

method	result
derivativedivides	$-2e^4\left(\frac{a\left(\frac{\left(\frac{1}{8}a^4+\frac{5}{4}a^2b^2+\frac{9}{8}b^4\right)(e\cot(dx+c))^{\frac{3}{2}}-\frac{ae\left(a^4-6a^2b^2-7b^4\right)\sqrt{e\cot(dx+c)}}{8b}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(a^4+18a^2b^2-15b^4\right)\arctan\left(\frac{b\sqrt{\ldots}}{\ldots}\right)}{8b\sqrt{ae\ldots}}\ldots\right)}{\left(a^2+b^2\right)^3e}\ldots\right)$
default	$-2e^4\left(\frac{a\left(\frac{\left(\frac{1}{8}a^4+\frac{5}{4}a^2b^2+\frac{9}{8}b^4\right)(e\cot(dx+c))^{\frac{3}{2}}-\frac{ae\left(a^4-6a^2b^2-7b^4\right)\sqrt{e\cot(dx+c)}}{8b}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(a^4+18a^2b^2-15b^4\right)\arctan\left(\frac{b\sqrt{\ldots}}{\ldots}\right)}{8b\sqrt{ae\ldots}}\ldots\right)}{\left(a^2+b^2\right)^3e}\ldots\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^4*(a/(a^2+b^2)^3/e*(((1/8*a^4+5/4*a^2*b^2+9/8*b^4)*(e*cot(d*x+c))^(3
/2)-1/8*a*e*(a^4-6*a^2*b^2-7*b^4)/b*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)*b+a
*e)^2+1/8*(a^4+18*a^2*b^2-15*b^4)/b/(a*e*b)^(1/2)*arctan(b*(e*cot(d*x+c))^(
1/2)/(a*e*b)^(1/2)))+1/e/(a^2+b^2)^3*(1/8*(-3*a^2*b*e+b^3*e)*(e^2)^(1/4)/e^
2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^
(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))
+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-a^3+3*a*b^2)/(e^2)^(1/4)*2^(1/2)*(l
n((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*co
t(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^
(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.53, size = 417, normalized size = 0.89

$$-\frac{\left(\frac{(a^5+18\,a^3b^2-15\,ab^4)\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\sqrt{ab}}-\frac{2\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)+\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\frac{a^4-7\,a^2b^2}{\sqrt{\tan\left(dx+c\right)}}-\frac{a^3b+9\,ab^3}{\tan\left(dx+c\right)^{\frac{3}{2}}}}{a^6b+2\,a^4b^3+a^2b^5+\frac{2\,(a^5b^2+2\,a^3b^4+ab^6)}{\tan\left(dx+c\right)}+\frac{a^4b^3+2\,a^2b^5+b^7}{\tan\left(dx+c\right)^2}}\right)e^{\frac{5}{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((a^5 + 18*a^3*b^2 - 15*a*b^4)*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c)))
)/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sqrt(a*b)) - (2*sqrt(2)*(a^3 + 3*a
^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c))))
+ 2*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) -
2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(sqrt(2
)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^3 - 3*a^2*b - 3*a*b
^2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6) - ((a^4 - 7*a^2*b^2)/sqrt(tan(d*x + c)) - (a^3*b
+ 9*a*b^3)/tan(d*x + c)^(3/2))/(a^6*b + 2*a^4*b^3 + a^2*b^5 + 2*(a^5*b^2 +
2*a^3*b^4 + a*b^6)/tan(d*x + c) + (a^4*b^3 + 2*a^2*b^5 + b^7)/tan(d*x + c)^
2))*e^(5/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)/(b*cot(d*x + c) + a)^3, x)
```

Mupad [**B**]
time = 6.51, size = 2500, normalized size = 5.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(5/2)/(a + b*cot(c + d*x))^3,x)
```

```
[Out] atan(((((10*a^16*b*d^2*e^18 - 2398*a^2*b^15*d^2*e^18 + 5238*a^4*b^13*d^2*e^
18 + 7386*a^6*b^11*d^2*e^18 - 8322*a^8*b^9*d^2*e^18 - 5498*a^10*b^7*d^2*e^1
8 + 2946*a^12*b^5*d^2*e^18 + 382*a^14*b^3*d^2*e^18)/(b^17*d^5 + a^16*b*d^5
+ 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*
a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - (((832*a*b^22*d^4*e^13 +
 5952*a^3*b^20*d^4*e^13 + 17664*a^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^4*e^13
 + 18816*a^9*b^14*d^4*e^13 - 2688*a^11*b^12*d^4*e^13 - 16128*a^13*b^10*d^4*
e^13 - 13056*a^15*b^8*d^4*e^13 - 4800*a^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*
e^13)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^
11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^
5) + ((e*cot(c + d*x))^(1/2)*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6
i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/
2)*(512*b^26*d^4*e^10 + 4608*a^2*b^24*d^4*e^10 + 17920*a^4*b^22*d^4*e^10 +
38400*a^6*b^20*d^4*e^10 + 46080*a^8*b^18*d^4*e^10 + 21504*a^10*b^16*d^4*e^1
0 - 21504*a^12*b^14*d^4*e^10 - 46080*a^14*b^12*d^4*e^10 - 38400*a^16*b^10*d
^4*e^10 - 17920*a^18*b^8*d^4*e^10 - 4608*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d
^4*e^10))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^
```

6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^
3*d^4))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15
*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x)
)^(1/2)*(8*a^19*b*d^2*e^15 - 1472*a*b^19*d^2*e^15 + 776*a^3*b^17*d^2*e^15 +
 11328*a^5*b^15*d^2*e^15 + 10208*a^7*b^13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15
 - 5328*a^11*b^9*d^2*e^15 + 4032*a^13*b^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15
 + 384*a^17*b^3*d^2*e^15))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4
*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^
5*d^4 + 8*a^14*b^3*d^4))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i +
a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2))*
(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4
*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*
(a^14*e^20 - 32*b^14*e^20 + 97*a^2*b^12*e^20 - 2082*a^4*b^10*e^20 + 3631*a^
6*b^8*e^20 - 2300*a^8*b^6*e^20 + 79*a^10*b^4*e^20 + 30*a^12*b^2*e^20))/(b^1
7*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 7
0*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-(e^5
1i)/(4(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 -
 a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*1i - (((10*a^16*b*d^2*e^18 - 239
8*a^2*b^15*d^2*e^18 + 5238*a^4*b^13*d^2*e^18 + 7386*a^6*b^11*d^2*e^18 - 832
2*a^8*b^9*d^2*e^18 - 5498*a^10*b^7*d^2*e^18 + 2946*a^12*b^5*d^2*e^18 + 382*
a^14*b^3*d^2*e^18)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^
5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 +
8*a^14*b^3*d^5) - (((832*a*b^22*d^4*e^13 + 5952*a^3*b^20*d^4*e^13 + 17664*a
^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^4*e^13 + 18816*a^9*b^14*d^4*e^13 - 2688
*a^11*b^12*d^4*e^13 - 16128*a^13*b^10*d^4*e^13 - 13056*a^15*b^8*d^4*e^13 -
4800*a^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*e^13)/(b^17*d^5 + a^16*b*d^5 + 8*
a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10
*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - ((e*cot(c + d*x))^(1/2)*(-(e
^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2
 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^26*d^4*e^10 + 4608*a^2*
b^24*d^4*e^10 + 17920*a^4*b^22*d^4*e^10 + 38400*a^6*b^20*d^4*e^10 + 46080*a
^8*b^18*d^4*e^10 + 21504*a^10*b^16*d^4*e^10 - 21504*a^12*b^14*d^4*e^10 - 46
080*a^14*b^12*d^4*e^10 - 38400*a^16*b^10*d^4*e^10 - 17920*a^18*b^8*d^4*e^10
 - 4608*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d^4*e^10))/(b^17*d^4 + a^16*b*d^4
+ 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*
a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-(e^5*1i)/(4*(b^6*d^2 -
a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i +
15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(8*a^19*b*d^2*e^15 - 1472
*a*b^19*d^2*e^15 + 776*a^3*b^17*d^2*e^15 + 11328*a^5*b^15*d^2*e^15 + 10208*
a^7*b^13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15 - 5328*a^11*b^9*d^2*e^15 + 4032*
a^13*b^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15 + 384*a^17*b^3*d^2*e^15))/(b^17*
d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*
a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4))*(-(e^5*1
i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a
^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2

```
+ a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b
^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(a^14*e^20 - 32*b^14*e^20 + 97*a^
2*b^12*e^20 - 2082*a^4*b^10*e^20 + 3631*a^6*b^8*e^20 - 2300*a^8*b^6*e^20 +
79*a^10*b^4*e^20 + 30*a^12*b^2*e^20))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d
^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8...
```

3.84 $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx$

Optimal. Leaf size=461

$$-\frac{(3a^4-26a^2b^2+3b^4)\,e^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4\sqrt{a}\,\sqrt{b}\,(a^2+b^2)^3\,d}-\frac{(a+b)\,(a^2-4ab+b^2)\,e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}}{}\right.}{\sqrt{2}\,(a^2+b^2)^3\,d}$$

```
[Out] -1/2*(a+b)*(a^2-4*a*b+b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^
(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*e^(3/2)*arctan(1+2^(
1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+4*a
*b+b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))
/(a^2+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x
+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(3*a^4-
26*a^2*b^2+3*b^4)*e^(3/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/
2))/(a^2+b^2)^3/d/a^(1/2)/b^(1/2)-1/2*a*e*(e*cot(d*x+c))^(1/2)/(a^2+b^2)/d/
(a+b*cot(d*x+c))^2-1/4*(3*a^2-5*b^2)*e*(e*cot(d*x+c))^(1/2)/(a^2+b^2)^2/d/(
a+b*cot(d*x+c))
```

Rubi [A]
time = 0.75, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$,
Rules used = {3648, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{e^{3/2}(a+b)\,(a^2-4ab+b^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\,(a^2+b^2)^3}+\frac{e^{3/2}(a+b)\,(a^2-4ab+b^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^3}-\frac{e^{3/2}(a-b)\,(a^2+4ab+b^2)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^3}+\frac{e^{3/2}(a-b)\,(a^2+4ab+b^2)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^3}-\frac{e(3a^2-5b^2)\,\sqrt{e\cot(c+dx)}}{4d\,(a^2+b^2)^2\,(a+b\cot(c+dx))}-\frac{ae\sqrt{e\cot(c+dx)}}{2d\,(a^2+b^2)\,(a+b\cot(c+dx))^2}-\frac{e^{3/2}(3a^4-26a^2b^2+3b^4)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4\sqrt{a}\,\sqrt{b}\,d\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -1/4*((3*a^4 - 26*a^2*b^2 + 3*b^4)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d
*x]])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*Sqrt[b]*(a^2 + b^2)^3*d) - ((a + b)*(a^2
 - 4*a*b + b^2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])
/(Sqrt[2]*(a^2 + b^2)^3*d) + ((a + b)*(a^2 - 4*a*b + b^2)*e^(3/2)*ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d) - (a*e
*Sqrt[e*Cot[c + d*x]])/(2*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2) - ((3*a^2 -
 5*b^2)*e*Sqrt[e*Cot[c + d*x]])/(4*(a^2 + b^2)^2*d*(a + b*Cot[c + d*x])) -
((a - b)*(a^2 + 4*a*b + b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - S
qrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) + ((a - b)*(a^2 +
 4*a*b + b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*C
ot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m
 + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx = -\frac{ae\sqrt{e\cot(c+dx)}}{2(a^2+b^2)\,d(a+b\cot(c+dx))^2} - \frac{\int \frac{\frac{ae^2}{2}-2be^2\cot(c+dx)-\frac{3}{2}ae^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^2}\,dx}{2(a^2+b^2)}$$

$$= -\frac{ae\sqrt{e\cot(c+dx)}}{2(a^2+b^2)\,d(a+b\cot(c+dx))^2} - \frac{(3a^2-5b^2)\,e\sqrt{e\cot(c+dx)}}{4(a^2+b^2)^2\,d(a+b\cot(c+dx))} + \frac{\int \frac{-\frac{1}{4}a}{}}{}$$

$$= -\frac{ae\sqrt{e\cot(c+dx)}}{2(a^2+b^2)\,d(a+b\cot(c+dx))^2} - \frac{(3a^2-5b^2)\,e\sqrt{e\cot(c+dx)}}{4(a^2+b^2)^2\,d(a+b\cot(c+dx))} + \frac{\int \frac{-2a}{}}{}$$

$$= -\frac{ae\sqrt{e\cot(c+dx)}}{2(a^2+b^2)\,d(a+b\cot(c+dx))^2} - \frac{(3a^2-5b^2)\,e\sqrt{e\cot(c+dx)}}{4(a^2+b^2)^2\,d(a+b\cot(c+dx))} + \frac{\text{Subst}}{}$$

$$= -\frac{ae\sqrt{e\cot(c+dx)}}{2(a^2+b^2)\,d(a+b\cot(c+dx))^2} - \frac{(3a^2-5b^2)\,e\sqrt{e\cot(c+dx)}}{4(a^2+b^2)^2\,d(a+b\cot(c+dx))} - \frac{((3a^4}{}$$

$$= -\frac{(3a^4-26a^2b^2+3b^4)\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4\sqrt{a}\,\sqrt{b}\,(a^2+b^2)^3\,d} - \frac{ae\sqrt{e\cot(}{2(a^2+b^2)\,d(a+}$$

$$= -\frac{(3a^4-26a^2b^2+3b^4)\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4\sqrt{a}\,\sqrt{b}\,(a^2+b^2)^3\,d} - \frac{ae\sqrt{e\cot(}{2(a^2+b^2)\,d(a+}$$

$$= -\frac{(3a^4-26a^2b^2+3b^4)\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4\sqrt{a}\,\sqrt{b}\,(a^2+b^2)^3\,d} - \frac{(a+b)(a^2-4ab}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 6.15, size = 518, normalized size = 1.12

$$-\frac{(e\cot(c+dx))^{3/2}\left(-\frac{2a(3a^2-b^2)\left(-\frac{\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}}+\sqrt{\cot(c+dx)}\right)}{(a^2+b^2)^3}+\frac{2b(3a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3(a^2+b^2)^3}-\frac{-\frac{3\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)\sqrt{\cot(c+dx)}}{\sqrt{a}}+\frac{2b^2\cot^2(c+dx)}{(a+b\cot(c+dx))^2}+\frac{3b\cot(c+dx)}{a+b\cot(c+dx)}}{4b(a^2+b^2)\sqrt{\cot(c+dx)}}-\frac{2b(3a^2-b^2)\left(\cot^{\frac{3}{2}}(c+dx)-\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)\right)}{3(a^2+b^2)^3}+\frac{4b^2\cot^{\frac{5}{2}}(c+dx)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{b\cot(c+dx)}{a}\right)}{5a(a^2+b^2)^2}+\frac{a(a^2-3b^2)\left(2\left(\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{4(a^2+b^2)^3}\right)}{d\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -(((e*Cot[c + d*x])^(3/2)*((-2*a*(3*a^2 - b^2)*(-((Sqrt[a]*ArcTan[(Sqrt[b]*
Sqrt[Cot[c + d*x]])/Sqrt[a]])/Sqrt[b]) + Sqrt[Cot[c + d*x]]))/(a^2 + b^2)^3
 + (2*b*(3*a^2 - b^2)*Cot[c + d*x]^(3/2))/(3*(a^2 + b^2)^3) - ((-3*Sqrt[b]*
ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]*Sqrt[Cot[c + d*x]])/Sqrt[a] +
(2*b^2*Cot[c + d*x]^2)/(a + b*Cot[c + d*x])^2 + (3*b*Cot[c + d*x])/(a + b*C
ot[c + d*x]))/(4*b*(a^2 + b^2)*Sqrt[Cot[c + d*x]]) - (2*b*(3*a^2 - b^2)*(Co
t[c + d*x]^(3/2) - Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c
 + d*x]^2]))/(3*(a^2 + b^2)^3) + (4*b^2*Cot[c + d*x]^(5/2)*Hypergeometric2F
1[2, 5/2, 7/2, -((b*Cot[c + d*x])/a)])/(5*a*(a^2 + b^2)^2) + (a*(a^2 - 3*b^
2)*(2*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - Sqrt[2]*ArcTan[1 +
Sqrt[2]*Sqrt[Cot[c + d*x]]]) + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[
2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c
+ d*x]] + Cot[c + d*x]]))/(4*(a^2 + b^2)^3)))/(d*Cot[c + d*x]^(3/2)))
```

Maple [A]
time = 0.65, size = 456, normalized size = 0.99

method	result
derivativedivides	$-\frac{2e^4\left(\frac{(-a^3e+3ab^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\,c\ldots}}{(\ldots}\right.\right.}{8e^2}\ldots\right.}{\ldots}$
default	$-\frac{2e^4\left(\frac{(-a^3e+3ab^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\,c\ldots}}{(\ldots}\right.\right.}{8e^2}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^4*(1/(a^2+b^2)^3/e^2*(1/8*(-a^3*e+3*a*b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)
*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e
*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan
(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)+1))+1/8*(3*a^2*b-b^3)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d
*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)+1)))+1/(a^2+b^2)^3/e^2*(((3/8*a^4*b-1/4*a^2*b^3-5/8*b^5)*(e*cot(d*x
+c))^(3/2)+1/8*a*e*(5*a^4+2*a^2*b^2-3*b^4)*(e*cot(d*x+c))^(1/2))/(e*cot(d*x
+c)*b+a*e)^2+1/8*(3*a^4-26*a^2*b^2+3*b^4)/(a*e*b)^(1/2)*arctan(b*(e*cot(d*x
+c))^(1/2)/(a*e*b)^(1/2))))
```

Maxima [A]
time = 0.51, size = 410, normalized size = 0.89

$$-\frac{\left(\frac{\left(3\,a^{4}-26\,a^{2}b^{2}+3\,b^{4}\right)\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)\sqrt{ab}}-\frac{2\,\sqrt{2}\,\left(a^{3}-3\,a^{2}b-3\,ab^{2}+b^{3}\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,\left(a^{3}-3\,a^{2}b-3\,ab^{2}+b^{3}\right)\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\,\left(a^{3}+3\,a^{2}b-3\,ab^{2}-b^{3}\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)-\sqrt{2}\,\left(a^{3}+3\,a^{2}b-3\,ab^{2}-b^{3}\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{\frac{5\,a^{3}-3\,ab^{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{3\,a^{2}b-5\,b^{3}}{\tan\left(dx+c\right)^{\frac{3}{2}}}}{a^{6}+2\,a^{4}b^{2}+a^{2}b^{4}+\frac{2\,\left(a^{5}b+2\,a^{3}b^{3}+ab^{5}\right)}{\tan\left(dx+c\right)}+\frac{a^{4}b^{2}+2\,a^{2}b^{4}+b^{6}}{\tan\left(dx+c\right)^{2}}}\right)e^{\frac{3}{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((3*a^4 - 26*a^2*b^2 + 3*b^4)*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))
/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a*b)) - (2*sqrt(2)*(a^3 - 3*a^2*
b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2
*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/s
qrt(tan(d*x + c)))) + sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(sqrt(2)/s
qrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2
- b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6) + ((5*a^3 - 3*a*b^2)/sqrt(tan(d*x + c)) + (3*a^2*b -
 5*b^3)/tan(d*x + c)^(3/2))/(a^6 + 2*a^4*b^2 + a^2*b^4 + 2*(a^5*b + 2*a^3*b
^3 + a*b^5)/tan(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)/tan(d*x + c)^2))*e^(
3/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}{\left(a+b\cot\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)/(a + b*cot(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)/(b*cot(d*x + c) + a)^3, x)
```

Mupad [B]
time = 6.21, size = 2500, normalized size = 5.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)/(a + b*cot(c + d*x))^3,x)
```

```
[Out] atan(((((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*d^2*e^15
+ 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d^2*e^15 -
 3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^16*d^5 + 8*a
^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*
b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + (((4224*a^4*b^18*d^4*e^12 - 3
20*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e^12 + 51072*
a^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*d^4*e^12 + 2
5344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*d^4*e^12 - 1
28*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d
^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 +
 8*a^14*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2
+ a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b
^2*d^2)))^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 + 17920*a^4*b^2
1*d^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^10 + 21504*a^10
*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*d^4*e^10 - 3840
0*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b^5*d^4*e^10 - 51
2*a^22*b^3*d^4*e^10))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d
^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 +
 8*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^
2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot
(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13 + 64*a^3*b^16*d^2*e^13 - 7456*a^5*b^
14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 + 19504*a^9*b^10*d^2*e^13 + 18880*a^11*
```

```
b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13 - 960*a^15*b^4*d^2*e^13 + 8*a^17*b^2*
d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6
*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2
*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a
^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2))*((e^3*1i)/(4*(b^6*d
^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*2
0i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(41*b^13*e^16 + 9*a^
12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e^16 - 4348*a^6*b^7*e^16 + 1671
*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 +
 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*
a^12*b^4*d^4 + 8*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2
*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(
1/2)*1i - (((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*d^2*e
^15 + 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d^2*e^
15 - 3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^16*d^5 +
 8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a
^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + (((4224*a^4*b^18*d^4*e^12
 - 320*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e^12 + 51
072*a^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*d^4*e^12
 + 25344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*d^4*e^12
 - 128*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^
12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d
^5 + 8*a^14*b^2*d^5) - ((e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^6*d^2 - a^6*
d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a
^4*b^2*d^2)))^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 + 17920*a^4
*b^21*d^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^10 + 21504*
a^10*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*d^4*e^10 -
38400*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b^5*d^4*e^10
- 512*a^22*b^3*d^4*e^10))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^
12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d
^4 + 8*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*
b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e
*cot(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13 + 64*a^3*b^16*d^2*e^13 - 7456*a^
5*b^14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 + 19504*a^9*b^10*d^2*e^13 + 18880*a
^11*b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13 - 960*a^15*b^4*d^2*e^13 + 8*a^17*
b^2*d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56
*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14
*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i -
15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2))*((e^3*1i)/(4*(b
^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d
^2*20i + 15*a^4*b^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(41*b^13*e^16 +
9*a^12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e^16 - 4348*a^6*b^7*e^16 +
1671*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d
^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 +
 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*((e^3*1i)/(...
```

3.85 $\int \frac{\sqrt{e \cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx$

Optimal. Leaf size=463

$$\frac{\sqrt{b}\,(15a^4-18a^2b^2-b^4)\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4a^{3/2}\,(a^2+b^2)^3\,d}+\frac{(a-b)\,(a^2+4ab+b^2)\,\sqrt{e}\,\text{ArcTan}\Big(1-\frac{\sqrt{2}}{}}{\sqrt{2}\,(a^2+b^2)^3\,d}$$

```
[Out] 1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^
(1/2)/(a^2+b^2)^3/d*2^(1/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*c
ot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*
b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/
(a^2+b^2)^3/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/
2)+2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/(a^2+b^2)^3/d*2^(1/2)+1/4*(15*a^4-
18*a^2*b^2-b^4)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*b^(1/2
)*e^(1/2)/a^(3/2)/(a^2+b^2)^3/d+1/2*b*(e*cot(d*x+c))^(1/2)/(a^2+b^2)/d/(a+b
*cot(d*x+c))^2+1/4*b*(7*a^2-b^2)*(e*cot(d*x+c))^(1/2)/a/(a^2+b^2)^2/d/(a+b*
cot(d*x+c))
```

Rubi [A]
time = 0.75, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3649, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{\sqrt{e}\,(a-b)\,(a^2+4ab+b^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\,(a^2+b^2)^3}-\frac{\sqrt{e}\,(a-b)\,(a^2+4ab+b^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\,(a^2+b^2)^3}+\frac{b(7a^2-b^2)\,\sqrt{e\cot(c+dx)}}{4ad\,(a^2+b^2)^2\,(a+b\cot(c+dx))}+\frac{b\sqrt{e\cot(c+dx)}}{2d\,(a^2+b^2)\,(a+b\cot(c+dx))^2}-\frac{\sqrt{e}\,(a+b)\,(a^2-4ab+b^2)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^3}+\frac{\sqrt{e}\,(a+b)\,(a^2-4ab+b^2)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\,(a^2+b^2)^3}+\frac{\sqrt{b}\,\sqrt{e}\,(15a^4-18a^2b^2-b^4)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4a^{3/2}d\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] (Sqrt[b]*(15*a^4 - 18*a^2*b^2 - b^4)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c +
 d*x]])/(Sqrt[a]*Sqrt[e])])/(4*a^(3/2)*(a^2 + b^2)^3*d) + ((a - b)*(a^2 + 4
*a*b + b^2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*(a^2 + b^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)*Sqrt[e]*ArcTan[1 + (S
qrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (b*Sqrt[
e*Cot[c + d*x]])/(2*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2) + (b*(7*a^2 - b^2
)*Sqrt[e*Cot[c + d*x]])/(4*a*(a^2 + b^2)^2*d*(a + b*Cot[c + d*x])) - ((a +
b)*(a^2 - 4*a*b + b^2)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]
*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) + ((a + b)*(a^2 - 4*a*b
 + b^2)*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c +
 d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx = \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} - \frac{\int \frac{-\frac{be}{2}-2ae\cot(c+dx)+\frac{3}{2}be\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx}{2(a^2+b^2)}$$

$$= \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4a(a^2+b^2)^2d(a+b\cot(c+dx))} + \frac{\int \frac{1}{4}b(9a \cdots}{\cdots}$$

$$= \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4a(a^2+b^2)^2d(a+b\cot(c+dx))} + \frac{\int \frac{2ab(3 \cdots}{\cdots}}{\cdots}$$

$$= \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4a(a^2+b^2)^2d(a+b\cot(c+dx))} + \frac{\text{Subst} \cdots}{\cdots}$$

$$= \frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4a(a^2+b^2)^2d(a+b\cot(c+dx))} + \frac{(b(15a \cdots}{\cdots}$$

$$= \frac{\sqrt{b}(15a^4-18a^2b^2-b^4)\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{3/2}(a^2+b^2)^3d} + \frac{b\sqrt{e\,\mathrm{co} \cdots}}{2(a^2+b^2)d(a \cdots}$$

$$= \frac{\sqrt{b}(15a^4-18a^2b^2-b^4)\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{3/2}(a^2+b^2)^3d} + \frac{b\sqrt{e\,\mathrm{co} \cdots}}{2(a^2+b^2)d(a \cdots}$$

$$= \frac{\sqrt{b}(15a^4-18a^2b^2-b^4)\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{3/2}(a^2+b^2)^3d} + \frac{(a-b)(a^2+4 \cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 6.19, size = 483, normalized size = 1.04

$$-\frac{\sqrt{e\cot(c+dx)}\left(-\frac{2\sqrt{a}\sqrt{b}(3a^2-b^2)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{(a^2+b^2)^3}+\frac{2b(3a^2-b^2)\sqrt{\cot(c+dx)}}{(a^2+b^2)^3}-\frac{2\sqrt{a}\sqrt{b}\left(-a\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)+\sqrt{a}\sqrt{b}\sqrt{\cot(c+dx)}-b\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)\cot(c+dx)\right)}{(a^2+b^2)^2(a+b\cot(c+dx))}+\frac{2a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)}{3(a^2+b^2)^3}+\frac{2b^2\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};-\frac{b\cot(c+dx)}{a}\right)}{3a^3(a^2+b^2)}-\frac{b(3a^2-b^2)\left(2\left(\sqrt{2}\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\right)+8\sqrt{\cot(c+dx)}+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{4(a^2+b^2)^3}\right)}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -((Sqrt[e*Cot[c + d*x]]*((-2*Sqrt[a]*Sqrt[b]*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*
Sqrt[Cot[c + d*x]])/Sqrt[a]])/(a^2 + b^2)^3 + (2*b*(3*a^2 - b^2)*Sqrt[Cot[c
 + d*x]])/(a^2 + b^2)^3 - (2*Sqrt[a]*Sqrt[b]*(-(a*ArcTan[(Sqrt[b]*Sqrt[Cot[
c + d*x]])/Sqrt[a]]) + Sqrt[a]*Sqrt[b]*Sqrt[Cot[c + d*x]] - b*ArcTan[(Sqrt[
b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]*Cot[c + d*x]))/((a^2 + b^2)^2*(a + b*Cot[c
+ d*x])) + (2*a*(a^2 - 3*b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1,
7/4, -Cot[c + d*x]^2])/(3*(a^2 + b^2)^3) + (2*b^2*Cot[c + d*x]^(3/2)*Hyperg
eometric2F1[3/2, 3, 5/2, -((b*Cot[c + d*x])/a)])/(3*a^3*(a^2 + b^2)) - (b*(
3*a^2 - b^2)*(2*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - Sqrt[2]*A
rcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]) + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log
[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*S
qrt[Cot[c + d*x]] + Cot[c + d*x]]))/(4*(a^2 + b^2)^3)))/(d*Sqrt[Cot[c + d*x
]]))
```

Maple [A]
time = 0.74, size = 460, normalized size = 0.99

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{b\left(\frac{\frac{b(7a^4+6a^2b^2-b^4)(e\cot(dx+c))^{\frac{3}{2}}}{8a}+\frac{e(9a^4+10a^2b^2+b^4)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{(15a^4-18a^2b^2-b^4)\arctan\left(\frac{b\sqrt{}}{}\right.}{8a\sqrt{aeb}}\right.}{e^3(a^2+b^2)^3}\right.}{}$
default	$-\frac{2e^4\left(-\frac{b\left(\frac{\frac{b(7a^4+6a^2b^2-b^4)(e\cot(dx+c))^{\frac{3}{2}}}{8a}+\frac{e(9a^4+10a^2b^2+b^4)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{(15a^4-18a^2b^2-b^4)\arctan\left(\frac{b\sqrt{}}{}\right.}{8a\sqrt{aeb}}\right.}{e^3(a^2+b^2)^3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^4*(-b/e^3/(a^2+b^2)^3*((1/8*b*(7*a^4+6*a^2*b^2-b^4)/a*(e*cot(d*x+c))
^(3/2)+1/8*e*(9*a^4+10*a^2*b^2+b^4)*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)*b+a
```

```
*e)^2+1/8*(15*a^4-18*a^2*b^2-b^4)/a/(a*e*b)^(1/2)*arctan(b*(e*cot(d*x+c))^(
1/2)/(a*e*b)^(1/2)))+1/(a^2+b^2)^3/e^3*(1/8*(3*a^2*b*e-b^3*e)*(e^2)^(1/4)/e
^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)
^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))
)+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(a^3-3*a*b^2)/(e^2)^(1/4)*2^(1/2)*(l
n((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*co
t(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^
(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.56, size = 419, normalized size = 0.90

$$\frac{\left(\frac{(15a^4b-18a^2b^3-b^5)\arctan\left(\frac{b}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^7+3a^5b^2+3a^3b^4+ab^6)\sqrt{ab}}-\frac{2\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(a^3+3a^2b-3ab^2-b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}(a^3-3a^2b-3ab^2+b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\frac{9a^3b+ab^3}{\sqrt{\tan(dx+c)}}+\frac{7a^2b^2-b^4}{\tan(dx+c)^{\frac{3}{2}}}}{a^7+2a^5b^2+a^3b^4+\frac{2(a^6b+2a^4b^3+a^2b^5)}{\tan(dx+c)}+\frac{a^5b^2+2a^3b^4+ab^6}{\tan(dx+c)^2}}\right)e^{\frac{1}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((15*a^4*b - 18*a^2*b^3 - b^5)*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c))))
/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sqrt(a*b)) - (2*sqrt(2)*(a^3 + 3*a^
2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) +
 2*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2
/sqrt(tan(d*x + c)))) - sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(sqrt(2)
/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^
2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6) + ((9*a^3*b + a*b^3)/sqrt(tan(d*x + c)) + (7*a^2*b
^2 - b^4)/tan(d*x + c)^(3/2))/(a^7 + 2*a^5*b^2 + a^3*b^4 + 2*(a^6*b + 2*a^4
*b^3 + a^2*b^5)/tan(d*x + c) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)/tan(d*x + c)^2
))*e^(1/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c))**3,x)

[Out] Integral(sqrt(e*cot(c + d*x))/(a + b*cot(c + d*x))**3, x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")

[Out] integrate(sqrt(e*cot(d*x + c))/(b*cot(d*x + c) + a)^3, x)

Mupad [**B**]
time = 6.13, size = 2500, normalized size = 5.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cot(c + d*x))^(1/2)/(a + b*cot(c + d*x))^3,x)

[Out] (((e*cot(c + d*x))^(1/2)*(b^3*e^2 + 9*a^2*b*e^2))/(4*(a^4 + b^4 + 2*a^2*b^2
)) + (b^2*e*(e*cot(c + d*x))^(3/2)*(7*a^2 - b^2))/(4*a*(a^4 + b^4 + 2*a^2*b
^2)))/(a^2*d*e^2 + b^2*d*e^2*cot(c + d*x)^2 + 2*a*b*d*e^2*cot(c + d*x)) - a
tan((((((64*a*b^23*d^4*e^11 + 1472*a^3*b^21*d^4*e^11 + 8832*a^5*b^19*d^4*e
^11 + 25344*a^7*b^17*d^4*e^11 + 40320*a^9*b^15*d^4*e^11 + 34944*a^11*b^13*d
^4*e^11 + 10752*a^13*b^11*d^4*e^11 - 8448*a^15*b^9*d^4*e^11 - 10176*a^17*b^
7*d^4*e^11 - 4160*a^19*b^5*d^4*e^11 - 640*a^21*b^3*d^4*e^11)/(a^18*d^5 + a^
2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b
^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) + ((e*cot(c +
d*x))^(1/2)*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a
^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*a^2*b^25*d^
4*e^10 + 4608*a^4*b^23*d^4*e^10 + 17920*a^6*b^21*d^4*e^10 + 38400*a^8*b^19*
d^4*e^10 + 46080*a^10*b^17*d^4*e^10 + 21504*a^12*b^15*d^4*e^10 - 21504*a^14
*b^13*d^4*e^10 - 46080*a^16*b^11*d^4*e^10 - 38400*a^18*b^9*d^4*e^10 - 17920
*a^20*b^7*d^4*e^10 - 4608*a^22*b^5*d^4*e^10 - 512*a^24*b^3*d^4*e^10))/(a^18
*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 +
70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-e/
(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i -
20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c + d*x))^(1/2)*(8*a*b
^20*d^2*e^11 - 1152*a^3*b^18*d^2*e^11 + 2528*a^5*b^16*d^2*e^11 + 15296*a^7*
b^14*d^2*e^11 + 14128*a^9*b^12*d^2*e^11 - 5056*a^11*b^10*d^2*e^11 - 9248*a^
13*b^8*d^2*e^11 + 64*a^15*b^6*d^2*e^11 + 1800*a^17*b^4*d^2*e^11 + 64*a^19*b

^2*d^2*e^11))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - (2*b^18*d^2*e^12 - 138*a^2*b^16*d^2*e^12 - 3046*a^4*b^14*d^2*e^12 + 4862*a^6*b^12*d^2*e^12 + 9222*a^8*b^10*d^2*e^12 - 5246*a^10*b^8*d^2*e^12 - 4290*a^12*b^6*d^2*e^12 + 2442*a^14*b^4*d^2*e^12 + 32*a^16*b^2*d^2*e^12)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(2*a^2*b^13*e^12 - b^15*e^12 + 49*a^4*b^11*e^12 + 2460*a^6*b^9*e^12 - 3631*a^8*b^7*e^12 + 1922*a^10*b^5*e^12 - 225*a^12*b^3*e^12))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*1i - (((((64*a*b^23*d^4*e^11 + 1472*a^3*b^21*d^4*e^11 + 8832*a^5*b^19*d^4*e^11 + 25344*a^7*b^17*d^4*e^11 + 40320*a^9*b^15*d^4*e^11 + 34944*a^11*b^13*d^4*e^11 + 10752*a^13*b^11*d^4*e^11 - 8448*a^15*b^9*d^4*e^11 - 10176*a^17*b^7*d^4*e^11 - 4160*a^19*b^5*d^4*e^11 - 640*a^21*b^3*d^4*e^11)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) - ((e*cot(c + d*x))^(1/2)*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*a^2*b^25*d^4*e^10 + 4608*a^4*b^23*d^4*e^10 + 17920*a^6*b^21*d^4*e^10 + 38400*a^8*b^19*d^4*e^10 + 46080*a^10*b^17*d^4*e^10 + 21504*a^12*b^15*d^4*e^10 - 21504*a^14*b^13*d^4*e^10 - 46080*a^16*b^11*d^4*e^10 - 38400*a^18*b^9*d^4*e^10 - 17920*a^20*b^7*d^4*e^10 - 4608*a^22*b^5*d^4*e^10 - 512*a^24*b^3*d^4*e^10))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) + ((e*cot(c + d*x))^(1/2)*(8*a*b^20*d^2*e^11 - 1152*a^3*b^18*d^2*e^11 + 2528*a^5*b^16*d^2*e^11 + 15296*a^7*b^14*d^2*e^11 + 14128*a^9*b^12*d^2*e^11 - 5056*a^11*b^10*d^2*e^11 - 9248*a^13*b^8*d^2*e^11 + 64*a^15*b^6*d^2*e^11 + 1800*a^17*b^4*d^2*e^11 + 64*a^19*b^2*d^2*e^11))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^16*b^2*d^4))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - (2*b^18*d^2*e^12 - 138*a^2*b^16*d^2*e^12 - 3046*a^4*b^14*d^2*e^12 + 4862*a^6*b^12*d^2*e^12 + 9222*a^8*b^10*d^2*e^12 - 5246*a^10*b^8*d^2*e^12 - 4290*a^12*b^6*d^2*e^12 + 2442*a^14*b^4*d^2*e^12 + 32*a^16*b^2*d^2*e^12)/(a^18*d^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 70*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 ...

3.86 $\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^3}\,dx$

Optimal. Leaf size=476

$$-\frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4a^{5/2}\,(a^2+b^2)^3\,d\sqrt{e}}+\frac{(a+b)\,(a^2-4ab+b^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot}}{\sqrt{e}}\right.}{\sqrt{2}\,(a^2+b^2)^3\,d\sqrt{e}}$$

```
[Out] -1/4*b^(3/2)*(35*a^4+6*a^2*b^2+3*b^4)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a
^(1/2)/e^(1/2))/a^(5/2)/(a^2+b^2)^3/d/e^(1/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*arc
tan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)/e^(1/2)-1
/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^
2+b^2)^3/d*2^(1/2)/e^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*
e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)/e^(1/2)-1/4*(a-
b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/
2))/(a^2+b^2)^3/d*2^(1/2)/e^(1/2)-1/2*b^2*(e*cot(d*x+c))^(1/2)/a/(a^2+b^2)/
d/e/(a+b*cot(d*x+c))^2-1/4*b^2*(11*a^2+3*b^2)*(e*cot(d*x+c))^(1/2)/a^2/(a^2
+b^2)^2/d/e/(a+b*cot(d*x+c))
```

Rubi [A]
time = 0.82, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3650, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a+b)\,(a^2-4ab+b^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,d\sqrt{e}\,(a^2+b^2)^3}-\frac{(a+b)\,(a^2-4ab+b^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,d\sqrt{e}\,(a^2+b^2)^3}-\frac{b^2(11a^2+3b^2)\,\sqrt{e\cot(c+dx)}}{4a^2de\,(a^2+b^2)^2\,(a+b\cot(c+dx))}-\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\,(a^2+b^2)\,(a+b\cot(c+dx))^2}+\frac{(a-b)\,(a^2+4ab+b^2)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}\,(a^2+b^2)^3}-\frac{(a-b)\,(a^2+4ab+b^2)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,d\sqrt{e}\,(a^2+b^2)^3}-\frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4a^{5/2}d\sqrt{e}\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^3),x]
```

```
[Out] -1/4*(b^(3/2)*(35*a^4 + 6*a^2*b^2 + 3*b^4)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d
*x]])/(Sqrt[a]*Sqrt[e])])/(a^(5/2)*(a^2 + b^2)^3*d*Sqrt[e]) + ((a + b)*(a^2
 - 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*(a^2 + b^2)^3*d*Sqrt[e]) - ((a + b)*(a^2 - 4*a*b + b^2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d*Sqrt[e]) - (b^2
*Sqrt[e*Cot[c + d*x]])/(2*a*(a^2 + b^2)*d*e*(a + b*Cot[c + d*x])^2) - (b^2*
(11*a^2 + 3*b^2)*Sqrt[e*Cot[c + d*x]])/(4*a^2*(a^2 + b^2)^2*d*e*(a + b*Cot[
c + d*x])) + ((a - b)*(a^2 + 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x
] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d*Sqrt[e]) - ((
a - b)*(a^2 + 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqr
t[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^3}\,dx = -\frac{b^2\sqrt{e\cot(c+dx)}}{2a\,(a^2+b^2)\,de(a+b\cot(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}(4a^2+3b^2)e+2abe\cot(c}{\sqrt{e\cot(c+dx)}}}{2a\,(a^2 -}$$

$$= -\frac{b^2\sqrt{e\cot(c+dx)}}{2a\,(a^2+b^2)\,de(a+b\cot(c+dx))^2} - \frac{b^2(11a^2+3b^2)\sqrt{e\,c}}{4a^2\,(a^2+b^2)^2\,de(a+b}$$

$$= -\frac{b^2\sqrt{e\cot(c+dx)}}{2a\,(a^2+b^2)\,de(a+b\cot(c+dx))^2} - \frac{b^2(11a^2+3b^2)\sqrt{e\,c}}{4a^2\,(a^2+b^2)^2\,de(a+b}$$

$$= -\frac{b^2\sqrt{e\cot(c+dx)}}{2a\,(a^2+b^2)\,de(a+b\cot(c+dx))^2} - \frac{b^2(11a^2+3b^2)\sqrt{e\,c}}{4a^2\,(a^2+b^2)^2\,de(a+b}$$

$$= -\frac{b^2\sqrt{e\cot(c+dx)}}{2a\,(a^2+b^2)\,de(a+b\cot(c+dx))^2} - \frac{b^2(11a^2+3b^2)\sqrt{e\,c}}{4a^2\,(a^2+b^2)^2\,de(a+b}$$

$$= -\frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4a^{5/2}\,(a^2+b^2)^3\,d\sqrt{e}} - \frac{}{2}$$

$$= -\frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4a^{5/2}\,(a^2+b^2)^3\,d\sqrt{e}} - \frac{}{2}$$

$$= -\frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4a^{5/2}\,(a^2+b^2)^3\,d\sqrt{e}} + \frac{(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 6.14, size = 411, normalized size = 0.86

$$-\frac{\sqrt{\cot(c+dx)}\left(\frac{2b^{3/2}(3a^2-b^2)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,(a^2+b^2)^3}+\frac{2b^2\sqrt{\cot(c+dx)}\left(\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\cot(c+dx)}}+\frac{a}{a+b\cot(c+dx)}\right)}{a(a^2+b^2)^2}+\frac{2b^2\sqrt{\cot(c+dx)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};-\frac{b\cot(c+dx)}{a}\right)}{a^3(a^2+b^2)}-\frac{2b(3a^2-b^2)\cot^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(c+dx)\right)}{3(a^2+b^2)^3}-\frac{a(a^2-3b^2)\left(4\left(\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)\right)+2\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-2\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{8(a^2+b^2)^3}\right)}{d\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^3),x]
```

```
[Out] -((Sqrt[Cot[c + d*x]]*((2*b^(3/2)*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c
+ d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)^3) + (2*b^2*Sqrt[Cot[c + d*x]]*((Sq
rt[a]*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]])/(Sqrt[b]*Sqrt[Cot[c + d
*x]]) + a/(a + b*Cot[c + d*x])))/(a*(a^2 + b^2)^2) + (2*b^2*Sqrt[Cot[c + d*
x]]*Hypergeometric2F1[1/2, 3, 3/2, -((b*Cot[c + d*x])/a)])/(a^3*(a^2 + b^2)
) - (2*b*(3*a^2 - b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -C
ot[c + d*x]^2])/(3*(a^2 + b^2)^3) - (a*(a^2 - 3*b^2)*(4*(Sqrt[2]*ArcTan[1 -
 Sqrt[2]*Sqrt[Cot[c + d*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]
]]) + 2*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 2*Sqrt
[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(8*(a^2 + b^2)^3))
)/(d*Sqrt[e*Cot[c + d*x]]))
```

Maple [A]
time = 0.71, size = 465, normalized size = 0.98

method	result
derivativedivides	$-\frac{2e^4\left(\frac{b^2\left(\frac{\frac{b(11a^4+14a^2b^2+3b^4)(e\cot(dx+c))^{\frac{3}{2}}}{8a^2}+\frac{e(13a^4+18a^2b^2+5b^4)\sqrt{e\cot(dx+c)}}{8a}}{(e\cot(dx+c)b+ae)^2}+\frac{(35a^4+6a^2b^2+3b^4)\arctan\left(\ldots\right.}{8a^2\sqrt{\ldots}}\right.}{e^4(a^2+b^2)^3}\ldots}{\ldots}$
default	$-\frac{2e^4\left(\frac{b^2\left(\frac{\frac{b(11a^4+14a^2b^2+3b^4)(e\cot(dx+c))^{\frac{3}{2}}}{8a^2}+\frac{e(13a^4+18a^2b^2+5b^4)\sqrt{e\cot(dx+c)}}{8a}}{(e\cot(dx+c)b+ae)^2}+\frac{(35a^4+6a^2b^2+3b^4)\arctan\left(\ldots\right.}{8a^2\sqrt{\ldots}}\right.}{e^4(a^2+b^2)^3}\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^4*(b^2/e^4/(a^2+b^2)^3*((1/8*b*(11*a^4+14*a^2*b^2+3*b^4)/a^2*(e*cot(
d*x+c))^(3/2)+1/8*e*(13*a^4+18*a^2*b^2+5*b^4)/a*(e*cot(d*x+c))^(1/2))/(e*co
t(d*x+c)*b+a*e)^2+1/8*(35*a^4+6*a^2*b^2+3*b^4)/a^2/(a*e*b)^(1/2)*arctan(b*(
e*cot(d*x+c))^(1/2)/(a*e*b)^(1/2)))+1/e^4/(a^2+b^2)^3*(1/8*(a^3*e-3*a*b^2*e
)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/
2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arc
tan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-3*a^2*b+b^3)/(e^2)^
(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)
/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

Maxima [A]
time = 0.52, size = 427, normalized size = 0.90

$$-\frac{\left(\frac{\left(35\,a^4b^2+6\,a^2b^4+3\,b^6\right)\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{\tan\left(dx+c\right)}}\right)}{\left(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6\right)\sqrt{ab}}+\frac{2\,\sqrt{2}\,\left(a^3-3\,a^2b-3\,ab^2+b^3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\,\sqrt{2}\,\left(a^3-3\,a^2b-3\,ab^2+b^3\right)\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+\sqrt{2}\,\left(a^3+3\,a^2b-3\,ab^2-b^3\right)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)-\sqrt{2}\,\left(a^3+3\,a^2b-3\,ab^2-b^3\right)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan\left(dx+c\right)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\frac{13\,a^3b^2+5\,ab^4}{\sqrt{\tan\left(dx+c\right)}}+\frac{11\,a^2b^3+3\,b^5}{\tan\left(dx+c\right)^{\frac{3}{2}}}}{a^8+2\,a^6b^2+a^4b^4+\frac{2\,\left(a^7b+2\,a^5b^3+a^3b^5\right)}{\tan\left(dx+c\right)}+\frac{a^6b^2+2\,a^4b^4+a^2b^6}{\tan\left(dx+c\right)^2}}\right)e^{\left(-\frac{1}{2}\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((35*a^4*b^2 + 6*a^2*b^4 + 3*b^6)*arctan(b/(sqrt(a*b)*sqrt(tan(d*x + c
))))/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*sqrt(a*b)) + (2*sqrt(2)*(a^3
- 3*a^2*b - 3*a*b^2 + b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c
)))) + 2*sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*arctan(-1/2*sqrt(2)*(sqrt(
2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*log(s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(a^3 + 3*a^2*b -
3*a*b^2 - b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((13*a^3*b^2 + 5*a*b^4)/sqrt(tan(d*x + c))
 + (11*a^2*b^3 + 3*b^5)/tan(d*x + c)^(3/2))/(a^8 + 2*a^6*b^2 + a^4*b^4 + 2*
(a^7*b + 2*a^5*b^3 + a^3*b^5)/tan(d*x + c) + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6
)/tan(d*x + c)^2))*e^(-1/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Integral(1/(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cot(d*x + c) + a)^3*sqrt(e*cot(d*x + c))), x)
```

Mupad [B]
time = 6.79, size = 2500, normalized size = 5.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))^3),x)
```

```
[Out] atan(((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i
+ 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*
d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e
 + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*d^2*e
- 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i
+ a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e
 - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i))
)^(1/2)*((192*a^2*b^24*d^4*e^10 + 1728*a^4*b^22*d^4*e^10 + 8320*a^6*b^20*d^
4*e^10 + 27264*a^8*b^18*d^4*e^10 + 62592*a^10*b^16*d^4*e^10 + 99456*a^12*b^
14*d^4*e^10 + 107520*a^14*b^12*d^4*e^10 + 76800*a^16*b^10*d^4*e^10 + 33984*
a^18*b^8*d^4*e^10 + 7872*a^20*b^6*d^4*e^10 + 384*a^22*b^4*d^4*e^10 - 128*a^
24*b^2*d^4*e^10)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^
5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5
+ 8*a^18*b^2*d^5) - ((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3
*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2
)*(e*cot(c + d*x))^(1/2)*(512*a^4*b^25*d^4*e^10 + 4608*a^6*b^23*d^4*e^10 +
```

17920*a^8*b^21*d^4*e^10 + 38400*a^10*b^19*d^4*e^10 + 46080*a^12*b^17*d^4*e^
10 + 21504*a^14*b^15*d^4*e^10 - 21504*a^16*b^13*d^4*e^10 - 46080*a^18*b^11*
d^4*e^10 - 38400*a^20*b^9*d^4*e^10 - 17920*a^22*b^7*d^4*e^10 - 4608*a^24*b^
5*d^4*e^10 - 512*a^26*b^3*d^4*e^10))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*
d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^
4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)) + ((e*cot(c + d*x))^(1/2)*(72*a*b^22
*d^2*e^9 + 576*a^3*b^20*d^2*e^9 + 5024*a^5*b^18*d^2*e^9 + 14272*a^7*b^16*d^
2*e^9 + 27824*a^9*b^14*d^2*e^9 + 53184*a^11*b^12*d^2*e^9 + 70240*a^13*b^10*
d^2*e^9 + 47680*a^15*b^8*d^2*e^9 + 12616*a^17*b^6*d^2*e^9 - 64*a^21*b^2*d^2
*e^9))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^1
0*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b
^2*d^4)) - (90*a*b^19*d^2*e^9 + 846*a^3*b^17*d^2*e^9 + 1714*a^5*b^15*d^2*e^
9 + 3606*a^7*b^13*d^2*e^9 - 14578*a^9*b^11*d^2*e^9 - 34486*a^11*b^9*d^2*e^9
 - 14970*a^13*b^7*d^2*e^9 + 2258*a^15*b^5*d^2*e^9 - 32*a^17*b^3*d^2*e^9)/(a
^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^
5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5))
 + ((e*cot(c + d*x))^(1/2)*(18*a^2*b^15*e^8 - 9*b^17*e^8 - 71*a^4*b^13*e^8 +
 892*a^6*b^11*e^8 + 857*a^8*b^9*e^8 + 6802*a^10*b^7*e^8 - 1257*a^12*b^5*e^8
))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^
10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d
^4))*1i - (1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*
20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(
b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d
^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*d^
2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e
*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d
^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*
6i)))^(1/2)*((192*a^2*b^24*d^4*e^10 + 1728*a^4*b^22*d^4*e^10 + 8320*a^6*b^2
0*d^4*e^10 + 27264*a^8*b^18*d^4*e^10 + 62592*a^10*b^16*d^4*e^10 + 99456*a^1
2*b^14*d^4*e^10 + 107520*a^14*b^12*d^4*e^10 + 76800*a^16*b^10*d^4*e^10 + 33
984*a^18*b^8*d^4*e^10 + 7872*a^20*b^6*d^4*e^10 + 384*a^22*b^4*d^4*e^10 - 12
8*a^24*b^2*d^4*e^10)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^1
2*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4*
d^5 + 8*a^18*b^2*d^5) + ((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e -
 a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^
(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^4*b^25*d^4*e^10 + 4608*a^6*b^23*d^4*e^1
0 + 17920*a^8*b^21*d^4*e^10 + 38400*a^10*b^19*d^4*e^10 + 46080*a^12*b^17*d^
4*e^10 + 21504*a^14*b^15*d^4*e^10 - 21504*a^16*b^13*d^4*e^10 - 46080*a^18*b
^11*d^4*e^10 - 38400*a^20*b^9*d^4*e^10 - 17920*a^22*b^7*d^4*e^10 - 4608*a^2
4*b^5*d^4*e^10 - 512*a^26*b^3*d^4*e^10))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b
^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^
6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)) - ((e*cot(c + d*x))^(1/2)*(72*a*
b^22*d^2*e^9 + 576*a^3*b^20*d^2*e^9 + 5024*a^5*b^18*d^2*e^9 + 14272*a^7*b^1
6*d^2*e^9 + 27824*a^9*b^14*d^2*e^9 + 53184*a^11*b^12*d^2*e^9 + 70240*a^13*b
^10*d^2*e^9 + 47680*a^15*b^8*d^2*e^9 + 12616*a^17*b^6*d^2*e^9 - 64*a^21*b^2

```
*d^2*e^9))/(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56
*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^
18*b^2*d^4)) - (90*a*b^19*d^2*e^9 + 846*a^3*b^17*d^2*e^9 + 1714*a^5*b^15*d^
2*e^9 + 3606*a^7*b^13*d^2*e^9 - 14578*a^9*b^11*d^2*e^9 - 34486*a^11*b^9*d^2
*e^9 - 14970*a^13*b^7*d^2*e^9 + 2258*a^15*b^5*d^2*e^9 - 32*a^17*b^3*d^2*e^9
)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 2...
```

3.87 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx$

Optimal. Leaf size=529

$$\frac{b^{5/2}(63a^4+46a^2b^2+15b^4)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4a^{7/2}\left(a^2+b^2\right)^3 de^{3/2}}-\frac{(a-b)\left(a^2+4ab+b^2\right)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot}}{}\right.}{\sqrt{2}\,\left(a^2+b^2\right)^3 de^{3/2}}$$

```
[Out] 1/4*b^(5/2)*(63*a^4+46*a^2*b^2+15*b^4)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/
a^(1/2)/e^(1/2))/a^(7/2)/(a^2+b^2)^3/d/e^(3/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*ar
ctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d/e^(3/2)*2^(1/2)+
1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a
^2+b^2)^3/d/e^(3/2)*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)
*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d/e^(3/2)*2^(1/2)-1/4*(a
+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1
/2))/(a^2+b^2)^3/d/e^(3/2)*2^(1/2)+1/4*(8*a^4+31*a^2*b^2+15*b^4)/a^3/(a^2+b
^2)^2/d/e/(e*cot(d*x+c))^(1/2)-1/2*b^2/a/(a^2+b^2)/d/e/(a+b*cot(d*x+c))^2/(
e*cot(d*x+c))^(1/2)-1/4*b^2*(13*a^2+5*b^2)/a^2/(a^2+b^2)^2/d/e/(a+b*cot(d*x
+c))/(e*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 1.13, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3650, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$-\frac{(a-b)\left(a^2+4ab+b^2\right)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,de^{3/2}\left(a^2+b^2\right)^3}+\frac{(a-b)\left(a^2+4ab+b^2\right)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\,de^{3/2}\left(a^2+b^2\right)^3}+\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\,\cot(c+dx)-\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}\left(a^2+b^2\right)^3}-\frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}\,\cot(c+dx)+\sqrt{2}\,\sqrt{e\cot(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}\,de^{3/2}\left(a^2+b^2\right)^3}-\frac{b^2(13a^2+5b^2)}{4a^2de\left(a^2+b^2\right)^2\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))}-\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^2}+\frac{b^{5/2}(63a^4+46a^2b^2+15b^4)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4a^{7/2}de^{3/2}\left(a^2+b^2\right)^3}+\frac{8a^4+31a^2b^2+15b^4}{4a^3de\left(a^2+b^2\right)^2\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^3),x]
```

```
[Out] (b^(5/2)*(63*a^4 + 46*a^2*b^2 + 15*b^4)*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]
])/(Sqrt[a]*Sqrt[e])])/(4*a^(7/2)*(a^2 + b^2)^3*d*e^(3/2)) - ((a - b)*(a^2
+ 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*(a^2 + b^2)^3*d*e^(3/2)) + ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d*e^(3/2)) + (8*a^
4 + 31*a^2*b^2 + 15*b^4)/(4*a^3*(a^2 + b^2)^2*d*e*Sqrt[e*Cot[c + d*x]]) - b
^2/(2*a*(a^2 + b^2)*d*e*Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2) - (b^2
*(13*a^2 + 5*b^2))/(4*a^2*(a^2 + b^2)^2*d*e*Sqrt[e*Cot[c + d*x]]*(a + b*Cot
[c + d*x])) + ((a + b)*(a^2 - 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*
x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d*e^(3/2)) - (
(a + b)*(a^2 - 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sq
rt[e*Cot[c + d*x]]])/(2*Sqrt[2]*(a^2 + b^2)^3*d*e^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
 NeQ[c^2 + d^2, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx = -\frac{b^2}{2a\left(a^2+b^2\right)de\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{b^2}{2a\left(a^2+b^2\right)de\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^2} - \frac{\cdots}{4a^2\left(a\cdots\right.}$$

$$= \frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2de\sqrt{e\cot(c+dx)}} - \frac{b^2\cdots}{2a\left(a^2+b^2\right)de\sqrt{e\cot(c+\cdots}}$$

$$= \frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2de\sqrt{e\cot(c+dx)}} - \frac{b^2\cdots}{2a\left(a^2+b^2\right)de\sqrt{e\cot(c+\cdots}}$$

$$= \frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2de\sqrt{e\cot(c+dx)}} - \frac{b^2\cdots}{2a\left(a^2+b^2\right)de\sqrt{e\cot(c+\cdots}}$$

$$= \frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2de\sqrt{e\cot(c+dx)}} - \frac{b^2\cdots}{2a\left(a^2+b^2\right)de\sqrt{e\cot(c+\cdots}}$$

$$= \frac{b^{5/2}\left(63a^4+46a^2b^2+15b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4a^{7/2}\left(a^2+b^2\right)^3de^{3/2}} + \frac{\cdots}{4\cdots}$$

$$= \frac{b^{5/2}\left(63a^4+46a^2b^2+15b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4a^{7/2}\left(a^2+b^2\right)^3de^{3/2}} + \frac{\cdots}{4\cdots}$$

$$= \frac{b^{5/2}\left(63a^4+46a^2b^2+15b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cot(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{4a^{7/2}\left(a^2+b^2\right)^3de^{3/2}} - \frac{(\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.90, size = 303, normalized size = 0.57

$$-\frac{-8a^2b^2(3a^2-b^2)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{b\cot(c+dx)}{a}\right)-16a^2b^2(a^2+b^2)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{b\cot(c+dx)}{a}\right)-8b^2(a^2+b^2)^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};-\frac{b\cot(c+dx)}{a}\right)-8a^4(a^2-3b^2)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(c+dx)\right)+\sqrt{2}\,a^3b(3a^2-b^2)\sqrt{\cot(c+dx)}\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\right)}{4a^3(a^2+b^2)^3\,de\sqrt{e\cot(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^3),x]
```

```
[Out] -1/4*(-8*a^2*b^2*(3*a^2 - b^2)*Hypergeometric2F1[-1/2, 1, 1/2, -((b*Cot[c +
 d*x])/a)] - 16*a^2*b^2*(a^2 + b^2)*Hypergeometric2F1[-1/2, 2, 1/2, -((b*Co
t[c + d*x])/a)] - 8*b^2*(a^2 + b^2)^2*Hypergeometric2F1[-1/2, 3, 1/2, -((b*
Cot[c + d*x])/a)] - 8*a^4*(a^2 - 3*b^2)*Hypergeometric2F1[-1/4, 1, 3/4, -Co
t[c + d*x]^2] + Sqrt[2]*a^3*b*(3*a^2 - b^2)*Sqrt[Cot[c + d*x]]*(2*ArcTan[1
- Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] +
Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[C
ot[c + d*x]] + Cot[c + d*x]]))/(a^3*(a^2 + b^2)^3*d*e*Sqrt[e*Cot[c + d*x]])
```

Maple [A]
time = 0.65, size = 480, normalized size = 0.91

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{b^3\left(\frac{\left(\frac{15}{8}a^4b+\frac{11}{4}a^2b^3+\frac{7}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}+\frac{ae\left(17a^4+26a^2b^2+9b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(63a^4+46a^2b^2+15b^4\right)\cdots}{\cdots}\right)}{a^3e^5\left(a^2+b^2\right)^3}\cdots\right)}{\cdots}$
default	$-\frac{2e^4\left(-\frac{b^3\left(\frac{\left(\frac{15}{8}a^4b+\frac{11}{4}a^2b^3+\frac{7}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}+\frac{ae\left(17a^4+26a^2b^2+9b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(63a^4+46a^2b^2+15b^4\right)\cdots}{\cdots}\right)}{a^3e^5\left(a^2+b^2\right)^3}\cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*e^4*(-b^3/a^3/e^5/(a^2+b^2)^3*(((15/8*a^4*b+11/4*a^2*b^3+7/8*b^5)*(e*c
ot(d*x+c))^(3/2)+1/8*a*e*(17*a^4+26*a^2*b^2+9*b^4)*(e*cot(d*x+c))^(1/2))/(e
```

```
*cot(d*x+c)*b+a*e)^2+1/8*(63*a^4+46*a^2*b^2+15*b^4)/(a*e*b)^(1/2)*arctan(b*
(e*cot(d*x+c))^(1/2)/(a*e*b)^(1/2)))+1/(a^2+b^2)^3/e^5*(1/8*(-3*a^2*b*e+b^3
*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-a^3+3*a*b^2)/(e^2
)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(
1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/
2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/a^3/e^5/(e*cot(d*x+c))^(1/2))
```

Maxima [A]
time = 0.53, size = 472, normalized size = 0.89

$$\frac{\left(\frac{(63\,a^4b^3+46\,a^2b^5+15\,b^7)\arctan\left(\frac{b}{\sqrt{ab}\sqrt{\tan\left(dx+c\right)}}\right)}{(a^9+3\,a^7b^2+3\,a^5b^4+a^3b^6)\sqrt{ab}}+\frac{2\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)+2\sqrt{2}\,(a^3+3\,a^2b-3\,ab^2-b^3)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan\left(dx+c\right)}}\right)\right)-\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\,(a^3-3\,a^2b-3\,ab^2+b^3)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan\left(dx+c\right)}}+\frac{1}{\tan(dx+c)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{8\,a^6+16\,a^4b^2+8\,a^2b^4+\frac{16\,a^5b+49\,a^3b^3+25\,ab^5}{\tan(dx+c)}+\frac{8\,a^4b^2+31\,a^2b^4+15\,b^6}{\tan(dx+c)^2}}{\frac{a^9+2\,a^7b^2+a^5b^4}{\sqrt{\tan\left(dx+c\right)}}+\frac{2\,(a^8b+2\,a^6b^3+a^4b^5)}{\tan(dx+c)^{\frac{3}{2}}}+\frac{a^7b^2+2\,a^5b^4+a^3b^6}{\tan(dx+c)^{\frac{5}{2}}}}\right)e^{\left(-\frac{3}{2}\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((63*a^4*b^3 + 46*a^2*b^5 + 15*b^7)*arctan(b/(sqrt(a*b)*sqrt(tan(d*x +
c))))/((a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*sqrt(a*b)) + (2*sqrt(2)*(a^3
 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x +
c)))) + 2*sqrt(2)*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*arctan(-1/2*sqrt(2)*(sqrt
(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*log(
sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(a^3 - 3*a^2*b -
 3*a*b^2 + b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (8*a^6 + 16*a^4*b^2 + 8*a^2*b^4 + (16*a^5
*b + 49*a^3*b^3 + 25*a*b^5)/tan(d*x + c) + (8*a^4*b^2 + 31*a^2*b^4 + 15*b^6
)/tan(d*x + c)^2)/((a^9 + 2*a^7*b^2 + a^5*b^4)/sqrt(tan(d*x + c)) + 2*(a^8*
b + 2*a^6*b^3 + a^4*b^5)/tan(d*x + c)^(3/2) + (a^7*b^2 + 2*a^5*b^4 + a^3*b^
6)/tan(d*x + c)^(5/2)))*e^(-3/2)/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}\left(a+b\cot\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(3/2)), x)
```

Mupad [B]
time = 10.00, size = 2500, normalized size = 4.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))^3),x)
```

```
[Out] ((2*e)/a + (e*cot(c + d*x)*(16*a^4*b + 25*b^5 + 49*a^2*b^3))/(4*a^2*(a^4 +
b^4 + 2*a^2*b^2)) + (b^2*e^2*cot(c + d*x)^2*(8*a^4 + 15*b^4 + 31*a^2*b^2))/
(4*a^3*(a^4*e + b^4*e + 2*a^2*b^2*e)))/(b^2*d*(e*cot(c + d*x))^(5/2) + a^2*
d*e^2*(e*cot(c + d*x))^(1/2) + 2*a*b*d*e*(e*cot(c + d*x))^(3/2)) + atan((((
-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 1
5*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2)*(((e*
cot(c + d*x))^(1/2)*(471859200*a^22*b^44*d^7*e^16 + 9500098560*a^24*b^42*d^
7*e^16 + 91857354752*a^26*b^40*d^7*e^16 + 564502986752*a^28*b^38*d^7*e^16 +
 2464648527872*a^30*b^36*d^7*e^16 + 8104469069824*a^32*b^34*d^7*e^16 + 2076
9933361152*a^34*b^32*d^7*e^16 + 42351565209600*a^36*b^30*d^7*e^16 + 6953494
5902592*a^38*b^28*d^7*e^16 + 92434029608960*a^40*b^26*d^7*e^16 + 9950871735
5008*a^42*b^24*d^7*e^16 + 86342935511040*a^44*b^22*d^7*e^16 + 5976709555814
4*a^46*b^20*d^7*e^16 + 32432589897728*a^48*b^18*d^7*e^16 + 13411815522304*a
^50*b^16*d^7*e^16 + 4030457708544*a^52*b^14*d^7*e^16 + 805425905664*a^54*b^
12*d^7*e^16 + 86608183296*a^56*b^10*d^7*e^16 + 1612709888*a^58*b^8*d^7*e^16
 + 16777216*a^60*b^6*d^7*e^16 + 167772160*a^62*b^4*d^7*e^16 + 16777216*a^64
*b^2*d^7*e^16) + (-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^
5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*
e^3)))^(1/2)*(251658240*a^24*b^45*d^8*e^18 - (e*cot(c + d*x))^(1/2)*(-1i/(4
*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*
b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2)*(134217728*
a^27*b^45*d^9*e^19 + 2550136832*a^29*b^43*d^9*e^19 + 22817013760*a^31*b^41*
```

d^9*e^19 + 127506841600*a^33*b^39*d^9*e^19 + 497276682240*a^35*b^37*d^9*e^1
9 + 1430626762752*a^37*b^35*d^9*e^19 + 3121367482368*a^39*b^33*d^9*e^19 + 5
202279137280*a^41*b^31*d^9*e^19 + 6502848921600*a^43*b^29*d^9*e^19 + 563580
2398720*a^45*b^27*d^9*e^19 + 2254320959488*a^47*b^25*d^9*e^19 - 22543209594
88*a^49*b^23*d^9*e^19 - 5635802398720*a^51*b^21*d^9*e^19 - 6502848921600*a^
53*b^19*d^9*e^19 - 5202279137280*a^55*b^17*d^9*e^19 - 3121367482368*a^57*b^
15*d^9*e^19 - 1430626762752*a^59*b^13*d^9*e^19 - 497276682240*a^61*b^11*d^9
*e^19 - 127506841600*a^63*b^9*d^9*e^19 - 22817013760*a^65*b^7*d^9*e^19 - 25
50136832*a^67*b^5*d^9*e^19 - 134217728*a^69*b^3*d^9*e^19) + 5049942016*a^26
*b^43*d^8*e^18 + 48368713728*a^28*b^41*d^8*e^18 + 293819383808*a^30*b^39*d^
8*e^18 + 1268458192896*a^32*b^37*d^8*e^18 + 4132731617280*a^34*b^35*d^8*e^1
8 + 10531192700928*a^36*b^33*d^8*e^18 + 21462823993344*a^38*b^31*d^8*e^18 +
35469618315264*a^40*b^29*d^8*e^18 + 47896904859648*a^42*b^27*d^8*e^18 + 52
983958077440*a^44*b^25*d^8*e^18 + 47896904859648*a^46*b^23*d^8*e^18 + 35090
285461504*a^48*b^21*d^8*e^18 + 20487396655104*a^50*b^19*d^8*e^18 + 92306229
16608*a^52*b^17*d^8*e^18 + 2994733056000*a^54*b^15*d^8*e^18 + 565576728576*
a^56*b^13*d^8*e^18 - 18572378112*a^58*b^11*d^8*e^18 - 50281316352*a^60*b^9*
d^8*e^18 - 16089350144*a^62*b^7*d^8*e^18 - 2516582400*a^64*b^5*d^8*e^18 - 1
67772160*a^66*b^3*d^8*e^18))*(-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2
*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*
a^4*b^2*d^2*e^3)))^(1/2) - 117964800*a^21*b^42*d^6*e^15 - 841482240*a^23*b^
40*d^6*e^15 + 3829399552*a^25*b^38*d^6*e^15 + 78068580352*a^27*b^36*d^6*e^1
5 + 497438162944*a^29*b^34*d^6*e^15 + 1899895980032*a^31*b^32*d^6*e^15 + 49
72695519232*a^33*b^30*d^6*e^15 + 9371195015168*a^35*b^28*d^6*e^15 + 1289072
0436224*a^37*b^26*d^6*e^15 + 12726089809920*a^39*b^24*d^6*e^15 + 8366961197
056*a^41*b^22*d^6*e^15 + 2597662490624*a^43*b^20*d^6*e^15 - 1171836108800*a
^45*b^18*d^6*e^15 - 1986881650688*a^47*b^16*d^6*e^15 - 1237583921152*a^49*b
^14*d^6*e^15 - 449507753984*a^51*b^12*d^6*e^15 - 97476149248*a^53*b^10*d^6*
e^15 - 11931222016*a^55*b^8*d^6*e^15 - 1006632960*a^57*b^6*d^6*e^15 - 13421
7728*a^59*b^4*d^6*e^15 - 8388608*a^61*b^2*d^6*e^15) - (e*cot(c + d*x))^(1/2
)*(7610564608*a^27*b^33*d^5*e^13 - 597688320*a^23*b^37*d^5*e^13 - 167143014
4*a^25*b^35*d^5*e^13 - 58982400*a^21*b^39*d^5*e^13 + 85774565376*a^29*b^31*
d^5*e^13 + 385487994880*a^31*b^29*d^5*e^13 + 1104303620096*a^33*b^27*d^5*e^
13 + 2240523796480*a^35*b^25*d^5*e^13 + 3345249468416*a^37*b^23*d^5*e^13 +
3717287903232*a^39*b^21*d^5*e^13 + 3053967114240*a^41*b^19*d^5*e^13 + 18074
74491392*a^43*b^17*d^5*e^13 + 726513221632*a^45*b^15*d^5*e^13 + 17076899020
8*a^47*b^13*d^5*e^13 + 10492051456*a^49*b^11*d^5*e^13 - 4917821440*a^51*b^9
*d^5*e^13 - 923009024*a^53*b^7*d^5*e^13 + 8388608*a^55*b^5*d^5*e^13))*(-1i/
(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^
2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2)*1i + ((-1
i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*
a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2)*(((e*co
t(c + d*x))^(1/2)*(471859200*a^22*b^44*d^7*e^16 + 9500098560*a^24*b^42*d^7*
e^16 + 91857354752*a^26*b^40*d^7*e^16 + 564502986752*a^28*b^38*d^7*e^16 + 2
464648527872*a^30*b^36*d^7*e^16 + 8104469069824...

3.88 $\int (a+b\cot(c+dx))^n\,dx$

Optimal. Leaf size=167

$$-\frac{b(a+b\cot(c+dx))^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\cot(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)d(1+n)}+\frac{b(a+b\cot(c+dx))^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\cot(c+dx)}{a+\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\left(a+\sqrt{-b^2}\right)d(1+n)}$$

```
[Out] -1/2*b*(a+b*cot(d*x+c))^(1+n)*hypergeom([1, 1+n],[2+n],(a+b*cot(d*x+c))/(a-
(-b^2)^(1/2)))/d/(1+n)/(a-(-b^2)^(1/2))/(-b^2)^(1/2)+1/2*b*(a+b*cot(d*x+c))
^(1+n)*hypergeom([1, 1+n],[2+n],(a+b*cot(d*x+c))/(a+(-b^2)^(1/2)))/d/(1+n)/
(-b^2)^(1/2)/(a+(-b^2)^(1/2))
```

Rubi [A]
time = 0.18, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3566, 726, 70}

$$\frac{b(a+b\cot(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\cot(c+dx)}{a+\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\,d(n+1)\left(a+\sqrt{-b^2}\right)}-\frac{b(a+b\cot(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\cot(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\,d(n+1)\left(a-\sqrt{-b^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])^n,x]
```

```
[Out] -1/2*(b*(a + b*Cot[c + d*x])^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a
+ b*Cot[c + d*x])/(a - Sqrt[-b^2])])/(Sqrt[-b^2]*(a - Sqrt[-b^2])*d*(1 + n)
) + (b*(a + b*Cot[c + d*x])^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a +
 b*Cot[c + d*x])/(a + Sqrt[-b^2])])/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*d*(1 + n
))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a+b\cot(c+dx))^n\,dx = -\frac{b\text{Subst}\left(\int \frac{(a+x)^n}{b^2+x^2}\,dx, x, b\cot(c+dx)\right)}{d}$$

$$= -\frac{b\text{Subst}\left(\int \left(\frac{\sqrt{-b^2}\,(a+x)^n}{2b^2\left(\sqrt{-b^2}-x\right)}+\frac{\sqrt{-b^2}\,(a+x)^n}{2b^2\left(\sqrt{-b^2}+x\right)}\right)dx, x, b\cot(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}-x}\,dx, x, b\cot(c+dx)\right)}{2\sqrt{-b^2}\,d}+\frac{b\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}+x}\,dx, x, b\cot(c+\right.}{2\sqrt{-b^2}\,d}$$

$$= -\frac{b(a+b\cot(c+dx))^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\cot(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)d(1+n)}+\frac{b(a+b\cot(c+dx}{2\sqrt{-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.33, size = 118, normalized size = 0.71

$$\frac{(a+b\cot(c+dx))^{1+n}\left((a+ib)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\cot(c+dx)}{a-ib}\right)-(a-ib)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\cot(c+dx)}{a+ib}\right)\right)}{2(a-ib)(-ia+b)d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])^n,x]
```

```
[Out] ((a + b*Cot[c + d*x])^(1 + n)*((a + I*b)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (a + b*Cot[c + d*x])/(a - I*b)] - (a - I*b)*Hypergeometric2F1[1, 1 + n, 2
+ n, (a + b*Cot[c + d*x])/(a + I*b)]))/(2*(a - I*b)*((-I)*a + b)*d*(1 + n))
```

Maple [F]
time = 0.54, size = 0, normalized size = 0.00

$$\int \left(a+b\cot\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))^n,x)
```

```
[Out] int((a+b*cot(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*cot(d*x + c) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*cot(d*x + c) + a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \cot (c + dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))**n,x)
[Out] Integral((a + b*cot(c + d*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \cot(c + d\,x))^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x))^n,x)
[Out] int((a + b*cot(c + d*x))^n, x)
```

3.89 $\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx$

Optimal. Leaf size=193

$$-\frac{F_1\left(1-n;-m,1;2-n;-\frac{b\cot(e+fx)}{a},-i\cot(e+fx)\right)\cot(e+fx)(a+b\cot(e+fx))^m\left(1+\frac{b\cot(e+fx)}{a}\right)^{-m}(}{2f(1-n)}$$

```
[Out] -1/2*AppellF1(1-n,1,-m,2-n,-I*cot(f*x+e),-b*cot(f*x+e)/a)*cot(f*x+e)*(a+b*c
ot(f*x+e))^m*(d*tan(f*x+e))^n/f/(1-n)/((1+b*cot(f*x+e)/a)^m)-1/2*AppellF1(1
-n,1,-m,2-n,I*cot(f*x+e),-b*cot(f*x+e)/a)*cot(f*x+e)*(a+b*cot(f*x+e))^m*(d*
tan(f*x+e))^n/f/(1-n)/((1+b*cot(f*x+e)/a)^m)
```

Rubi [A]

time = 0.20, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4327, 3656, 926, 140, 138}

$$-\frac{\cot(e+fx)(d\tan(e+fx))^n(a+b\cot(e+fx))^m\left(\frac{b\cot(e+fx)}{a}+1\right)^{-m}F_1\left(1-n;-m,1;2-n;-\frac{b\cot(e+fx)}{a},-i\cot(e+fx)\right)}{2f(1-n)}-\frac{\cot(e+fx)(d\tan(e+fx))^n(a+b\cot(e+fx))^m\left(\frac{b\cot(e+fx)}{a}+1\right)^{-m}F_1\left(1-n;-m,1;2-n;-\frac{b\cot(e+fx)}{a},i\cot(e+fx)\right)}{2f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[e + f*x])^m*(d*Tan[e + f*x])^n,x]
```

```
[Out] -1/2*(AppellF1[1 - n, -m, 1, 2 - n, -((b*Cot[e + f*x])/a), (-I)*Cot[e + f*x
]]*Cot[e + f*x]*(a + b*Cot[e + f*x])^m*(d*Tan[e + f*x])^n)/(f*(1 - n)*(1 +
(b*Cot[e + f*x])/a)^m) - (AppellF1[1 - n, -m, 1, 2 - n, -((b*Cot[e + f*x])/
a), I*Cot[e + f*x]]*Cot[e + f*x]*(a + b*Cot[e + f*x])^m*(d*Tan[e + f*x])^n)
/(2*f*(1 - n)*(1 + (b*Cot[e + f*x])/a)^m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
```

```
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3656

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4327

```
Int[(u_)*((c_.)*tan[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Cot[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownCotangentIntegrandQ[u,
 x]
```

Rubi steps

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx = ((d\cot(e+fx))^n(d\tan(e+fx))^n)\int (d\cot(e+fx))^{-n}(a+b$$

$$= -\frac{((d\cot(e+fx))^n(d\tan(e+fx))^n)\,\text{Subst}\left(\int \frac{(dx)^{-n}(a+bx)^m}{1+x^2}\,dx\right.}{f}$$

$$= -\frac{((d\cot(e+fx))^n(d\tan(e+fx))^n)\,\text{Subst}\left(\int \left(\frac{i(dx)^{-n}(a+bx)^m}{2(i-x)}\right.\right.}{f}$$

$$= -\frac{(i(d\cot(e+fx))^n(d\tan(e+fx))^n)\,\text{Subst}\left(\int \frac{(dx)^{-n}(a+bx)^m}{i-x}\,d\right.}{2f}$$

$$= -\frac{\left(i(d\cot(e+fx))^n(a+b\cot(e+fx))^m\left(1+\frac{b\cot(e+fx)}{a}\right)^{-m}\right.}{}$$

$$= -\frac{F_1\left(1-n;-m,1;2-n;-\frac{b\cot(e+fx)}{a},-i\cot(e+fx)\right)\cot(e}{2f(}$$

Mathematica [F]
time = 3.30, size = 0, normalized size = 0.00

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Cot[e + f*x])^m*(d*Tan[e + f*x])^n,x]
```

```
[Out] Integrate[(a + b*Cot[e + f*x])^m*(d*Tan[e + f*x])^n, x]
```

Maple [F]
time = 0.77, size = 0, normalized size = 0.00

$$\int \left(a+b\cot\left(fx+e\right)\right)^{m}\left(d\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x)
```

```
[Out] int((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cot(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(e+fx\right)\right)^{n}\left(a+b\cot\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))**m*(d*tan(f*x+e))**n,x)
```

```
[Out] Integral((d*tan(e + f*x))**n*(a + b*cot(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*cot(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+f\,x))^n\,(a+b\cot(e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^n*(a + b*cot(e + f*x))^m,x)
[Out] int((d*tan(e + f*x))^n*(a + b*cot(e + f*x))^m, x)
```

3.90 $\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

Optimal. Leaf size=45

$$\frac{2i\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d}$$

```
[Out] 2*I*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3618, 65, 214}

$$\frac{2i\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + I*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

```
[Out] ((2*I)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = -\frac{i\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\cot(c+dx)\right)}{d}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd}$$
$$= \frac{2i\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d}$$

Mathematica [A]
time = 0.10, size = 45, normalized size = 1.00

$$\frac{2i\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
[Out] ((2*I)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 730 vs. 2(36) = 72.
time = 0.96, size = 731, normalized size = 16.24

method	result
derivativedivides	$-\frac{-\frac{\left(-2i\sqrt{a^2+b^2}\,a^2-i\sqrt{a^2+b^2}\,b^2-2ia^3-2iab^2+\sqrt{a^2+b^2}\,ab+a^2b+b^3\right)\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{}\right.}{2}}{}$
default	$-\frac{-\frac{\left(-2i\sqrt{a^2+b^2}\,a^2-i\sqrt{a^2+b^2}\,b^2-2ia^3-2iab^2+\sqrt{a^2+b^2}\,ab+a^2b+b^3\right)\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{}\right.}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)/((a^2+b^2)^(1/2)*a+a^
2+b^2)*(-1/2*(-2*I*(a^2+b^2)^(1/2)*a^2-I*(a^2+b^2)^(1/2)*b^2-2*I*a^3-2*I*a*
b^2+(a^2+b^2)^(1/2)*a*b+a^2*b+b^3)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))+2*(I*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*(a^2+b^2)^(1/2)*a^2+I*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+I*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*a*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*
b-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^3+1/2
*(-2*I*(a^2+b^2)^(1/2)*a^2-I*(a^2+b^2)^(1/2)*b^2-2*I*a^3-2*I*a*b^2+(a^2+b^2
)^(1/2)*a*b+a^2*b+b^3)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2
*(a^2+b^2)^(1/2)-2*a)^(1/2)))-1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/
2)*(1/2*(-I*(a^2+b^2)^(1/2)-I*a+b)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-I*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*a+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b-1/2*(-I*(a^2+b^2)^(1/2)-I*a+b)*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*
cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((I*cot(d*x + c) + 1)/sqrt(b*cot(d*x + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(33) = 66$.
time = 2.36, size = 159, normalized size = 3.53

$$-\frac{1}{2}\sqrt{-\frac{4i}{(ia+b)d^2}}\log\left(\frac{1}{2}(ia+b)d\sqrt{-\frac{4i}{(ia+b)d^2}}+\sqrt{\frac{(a+ib)e^{(2idx+2ic)}-a+ib}{e^{(2idx+2ic)}-1}}\right)+\frac{1}{2}\sqrt{-\frac{4i}{(ia+b)d^2}}\log\left(\frac{1}{2}(-ia-b)d\sqrt{-\frac{4i}{(ia+b)d^2}}+\sqrt{\frac{(a+ib)e^{(2idx+2ic)}-a+ib}{e^{(2idx+2ic)}-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-4*I/((I*a + b)*d^2))*log(1/2*(I*a + b)*d*sqrt(-4*I/((I*a + b)*d^
2)) + sqrt(((a + I*b)*e^(2*I*d*x + 2*I*c) - a + I*b)/(e^(2*I*d*x + 2*I*c) -
 1))) + 1/2*sqrt(-4*I/((I*a + b)*d^2))*log(1/2*(-I*a - b)*d*sqrt(-4*I/((I*a
```

```
 + b)*d^2)) + sqrt(((a + I*b)*e^(2*I*d*x + 2*I*c) - a + I*b)/(e^(2*I*d*x +
2*I*c) - 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$i\left(\int\left(-\frac{i}{\sqrt{a+b\cot(c+dx)}}\right)dx+\int\frac{\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*cot(d*x+c))/(a+b*cot(d*x+c))**(1/2),x)
[Out] I*(Integral(-I/sqrt(a + b*cot(c + d*x)), x) + Integral(cot(c + d*x)/sqrt(a
+ b*cot(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((I*cot(d*x + c) + 1)/sqrt(b*cot(d*x + c) + a), x)
```

Mupad [B]
time = 2.54, size = 1410, normalized size = 31.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)*1i + 1)/(a + b*cot(c + d*x))^(1/2),x)
[Out] (log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*cot(c + d*x))^(1/2) + 1i)*(-1/(a*
d^2 - b*d^2*1i))^(1/2))/2 - log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*cot(c
+ d*x))^(1/2)*1i + 1)*(-1/(4*(a*d^2 - b*d^2*1i)))^(1/2) + (log(16*b^3*d*(-1
/(d^2*(a - b*1i)))^(1/2) - 16*b^2*(a + b*cot(c + d*x))^(1/2) + (16*a*b^2*(a
 + b*cot(c + d*x))^(1/2))/(a - b*1i))*(-1/(a*d^2 - b*d^2*1i))^(1/2))/2 - lo
g(16*b^2*(a + b*cot(c + d*x))^(1/2) + 16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/2)
- (16*a*b^2*(a + b*cot(c + d*x))^(1/2))/(a - b*1i))*(-1/(4*(a*d^2 - b*d^2*1
i)))^(1/2) - 2*atanh((32*b^2*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2
+ 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((b^4*d^2*64i)/(4*a^2*d^3
+ 4*b^2*d^3) - (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a*b^3*(a + b*cot(
c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2)
)^(1/2)*128i)/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/
(4*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*
```

```
a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2*(a + b*cot(c + d*x))^(1/
2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((b^
6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2
*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a*b^5*d^2)/(4*a^2*
d^3 + 4*b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) - 2*atanh((3
2*b^2*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2
*d^2 + 4*b^2*d^2))^(1/2))/((a^2*b^2*d^2*64i)/(4*a^2*d^3 + 4*b^2*d^3) - (b^2
*16i)/d + (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2*(a + b*cot
(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2
))^(1/2))/((a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (a^2*b^2*64i)/d - (
b^4*64i)/d + (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a^4*b^2*d^2*256i)
/(4*a^2*d^3 + 4*b^2*d^3) + (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a*b^
3*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2
 + 4*b^2*d^2))^(1/2)*128i)/((a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (a
^2*b^2*64i)/d - (b^4*64i)/d + (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (
a^4*b^2*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (256*a*b^5*d^2)/(4*a^2*d^3 + 4*
b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)
```

3.91 $\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

Optimal. Leaf size=45

$$-\frac{2i\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d}$$

[Out] -2*I*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d/(a+I*b)^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3618, 65, 214}

$$-\frac{2i\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

Antiderivative was successfully verified.

[In] Int[(1 - I*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]

[Out] ((-2*I)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{i\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx, x, -i\cot(c+dx)\right)}{d}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd}$$
$$= -\frac{2i\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d}$$

Mathematica [A]
time = 1.43, size = 70, normalized size = 1.56

$$-\frac{2i\tanh^{-1}\left(\frac{\sqrt{a+\frac{ib\left(1+e^{2i(c+dx)}\right)}{-1+e^{2i(c+dx)}}}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - I*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
[Out] ((-2*I)*ArcTanh[Sqrt[a + (I*b*(1 + E^((2*I)*(c + d*x))))/(-1 + E^((2*I)*(c
+ d*x)))]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 738 vs. 2(36) = 72.
time = 0.47, size = 739, normalized size = 16.42

method	result
derivativedivides	$-\frac{\left(-2i\sqrt{a^2+b^2}\,a^2-i\sqrt{a^2+b^2}\,b^2-2ia^3-2iab^2-\sqrt{a^2+b^2}\,ab-a^2b-b^3\right)\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{}}\right.}{2}$

default	$-\frac{\left(-2i\sqrt{a^2+b^2}\,a^2-i\sqrt{a^2+b^2}\,b^2-2ia^3-2ia\,b^2-\sqrt{a^2+b^2}\,ab-a^2b-b^3\right)\ln\left(\sqrt{a+b\cot\left(dx+c\right)}\,\sqrt{2}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)/((a^2+b^2)^(1/2)*a+a^2
+b^2)*(-1/2*(-2*I*(a^2+b^2)^(1/2)*a^2-I*(a^2+b^2)^(1/2)*b^2-2*I*a^3-2*I*a*b
^2-(a^2+b^2)^(1/2)*a*b-a^2*b-b^3)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))+2*(I*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*(a^2+b^2)^(1/2)*a^2+I*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+I*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a*b^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^3+1/2*
(-2*I*(a^2+b^2)^(1/2)*a^2-I*(a^2+b^2)^(1/2)*b^2-2*I*a^3-2*I*a*b^2-(a^2+b^2)
^(1/2)*a*b-a^2*b-b^3)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a
)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*
(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2
)*(1/2*(-I*(a^2+b^2)^(1/2)-I*a-b)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-I*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*a-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b-1/2*(-I*(a^2+b^2)^(1/2)-I*a-b)*(2
*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*c
ot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/
2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-I*cot(d*x + c) + 1)/sqrt(b*cot(d*x + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(33) = 66$.
time = 2.53, size = 159, normalized size = 3.53

$$\frac{1}{2}\sqrt{\frac{4i}{(-ia+b)d^2}}\log\left(\frac{1}{2}(ia-b)d\sqrt{\frac{4i}{(-ia+b)d^2}}+\sqrt{\frac{(a+ib)e^{(2i\,dx+2i\,c)}-a+ib}{e^{(2i\,dx+2i\,c)}-1}}\right)-\frac{1}{2}\sqrt{\frac{4i}{(-ia+b)d^2}}\log\left(\frac{1}{2}(-ia+b)d\sqrt{\frac{4i}{(-ia+b)d^2}}+\sqrt{\frac{(a+ib)e^{(2i\,dx+2i\,c)}-a+ib}{e^{(2i\,dx+2i\,c)}-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I/((-I*a + b)*d^2))*log(1/2*(I*a - b)*d*sqrt(4*I/((-I*a + b)*d^2
)) + sqrt(((a + I*b)*e^(2*I*d*x + 2*I*c) - a + I*b)/(e^(2*I*d*x + 2*I*c) -
1))) - 1/2*sqrt(4*I/((-I*a + b)*d^2))*log(1/2*(-I*a + b)*d*sqrt(4*I/((-I*a
+ b)*d^2)) + sqrt(((a + I*b)*e^(2*I*d*x + 2*I*c) - a + I*b)/(e^(2*I*d*x + 2
*I*c) - 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-i\left(\int \frac{i}{\sqrt{a+b\cot\left(c+dx\right)}}\,dx+\int \frac{\cot\left(c+dx\right)}{\sqrt{a+b\cot\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-I*cot(d*x+c))/(a+b*cot(d*x+c))**(1/2),x)
```

```
[Out] -I*(Integral(I/sqrt(a + b*cot(c + d*x)), x) + Integral(cot(c + d*x)/sqrt(a
+ b*cot(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-I*cot(d*x + c) + 1)/sqrt(b*cot(d*x + c) + a), x)
```

Mupad [B]
time = 1.40, size = 1410, normalized size = 31.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cot(c + d*x)*1i - 1)/(a + b*cot(c + d*x))^(1/2),x)
```

```
[Out] (log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*1i + 1)*(-1/(
a*d^2 - b*d^2*1i))^(1/2))/2 - log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*cot(
c + d*x))^(1/2) + 1i)*(-1/(4*(a*d^2 - b*d^2*1i)))^(1/2) + (log(16*b^3*d*(-1
/(d^2*(a - b*1i)))^(1/2) - 16*b^2*(a + b*cot(c + d*x))^(1/2) + (16*a*b^2*(a
 + b*cot(c + d*x))^(1/2))/(a - b*1i))*(-1/(a*d^2 - b*d^2*1i))^(1/2))/2 - lo
g(16*b^2*(a + b*cot(c + d*x))^(1/2) + 16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/2)
- (16*a*b^2*(a + b*cot(c + d*x))^(1/2))/(a - b*1i))*(-1/(4*(a*d^2 - b*d^2*1
i)))^(1/2) - 2*atanh((32*b^2*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2
```

```
+ 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((b^4*d^2*64i)/(4*a^2*d^3
+ 4*b^2*d^3) - (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a*b^3*(a + b*cot(
c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2)
)^(1/2)*128i)/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/
(4*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*
a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2*(a + b*cot(c + d*x))^(1/
2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((b^
6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2
*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a*b^5*d^2)/(4*a^2*
d^3 + 4*b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2) + 2*atanh((3
2*b^2*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2
*d^2 + 4*b^2*d^2))^(1/2))/((a^2*b^2*d^2*64i)/(4*a^2*d^3 + 4*b^2*d^3) - (b^2
*16i)/d + (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2*(a + b*cot
(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2
))^(1/2))/((a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (a^2*b^2*64i)/d - (
b^4*64i)/d + (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a^4*b^2*d^2*256i)
/(4*a^2*d^3 + 4*b^2*d^3) + (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a*b^
3*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2
 + 4*b^2*d^2))^(1/2)*128i)/((a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (a
^2*b^2*64i)/d - (b^4*64i)/d + (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (
a^4*b^2*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (256*a*b^5*d^2)/(4*a^2*d^3 + 4*
b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)
```

3.92 $\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{(aA+bB)x}{a^2+b^2}-\frac{(Ab-aB)\log(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)\,d}$$

```
[Out] (A*a+B*b)*x/(a^2+b^2)-(A*b-B*a)*ln(b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {3612, 3611}

$$\frac{x(aA+bB)}{a^2+b^2}-\frac{(Ab-aB)\log(a\sin(c+dx)+b\cos(c+dx))}{d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x]),x]
```

```
[Out] ((a*A + b*B)*x)/(a^2 + b^2) - ((A*b - a*B)*Log[b*Cos[c + d*x] + a*Sin[c + d
*x]])/((a^2 + b^2)*d)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx&=\frac{(aA+bB)x}{a^2+b^2}-\frac{(Ab-aB)\int \frac{-b+a\cot(c+dx)}{a+b\cot(c+dx)}\,dx}{a^2+b^2}\\&=\frac{(aA+bB)x}{a^2+b^2}-\frac{(Ab-aB)\log(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)\,d}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 67, normalized size = 1.14

$$-\frac{2(aA+bB)\text{ArcTan}(\cot(c+dx))+(Ab-aB)\left(2\log(a+b\cot(c+dx))-\log\left(\csc^2(c+dx)\right)\right)}{2\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x]),x]
```

```
[Out] -1/2*(2*(a*A + b*B)*ArcTan[Cot[c + d*x]] + (A*b - a*B)*(2*Log[a + b*Cot[c +
 d*x]] - Log[Csc[c + d*x]^2]))/((a^2 + b^2)*d)
```

Maple [A]
time = 0.29, size = 91, normalized size = 1.54

method	result
norman	$\frac{(Aa+Bb)x}{a^2+b^2}+\frac{(Ab-Ba)\ln\left(1+\tan^2(dx+c)\right)}{2d(a^2+b^2)}-\frac{(Ab-Ba)\ln(a\tan(dx+c)+b)}{d(a^2+b^2)}$
derivativedivides	$\frac{-\frac{(Ab-Ba)\ln(a+b\cot(dx+c))}{a^2+b^2}+\frac{\frac{(Ab-Ba)\ln\left(\cot^2(dx+c)+1\right)}{2}+(-Aa-Bb)\left(\frac{\pi}{2}-\mathrm{arccot}(\cot(dx+c))\right)}{a^2+b^2}}{d}$
default	$\frac{-\frac{(Ab-Ba)\ln(a+b\cot(dx+c))}{a^2+b^2}+\frac{\frac{(Ab-Ba)\ln\left(\cot^2(dx+c)+1\right)}{2}+(-Aa-Bb)\left(\frac{\pi}{2}-\mathrm{arccot}(\cot(dx+c))\right)}{a^2+b^2}}{d}$
risch	$\frac{ixB}{ib+a}+\frac{xA}{ib+a}+\frac{2iAbx}{a^2+b^2}-\frac{2iBax}{a^2+b^2}+\frac{2iAbc}{d(a^2+b^2)}-\frac{2iBac}{d(a^2+b^2)}-\frac{\ln\left(e^{2i(dx+c)}+\frac{ib-a}{ib+a}\right)Ab}{d(a^2+b^2)}+\frac{\ln\left(e^{2i(dx+c)}+\frac{ib-a}{ib+a}\right)}{d(a^2+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(A*b-B*a)/(a^2+b^2)*ln(a+b*cot(d*x+c))+1/(a^2+b^2)*(1/2*(A*b-B*a)*ln(
cot(d*x+c)^2+1)+(-A*a-B*b)*(1/2*Pi-arccot(cot(d*x+c)))))
```

Maxima [A]
time = 0.54, size = 89, normalized size = 1.51

$$\frac{\frac{2(Aa+Bb)(dx+c)}{a^2+b^2}+\frac{2(Ba-Ab)\log(a\tan(dx+c)+b)}{a^2+b^2}-\frac{(Ba-Ab)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(A*a + B*b)*(d*x + c)/(a^2 + b^2) + 2*(B*a - A*b)*log(a*tan(d*x + c)
 + b)/(a^2 + b^2) - (B*a - A*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Fricas [A]
time = 2.69, size = 79, normalized size = 1.34

$$\frac{2\,(Aa+Bb)dx+(Ba-Ab)\log\left(ab\sin\left(2\,dx+2\,c\right)+\frac{1}{2}\,a^2+\frac{1}{2}\,b^2-\frac{1}{2}\,(a^2-b^2)\cos\left(2\,dx+2\,c\right)\right)}{2\,(a^2+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(A*a + B*b)*d*x + (B*a - A*b)*log(a*b*sin(2*d*x + 2*c) + 1/2*a^2 + 1
/2*b^2 - 1/2*(a^2 - b^2)*cos(2*d*x + 2*c)))/((a^2 + b^2)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 524, normalized size = 8.88

$$\begin{cases} \frac{\tilde{\infty}x(A+B\cot(c))}{\cot(c)} & \text{for}\: a=0\wedge b=0\wedge d=0 \\ \frac{iAdx\cot(c+dx)}{2bd\cot(c+dx)-2ibd}+\frac{Adx}{2bd\cot(c+dx)-2ibd}-\frac{iA}{2bd\cot(c+dx)-2ibd}+\frac{Bdx\cot(c+dx)}{2bd\cot(c+dx)-2ibd}-\frac{iBdx}{2bd\cot(c+dx)-2ibd}+\frac{B}{2bd\cot(c+dx)-2ibd} & \text{for}\: a=-ib \\ -\frac{iAdx\cot(c+dx)}{2bd\cot(c+dx)+2ibd}+\frac{Adx}{2bd\cot(c+dx)+2ibd}+\frac{iA}{2bd\cot(c+dx)+2ibd}+\frac{Bdx\cot(c+dx)}{2bd\cot(c+dx)+2ibd}+\frac{iBdx}{2bd\cot(c+dx)+2ibd}+\frac{B}{2bd\cot(c+dx)+2ibd} & \text{for}\: a=ib \\ \frac{x(A+B\cot(c))}{a+b\cot(c)} & \text{for}\: d=0 \\ \frac{\frac{A\log\left(\tan^2(c+dx)+1\right)}{2d}+Bx}{b} & \text{for}\: a=0 \\ \frac{2Aadx}{2a^2d+2b^2d}-\frac{2Ab\log\left(\tan(c+dx)+\frac{b}{a}\right)}{2a^2d+2b^2d}+\frac{Ab\log\left(\tan^2(c+dx)+1\right)}{2a^2d+2b^2d}+\frac{2Ba\log\left(\tan(c+dx)+\frac{b}{a}\right)}{2a^2d+2b^2d}-\frac{Ba\log\left(\tan^2(c+dx)+1\right)}{2a^2d+2b^2d}+\frac{2Bbdx}{2a^2d+2b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cot(c))/cot(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (I
*A*d*x*cot(c + d*x)/(2*b*d*cot(c + d*x) - 2*I*b*d) + A*d*x/(2*b*d*cot(c + d
*x) - 2*I*b*d) - I*A/(2*b*d*cot(c + d*x) - 2*I*b*d) + B*d*x*cot(c + d*x)/(2
*b*d*cot(c + d*x) - 2*I*b*d) - I*B*d*x/(2*b*d*cot(c + d*x) - 2*I*b*d) + B/(
2*b*d*cot(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (-I*A*d*x*cot(c + d*x)/(2*b*d*
cot(c + d*x) + 2*I*b*d) + A*d*x/(2*b*d*cot(c + d*x) + 2*I*b*d) + I*A/(2*b*d
*cot(c + d*x) + 2*I*b*d) + B*d*x*cot(c + d*x)/(2*b*d*cot(c + d*x) + 2*I*b*d
) + I*B*d*x/(2*b*d*cot(c + d*x) + 2*I*b*d) + B/(2*b*d*cot(c + d*x) + 2*I*b*
d), Eq(a, I*b)), (x*(A + B*cot(c))/(a + b*cot(c)), Eq(d, 0)), ((A*log(tan(c
 + d*x)**2 + 1)/(2*d) + B*x)/b, Eq(a, 0)), (2*A*a*d*x/(2*a**2*d + 2*b**2*d)
 - 2*A*b*log(tan(c + d*x) + b/a)/(2*a**2*d + 2*b**2*d) + A*b*log(tan(c + d*
x)**2 + 1)/(2*a**2*d + 2*b**2*d) + 2*B*a*log(tan(c + d*x) + b/a)/(2*a**2*d
+ 2*b**2*d) - B*a*log(tan(c + d*x)**2 + 1)/(2*a**2*d + 2*b**2*d) + 2*B*b*d*
x/(2*a**2*d + 2*b**2*d), True))
```

Giac [A]
time = 0.47, size = 95, normalized size = 1.61

$$\frac{\frac{2\,(Aa+Bb)(dx+c)}{a^2+b^2}-\frac{(Ba-Ab)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,(Ba^2-Aab)\log(|a\tan(dx+c)+b|)}{a^3+ab^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a + B*b)*(d*x + c)/(a^2 + b^2) - (B*a - A*b)*log(tan(d*x + c)^2 +
 1)/(a^2 + b^2) + 2*(B*a^2 - A*a*b)*log(abs(a*tan(d*x + c) + b))/(a^3 + a*b
^2))/d
```

Mupad [B]
time = 1.00, size = 155, normalized size = 2.63

$$\frac{A\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)}{2\,\left(b\,d+a\,d\,1\mathrm{i}\right)}-\frac{B\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)}{2\,\left(a\,d-b\,d\,1\mathrm{i}\right)}-\frac{A\,b\,\ln\left(a+b\cot(c+d\,x)\right)}{d\,\left(a^2+b^2\right)}+\frac{B\,a\,\ln\left(a+b\cot(c+d\,x)\right)}{d\,\left(a^2+b^2\right)}+\frac{A\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(a\,d+b\,d\,1\mathrm{i}\right)}-\frac{B\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(-b\,d+a\,d\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cot(c + d*x))/(a + b*cot(c + d*x)),x)
```

```
[Out] (A*log(cot(c + d*x) - 1i)*1i)/(2*(a*d + b*d*1i)) + (A*log(cot(c + d*x) + 1i
))/(2*(a*d*1i + b*d)) - (B*log(cot(c + d*x) + 1i))/(2*(a*d - b*d*1i)) - (B*
log(cot(c + d*x) - 1i)*1i)/(2*(a*d*1i - b*d)) - (A*b*log(a + b*cot(c + d*x)
))/(d*(a^2 + b^2)) + (B*a*log(a + b*cot(c + d*x)))/(d*(a^2 + b^2))
```

3.93 $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx$

Optimal. Leaf size=111

$$\frac{(a^2A-Ab^2+2abB)\,x}{(a^2+b^2)^2}+\frac{Ab-aB}{(a^2+b^2)\,d(a+b\cot(c+dx))}-\frac{(2aAb-a^2B+b^2B)\log(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)^2\,d}$$

```
[Out] (A*a^2-A*b^2+2*B*a*b)*x/(a^2+b^2)^2+(A*b-B*a)/(a^2+b^2)/d/(a+b*cot(d*x+c))-
(2*A*a*b-B*a^2+B*b^2)*ln(b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)^2/d
```

Rubi [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3610, 3612, 3611}

$$\frac{Ab-aB}{d\,(a^2+b^2)\,(a+b\cot(c+dx))}-\frac{(a^2(-B)+2aAb+b^2B)\log(a\sin(c+dx)+b\cos(c+dx))}{d\,(a^2+b^2)^2}+\frac{x(a^2A+2abB-Ab^2)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] ((a^2*A - A*b^2 + 2*a*b*B)*x)/(a^2 + b^2)^2 + (A*b - a*B)/((a^2 + b^2)*d*(a
 + b*Cot[c + d*x])) - ((2*a*A*b - a^2*B + b^2*B)*Log[b*Cos[c + d*x] + a*Sin
[c + d*x]])/((a^2 + b^2)^2*d)
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
```

```
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx = \frac{Ab-aB}{(a^2+b^2)\,d(a+b\cot(c+dx))} + \frac{\int \frac{aA+bB-(Ab-aB)\cot(c+dx)}{a+b\cot(c+dx)}\,dx}{a^2+b^2}$$
$$= \frac{(a^2A-Ab^2+2abB)\,x}{(a^2+b^2)^2} + \frac{Ab-aB}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{(2aAb-a^2B+b^2B}{(a^2+}$$
$$= \frac{(a^2A-Ab^2+2abB)\,x}{(a^2+b^2)^2} + \frac{Ab-aB}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{(2aAb-a^2B+b^2B}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.02, size = 144, normalized size = 1.30

$$\frac{-\frac{(iA+B)\log(i-\tan(c+dx))}{(a-ib)^2}+\frac{i(A+iB)\log(i+\tan(c+dx))}{(a+ib)^2}+\frac{2(-2aAb+a^2B-b^2B)\log(b+a\tan(c+dx))}{(a^2+b^2)^2}+\frac{2b(-Ab+aB)}{a(a^2+b^2)(b+a\tan(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^2,x]
```

```
[Out] (-(((I*A + B)*Log[I - Tan[c + d*x]])/(a - I*b)^2) + (I*(A + I*B)*Log[I + Ta
n[c + d*x]])/(a + I*b)^2 + (2*(-2*a*A*b + a^2*B - b^2*B)*Log[b + a*Tan[c +
d*x]])/(a^2 + b^2)^2 + (2*b*(-(A*b) + a*B))/(a*(a^2 + b^2)*(b + a*Tan[c + d
*x]))/(2*d)
```

Maple [A]
time = 0.35, size = 147, normalized size = 1.32

method	result
derivativedivides	$\frac{\frac{Ab-Ba}{(a^2+b^2)(a+b\cot(dx+c))}-\frac{(2Aab-B\,a^2+B\,b^2)\ln(a+b\cot(dx+c))}{(a^2+b^2)^2}+\frac{\frac{(2Aab-B\,a^2+B\,b^2)\ln(\cot^2(dx+c)+1)}{2}+(-A\,a^2+A\,b^2-2Bal}{(a^2+b^2)^2}}{d}$
default	$\frac{\frac{Ab-Ba}{(a^2+b^2)(a+b\cot(dx+c))}-\frac{(2Aab-B\,a^2+B\,b^2)\ln(a+b\cot(dx+c))}{(a^2+b^2)^2}+\frac{\frac{(2Aab-B\,a^2+B\,b^2)\ln(\cot^2(dx+c)+1)}{2}+(-A\,a^2+A\,b^2-2Bal}{(a^2+b^2)^2}}{d}$
norman	$\frac{\frac{b(A\,a^2-A\,b^2+2Bab)x}{a^4+2a^2b^2+b^4}+\frac{a(A\,a^2-A\,b^2+2Bab)x\tan(dx+c)}{a^4+2a^2b^2+b^4}-\frac{(Ab-Ba)b}{ad(a^2+b^2)}}{a\tan(dx+c)+b}+\frac{(2Aab-B\,a^2+B\,b^2)\ln(1+\tan^2(dx+c))}{2d(a^4+2a^2b^2+b^4)}-\frac{(2A}{}$
risch	$\frac{ixB}{2iab+a^2-b^2}+\frac{xA}{2iab+a^2-b^2}+\frac{4iAabx}{a^4+2a^2b^2+b^4}-\frac{2iB\,a^2x}{a^4+2a^2b^2+b^4}+\frac{2iB\,b^2x}{a^4+2a^2b^2+b^4}+\frac{4iAabc}{d(a^4+2a^2b^2+b^4)}-\frac{2i}{d(a^4+2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((A*b-B*a)/(a^2+b^2)/(a+b*cot(d*x+c))-(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2
*ln(a+b*cot(d*x+c))+1/(a^2+b^2)^2*(1/2*(2*A*a*b-B*a^2+B*b^2)*ln(cot(d*x+c)^
2+1)+(-A*a^2+A*b^2-2*B*a*b)*(1/2*Pi-arccot(cot(d*x+c)))))
```

Maxima [A]
time = 0.53, size = 185, normalized size = 1.67

$$\frac{\frac{2\,(Aa^2+2\,Bab-Ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Ba^2-2\,Aab-Bb^2)\log(a\tan(dx+c)+b)}{a^4+2\,a^2b^2+b^4}-\frac{(Ba^2-2\,Aab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Bab-Ab^2)}{a^3b+ab^3+(a^4+a^2b^2)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(B*a
^2 - 2*A*a*b - B*b^2)*log(a*tan(d*x + c) + b)/(a^4 + 2*a^2*b^2 + b^4) - (B*
a^2 - 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*
(B*a*b - A*b^2)/(a^3*b + a*b^3 + (a^4 + a^2*b^2)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(111) = 222$.
time = 3.32, size = 340, normalized size = 3.06

$$\frac{2\,Ba^2b-2\,Aab^2+2\,(Aa^2b+2\,Bab^2-Ab^3)dx+2\,(Ba^2b-Aab^2+(Aa^2b+2\,Bab^2-Ab^3)dx)\cos(2\,dx+2\,c)+(Ba^2b-2\,Aab^2-Bb^3+(Ba^2b-2\,Aab^2-Bb^3)\cos(2\,dx+2\,c)+(Ba^3-2\,Aa^2b-Bab^2)\sin(2\,dx+2\,c))\log\left(ab\sin(2\,dx+2\,c)+\frac{1}{2}\,a^2+\frac{1}{2}\,b^2-\frac{1}{2}\,(a^2-b^2)\cos(2\,dx+2\,c)\right)-2\,(Bab^2-Ab^3-(Aa^3+2\,Ba^2b-Aab^2)dx)\sin(2\,dx+2\,c)}{2\,((a^4b+2\,a^2b^3+b^5)d\cos(2\,dx+2\,c)+(a^5+2\,a^3b^2+ab^4)d\sin(2\,dx+2\,c)+(a^4b+2\,a^2b^3+b^5)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*B*a^2*b - 2*A*a*b^2 + 2*(A*a^2*b + 2*B*a*b^2 - A*b^3)*d*x + 2*(B*a^2
*b - A*a*b^2 + (A*a^2*b + 2*B*a*b^2 - A*b^3)*d*x)*cos(2*d*x + 2*c) + (B*a^2
*b - 2*A*a*b^2 - B*b^3 + (B*a^2*b - 2*A*a*b^2 - B*b^3)*cos(2*d*x + 2*c) + (
B*a^3 - 2*A*a^2*b - B*a*b^2)*sin(2*d*x + 2*c))*log(a*b*sin(2*d*x + 2*c) + 1
/2*a^2 + 1/2*b^2 - 1/2*(a^2 - b^2)*cos(2*d*x + 2*c)) - 2*(B*a*b^2 - A*b^3 -
 (A*a^3 + 2*B*a^2*b - A*a*b^2)*d*x)*sin(2*d*x + 2*c))/((a^4*b + 2*a^2*b^3 +
 b^5)*d*cos(2*d*x + 2*c) + (a^5 + 2*a^3*b^2 + a*b^4)*d*sin(2*d*x + 2*c) + (
a^4*b + 2*a^2*b^3 + b^5)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.59, size = 3966, normalized size = 35.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x*(A + B*cot(c))/cot(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)),
 ((-A*x + A*tan(c + d*x)/d + B*log(tan(c + d*x)**2 + 1)/(2*d))/b**2, Eq(a,
0)), (-A*d*x*cot(c + d*x)**2/(4*b**2*d*cot(c + d*x)**2 - 8*I*b**2*d*cot(c +
 d*x) - 4*b**2*d) + 2*I*A*d*x*cot(c + d*x)/(4*b**2*d*cot(c + d*x)**2 - 8*I*
b**2*d*cot(c + d*x) - 4*b**2*d) + A*d*x/(4*b**2*d*cot(c + d*x)**2 - 8*I*b**
2*d*cot(c + d*x) - 4*b**2*d) + A*cot(c + d*x)/(4*b**2*d*cot(c + d*x)**2 - 8
*I*b**2*d*cot(c + d*x) - 4*b**2*d) - 2*I*A/(4*b**2*d*cot(c + d*x)**2 - 8*I*
b**2*d*cot(c + d*x) - 4*b**2*d) + I*B*d*x*cot(c + d*x)**2/(4*b**2*d*cot(c +
 d*x)**2 - 8*I*b**2*d*cot(c + d*x) - 4*b**2*d) + 2*B*d*x*cot(c + d*x)/(4*b*
*2*d*cot(c + d*x)**2 - 8*I*b**2*d*cot(c + d*x) - 4*b**2*d) - I*B*d*x/(4*b**
2*d*cot(c + d*x)**2 - 8*I*b**2*d*cot(c + d*x) - 4*b**2*d) - I*B*cot(c + d*x
)/(4*b**2*d*cot(c + d*x)**2 - 8*I*b**2*d*cot(c + d*x) - 4*b**2*d), Eq(a, -I
*b)), (-A*d*x*cot(c + d*x)**2/(4*b**2*d*cot(c + d*x)**2 + 8*I*b**2*d*cot(c
+ d*x) - 4*b**2*d) - 2*I*A*d*x*cot(c + d*x)/(4*b**2*d*cot(c + d*x)**2 + 8*I
*b**2*d*cot(c + d*x) - 4*b**2*d) + A*d*x/(4*b**2*d*cot(c + d*x)**2 + 8*I*b*
*2*d*cot(c + d*x) - 4*b**2*d) + A*cot(c + d*x)/(4*b**2*d*cot(c + d*x)**2 +
8*I*b**2*d*cot(c + d*x) - 4*b**2*d) + 2*I*A/(4*b**2*d*cot(c + d*x)**2 + 8*I
*b**2*d*cot(c + d*x) - 4*b**2*d) - I*B*d*x*cot(c + d*x)**2/(4*b**2*d*cot(c
+ d*x)**2 + 8*I*b**2*d*cot(c + d*x) - 4*b**2*d) + 2*B*d*x*cot(c + d*x)/(4*b
**2*d*cot(c + d*x)**2 + 8*I*b**2*d*cot(c + d*x) - 4*b**2*d) + I*B*d*x/(4*b*
*2*d*cot(c + d*x)**2 + 8*I*b**2*d*cot(c + d*x) - 4*b**2*d) + I*B*cot(c + d*
x)/(4*b**2*d*cot(c + d*x)**2 + 8*I*b**2*d*cot(c + d*x) - 4*b**2*d), Eq(a, I
*b)), (A*d*x*tan(c + d*x)**2*cot(c + d*x)**2/(8*b**2*d*tan(c + d*x)**2*cot(
c + d*x)**2 - 16*b**2*d*tan(c + d*x)*cot(c + d*x) + 8*b**2*d) + 3*A*d*x*tan
(c + d*x)**2/(8*b**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*b**2*d*tan(c +
d*x)*cot(c + d*x) + 8*b**2*d) + 4*A*d*x*tan(c + d*x)*cot(c + d*x)/(8*b**2*d
*tan(c + d*x)**2*cot(c + d*x)**2 - 16*b**2*d*tan(c + d*x)*cot(c + d*x) + 8*
b**2*d) + 3*A*d*x*cot(c + d*x)**2/(8*b**2*d*tan(c + d*x)**2*cot(c + d*x)**2
 - 16*b**2*d*tan(c + d*x)*cot(c + d*x) + 8*b**2*d) + A*d*x/(8*b**2*d*tan(c
+ d*x)**2*cot(c + d*x)**2 - 16*b**2*d*tan(c + d*x)*cot(c + d*x) + 8*b**2*d)
 + A*tan(c + d*x)*cot(c + d*x)**2/(8*b**2*d*tan(c + d*x)**2*cot(c + d*x)**2
 - 16*b**2*d*tan(c + d*x)*cot(c + d*x) + 8*b**2*d) - 5*A*tan(c + d*x)/(8*b*
*2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*b**2*d*tan(c + d*x)*cot(c + d*x)
+ 8*b**2*d) - 4*A*cot(c + d*x)/(8*b**2*d*tan(c + d*x)**2*cot(c + d*x)**2 -
16*b**2*d*tan(c + d*x)*cot(c + d*x) + 8*b**2*d) + 2*B*d*x*tan(c + d*x)**2*c
ot(c + d*x)/(8*b**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*b**2*d*tan(c + d
*x)*cot(c + d*x) + 8*b**2*d) + 2*B*d*x*tan(c + d*x)*cot(c + d*x)**2/(8*b**2
*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*b**2*d*tan(c + d*x)*cot(c + d*x) +
8*b**2*d) - 2*B*d*x*tan(c + d*x)/(8*b**2*d*tan(c + d*x)**2*cot(c + d*x)**2
- 16*b**2*d*tan(c + d*x)*cot(c + d*x) + 8*b**2*d) - 2*B*d*x*cot(c + d*x)/(8
*b**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*b**2*d*tan(c + d*x)*cot(c + d*
```

```
x) + 8*b**2*d) + 3*B*tan(c + d*x)**2/(8*b**2*d*tan(c + d*x)**2*cot(c + d*x)
**2 - 16*b**2*d*tan(c + d*x)*cot(c + d*x) + 8*b**2*d) + B*cot(c + d*x)**2/(
8*b**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*b**2*d*tan(c + d*x)*cot(c + d
*x) + 8*b**2*d) + 2*B/(8*b**2*d*tan(c + d*x)**2*cot(c + d*x)**2 - 16*b**2*d
*tan(c + d*x)*cot(c + d*x) + 8*b**2*d), Eq(a, -b/tan(c + d*x))), (x*(A + B*
cot(c))/(a + b*cot(c))**2, Eq(d, 0)), (2*A*a**4*d*x*tan(c + d*x)/(2*a**6*d*
tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*
a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + 2*A*a**3*b*d*x/(2*a**6*d*tan(c + d
*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4
*d*tan(c + d*x) + 2*a*b**5*d) - 4*A*a**3*b*log(tan(c + d*x) + b/a)*tan(c +
d*x)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a
**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + 2*A*a**3*b*log(tan(
c + d*x)**2 + 1)*tan(c + d*x)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*
b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5
*d) - 2*A*a**2*b**2*d*x*tan(c + d*x)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d +
4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2
*a*b**5*d) - 4*A*a**2*b**2*log(tan(c + d*x) + b/a)/(2*a**6*d*tan(c + d*x) +
 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*ta
n(c + d*x) + 2*a*b**5*d) + 2*A*a**2*b**2*log(tan(c + d*x)**2 + 1)/(2*a**6*d
*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2
*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) - 2*A*a**2*b**2/(2*a**6*d*tan(c + d
*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4
*d*tan(c + d*x) + 2*a*b**5*d) - 2*A*a*b**3*d*x/(2*a**6*d*tan(c + d*x) + 2*a
**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c
+ d*x) + 2*a*b**5*d) - 2*A*b**4/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**
4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(111) = 222.

time = 0.52, size = 241, normalized size = 2.17

$$\frac{\frac{2\,(Aa^2+2\,Bab-Ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{(Ba^2-2\,Aab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Ba^3-2\,Aa^2b-Bab^2)\log(|a\tan(dx+c)+b|)}{a^5+2\,a^3b^2+ab^4}-\frac{2\,(Ba^4\tan(dx+c)-2\,Aa^3b\tan(dx+c)-Ba^2b^2\tan(dx+c)-Aa^2b^2-2\,Bab^3+Ab^4)}{(a^5+2\,a^3b^2+ab^4)(a\tan(dx+c)+b)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (B*a^2
 - 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(B*
a^3 - 2*A*a^2*b - B*a*b^2)*log(abs(a*tan(d*x + c) + b))/(a^5 + 2*a^3*b^2 +
a*b^4) - 2*(B*a^4*tan(d*x + c) - 2*A*a^3*b*tan(d*x + c) - B*a^2*b^2*tan(d*x
 + c) - A*a^2*b^2 - 2*B*a*b^3 + A*b^4)/((a^5 + 2*a^3*b^2 + a*b^4)*(a*tan(d*
x + c) + b)))/d
```

Mupad [B]

time = 1.46, size = 268, normalized size = 2.41

$$\ln\left(a+b\cot(c+d\,x)\right)\,\left(\frac{B}{d\,\left(a^2+b^2\right)}-\frac{2\,B\,b^2}{d\,{\left(a^2+b^2\right)}^2}\right)+\frac{A\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)}{2\,\left(-1\mathrm{i}\,d\,a^2+2\,d\,a\,b+1\mathrm{i}\,d\,b^2\right)}-\frac{B\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)}{2\,\left(d\,a^2+2\mathrm{i}\,d\,a\,b-d\,b^2\right)}+\frac{A\,b}{\left(a\,d+b\,d\cot\left(c+d\,x\right)\right)\,\left(a^2+b^2\right)}-\frac{B\,a}{\left(a\,d+b\,d\cot\left(c+d\,x\right)\right)\,\left(a^2+b^2\right)}-\frac{2\,A\,a\,b\,\ln\left(a+b\cot(c+d\,x)\right)}{d\,{\left(a^2+b^2\right)}^2}+\frac{A\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(-d\,a^2+2\mathrm{i}\,d\,a\,b+d\,b^2\right)}-\frac{B\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(1\mathrm{i}\,d\,a^2+2\,d\,a\,b-1\mathrm{i}\,d\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^2,x)
```

```
[Out] log(a + b*cot(c + d*x))*(B/(d*(a^2 + b^2)) - (2*B*b^2)/(d*(a^2 + b^2)^2)) +
 (A*log(cot(c + d*x) + 1i)*1i)/(2*(b^2*d - a^2*d + a*b*d*2i)) + (A*log(cot(
c + d*x) - 1i))/(2*(b^2*d*1i - a^2*d*1i + 2*a*b*d)) - (B*log(cot(c + d*x) -
 1i))/(2*(a^2*d - b^2*d + a*b*d*2i)) - (B*log(cot(c + d*x) + 1i)*1i)/(2*(a^
2*d*1i - b^2*d*1i + 2*a*b*d)) + (A*b)/((a*d + b*d*cot(c + d*x))*(a^2 + b^2)
) - (B*a)/((a*d + b*d*cot(c + d*x))*(a^2 + b^2)) - (2*A*a*b*log(a + b*cot(c
 + d*x)))/(d*(a^2 + b^2)^2)
```

3.94 $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx$

Optimal. Leaf size=175

$$\frac{(a^3A-3aAb^2+3a^2bB-b^3B)\,x}{(a^2+b^2)^3}+\frac{Ab-aB}{2\,(a^2+b^2)\,d(a+b\cot(c+dx))^2}+\frac{2aAb-a^2B+b^2B}{(a^2+b^2)^2\,d(a+b\cot(c+dx))}-\frac{(3a^2Ab$$

```
[Out] (A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*x/(a^2+b^2)^3+1/2*(A*b-B*a)/(a^2+b^2)/d/(
a+b*cot(d*x+c))^2+(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2/d/(a+b*cot(d*x+c))-(3*A
*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*ln(b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)^3/d
```

Rubi [A]

time = 0.19, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3610, 3612, 3611}

$$\frac{Ab-aB}{2d\,(a^2+b^2)\,(a+b\cot(c+dx))^2}+\frac{a^2(-B)+2aAb+b^2B}{d\,(a^2+b^2)^2\,(a+b\cot(c+dx))}-\frac{(a^3(-B)+3a^2Ab+3ab^2B-Ab^3)\log(a\sin(c+dx)+b\cos(c+dx))}{d\,(a^2+b^2)^3}+\frac{x(a^3A+3a^2bB-3aAb^2-b^3B)}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] ((a^3*A - 3*a*A*b^2 + 3*a^2*b*B - b^3*B)*x)/(a^2 + b^2)^3 + (A*b - a*B)/(2*
(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2) + (2*a*A*b - a^2*B + b^2*B)/((a^2 + b
^2)^2*d*(a + b*Cot[c + d*x])) - ((3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)*Lo
g[b*Cos[c + d*x] + a*Sin[c + d*x]])/((a^2 + b^2)^3*d)
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx = \frac{Ab-aB}{2\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{\int \frac{aA+bB-(Ab-aB)\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx}{a^2+b^2}$$

$$= \frac{Ab-aB}{2\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{2aAb-a^2B+b^2B}{\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))} + \frac{\int \frac{a^2A-Ab^2}{\cdots}}{\cdots}$$

$$= \frac{\left(a^3A-3aAb^2+3a^2bB-b^3B\right)x}{\left(a^2+b^2\right)^3} + \frac{Ab-aB}{2\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{2\cdots}{\left(a^2+b\cdots\right.}$$

$$= \frac{\left(a^3A-3aAb^2+3a^2bB-b^3B\right)x}{\left(a^2+b^2\right)^3} + \frac{Ab-aB}{2\left(a^2+b^2\right)d(a+b\cot(c+dx))^2} + \frac{2\cdots}{\left(a^2+b\cdots\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.21, size = 202, normalized size = 1.15

$$\frac{-\frac{i(A-iB)\log(i-\tan(c+dx))}{(a-ib)^3} + \frac{i(A+iB)\log(i+\tan(c+dx))}{(a+ib)^3} + \frac{2\left(-3a^2Ab+Ab^3+a^3B-3ab^2B\right)\log(b+a\tan(c+dx)) - \frac{b\left(a^2+b^2\right)\left(b\left(5a^2Ab+Ab^3-3a^3B+ab^2B\right)+\left(6a^3Ab+2aAb^3-4a^4B\right)\tan(c+dx)\right)}{a^2(b+a\tan(c+dx))^2}}{\left(a^2+b^2\right)^3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] (((-I)*(A - I*B)*Log[I - Tan[c + d*x]])/(a - I*b)^3 + (I*(A + I*B)*Log[I +
Tan[c + d*x]])/(a + I*b)^3 + (2*(-3*a^2*A*b + A*b^3 + a^3*B - 3*a*b^2*B)*Lo
g[b + a*Tan[c + d*x]] - (b*(a^2 + b^2)*(b*(5*a^2*A*b + A*b^3 - 3*a^3*B + a*
b^2*B) + (6*a^3*A*b + 2*a*A*b^3 - 4*a^4*B)*Tan[c + d*x]))/(a^2*(b + a*Tan[c
 + d*x])^2))/(a^2 + b^2)^3)/(2*d)
```

Maple [A]
time = 0.40, size = 216, normalized size = 1.23

method	result
derivativedivides	$\frac{\frac{\frac{\left(3A\,a^2b-A\,b^3-B\,a^3+3Ba\,b^2\right)\ln\left(\cot^2(dx+c)+1\right)}{2}+\left(-A\,a^3+3Aa\,b^2-3B\,a^2b+B\,b^3\right)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{\left(a^2+b^2\right)^3}-\frac{\left(3A\,a^2b-A\,b^3-B\,a^3\cdots\right.}{\left(\cdots\right.}}{d}$
default	$\frac{\frac{\frac{\left(3A\,a^2b-A\,b^3-B\,a^3+3Ba\,b^2\right)\ln\left(\cot^2(dx+c)+1\right)}{2}+\left(-A\,a^3+3Aa\,b^2-3B\,a^2b+B\,b^3\right)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{\left(a^2+b^2\right)^3}-\frac{\left(3A\,a^2b-A\,b^3-B\,a^3\cdots\right.}{\left(\cdots\right.}}{d}$

norman	$\frac{\frac{b^2\left(A\,a^3-3Aa\,b^2+3B\,a^2b-B\,b^3\right)x}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{\left(A\,a^3-3Aa\,b^2+3B\,a^2b-B\,b^3\right)a^2x\left(\tan^2(dx+c)\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}-\frac{b^2\left(5A\,a^2b+A\,b^3-3B\,a^3+Ba\,b^2\right)}{2d\,a^2\left(a^4+2a^2b^2+b^4\right)}-\frac{b\left(3A\,a^2b+\right.}{da}}{(a\tan(dx+c)+b)^2}$
risch	$\frac{ixB}{3ia^2b-ib^3+a^3-3a\,b^2}+\frac{xA}{3ia^2b-ib^3+a^3-3a\,b^2}+\frac{6iA\,a^2bx}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2iA\,b^3x}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2iB\,a^3x}{a^6+3a^4b^2+3a^2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^3*(1/2*(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*ln(cot(d*x+c)^2+1
)+(-A*a^3+3*A*a*b^2-3*B*a^2*b+B*b^3)*(1/2*Pi-arccot(cot(d*x+c))))-(3*A*a^2*
b-A*b^3-B*a^3+3*B*a*b^2)/(a^2+b^2)^3*ln(a+b*cot(d*x+c))+1/2*(A*b-B*a)/(a^2+
b^2)/(a+b*cot(d*x+c))^2+(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2/(a+b*cot(d*x+c)))
```

Maxima [A]
time = 0.54, size = 337, normalized size = 1.93

$$\frac{\frac{2\,(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)\log(a\tan(dx+c)+b)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{3\,Ba^3b^2-5\,Aa^2b^3-Bab^4-Ab^5+2\,(2\,Ba^4b-3\,Aa^3b^2-Aab^4)\tan(dx+c)}{a^6b^2+2\,a^4b^4+a^2b^6+(a^8+2\,a^6b^2+a^4b^4)\tan(dx+c)^2+2\,(a^7b+2\,a^5b^3+a^3b^5)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) + 2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(a*tan(d*x
 + c) + b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (B*a^3 - 3*A*a^2*b - 3*B*a
*b^2 + A*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 (3*B*a^3*b^2 - 5*A*a^2*b^3 - B*a*b^4 - A*b^5 + 2*(2*B*a^4*b - 3*A*a^3*b^2
- A*a*b^4)*tan(d*x + c))/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6 + (a^8 + 2*a^6*b^2
+ a^4*b^4)*tan(d*x + c)^2 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*tan(d*x + c)))/
d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(171) = 342.
time = 2.86, size = 549, normalized size = 3.14

$$\frac{2\,Ba^3b^2-2\,Aa^2b^3+2\,Bab^4-2\,Ab^5-2\,(Aa^5+3\,Ba^4b-2\,Aa^3b^2+2\,Ba^2b^3-3\,Aab^4-Bb^5)dx-2\,(4\,Ba^3b^2-6\,Aa^2b^3-2\,Bab^4-(Aa^5+3\,Ba^4b-4\,Aa^3b^2-4\,Ba^2b^3+3\,Aab^4+Bb^5)dx)\cos(2\,dx+2\,c)-(Ba^5-3\,Aa^4b-2\,Ba^3b^2-2\,Aa^2b^3-3\,Bab^4+Ab^5-(Ba^5-3\,Aa^4b-4\,Ba^3b^2+4\,Aa^2b^3+3\,Bab^4-Ab^5)\cos(2\,dx+2\,c)+2\,(Ba^4b-3\,Aa^3b^2-3\,Ba^2b^3+Aab^4)\sin(2\,dx+2\,c))\log\left(ab\sin(2\,dx+2\,c)+\frac{1}{2}\,a^2+\frac{1}{2}\,b^2-\frac{1}{2}\,(a^2-b^2)\cos(2\,dx+2\,c)\right)-2\,(2\,Ba^4b-3\,Aa^3b^2-3\,Ba^2b^3+3\,Aab^4+Bb^5+2\,(Aa^4b+3\,Ba^3b^2-3\,Aa^2b^3-Bab^4)dx)\sin(2\,dx+2\,c)}{2\,((a^8+2\,a^6b^2-2\,a^2b^6-b^8)d\cos(2\,dx+2\,c)-2\,(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7)d\sin(2\,dx+2\,c)-(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*B*a^3*b^2 - 2*A*a^2*b^3 + 2*B*a*b^4 - 2*A*b^5 - 2*(A*a^5 + 3*B*a^4*b
 - 2*A*a^3*b^2 + 2*B*a^2*b^3 - 3*A*a*b^4 - B*b^5)*d*x - 2*(4*B*a^3*b^2 - 6*
A*a^2*b^3 - 2*B*a*b^4 - (A*a^5 + 3*B*a^4*b - 4*A*a^3*b^2 - 4*B*a^2*b^3 + 3*
A*a*b^4 + B*b^5)*d*x)*cos(2*d*x + 2*c) - (B*a^5 - 3*A*a^4*b - 2*B*a^3*b^2 -
 2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - (B*a^5 - 3*A*a^4*b - 4*B*a^3*b^2 + 4*A*a
```

```
^2*b^3 + 3*B*a*b^4 - A*b^5)*cos(2*d*x + 2*c) + 2*(B*a^4*b - 3*A*a^3*b^2 - 3
*B*a^2*b^3 + A*a*b^4)*sin(2*d*x + 2*c))*log(a*b*sin(2*d*x + 2*c) + 1/2*a^2
+ 1/2*b^2 - 1/2*(a^2 - b^2)*cos(2*d*x + 2*c)) - 2*(2*B*a^4*b - 3*A*a^3*b^2
- 3*B*a^2*b^3 + 3*A*a*b^4 + B*b^5 + 2*(A*a^4*b + 3*B*a^3*b^2 - 3*A*a^2*b^3
- B*a*b^4)*d*x)*sin(2*d*x + 2*c))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d*co
s(2*d*x + 2*c) - 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*sin(2*d*x + 2*
c) - (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(171) = 342.
time = 0.58, size = 412, normalized size = 2.35

$$\frac{\frac{2\,(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(Ba^4-3\,Aa^3b-3\,Ba^2b^2+Aab^3)\log(|a\tan(dx+c)+b|)}{a^7+3\,a^5b^2+3\,a^3b^4+ab^6}-\frac{3\,Ba^7\tan(dx+c)^2-9\,Aa^6b\tan(dx+c)^2-9\,Ba^5b^2\tan(dx+c)^2+3\,Aa^4b^3\tan(dx+c)^2+2\,Ba^6b\tan(dx+c)-12\,Aa^5b^2\tan(dx+c)-22\,Ba^4b^3\tan(dx+c)+14\,Aa^3b^4\tan(dx+c)+2\,Aab^6\tan(dx+c)-4\,Aa^4b^3-11\,Ba^3b^4+9\,Aa^2b^5+Bab^6+Ab^7}{(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6)(a\tan(dx+c)+b)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) - (B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(tan(d*x + c
)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(B*a^4 - 3*A*a^3*b - 3*B*a
^2*b^2 + A*a*b^3)*log(abs(a*tan(d*x + c) + b))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4
 + a*b^6) - (3*B*a^7*tan(d*x + c)^2 - 9*A*a^6*b*tan(d*x + c)^2 - 9*B*a^5*b^
2*tan(d*x + c)^2 + 3*A*a^4*b^3*tan(d*x + c)^2 + 2*B*a^6*b*tan(d*x + c) - 12
*A*a^5*b^2*tan(d*x + c) - 22*B*a^4*b^3*tan(d*x + c) + 14*A*a^3*b^4*tan(d*x
+ c) + 2*A*a*b^6*tan(d*x + c) - 4*A*a^4*b^3 - 11*B*a^3*b^4 + 9*A*a^2*b^5 +
B*a*b^6 + A*b^7)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*(a*tan(d*x + c) +
 b)^2))/d
```

Mupad [**B**]
time = 2.60, size = 481, normalized size = 2.75

$$\frac{\frac{5\,A\,a^2\,b+A\,b^3}{2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{2\,A\,a\,b^2\,\cot(c+d\,x)}{a^4+2\,a^2\,b^2+b^4}}{d\,a^2+2\,d\,a\,b\cot\left(c+d\,x\right)+d\,b^2\cot\left(c+d\,x\right)^2}-\ln\left(a+b\cot(c+d\,x)\right)\left(\frac{3\,A\,b}{d\,{(a^2+b^2)}^2}-\frac{4\,A\,b^3}{d\,{(a^2+b^2)}^3}\right)-\frac{\frac{3\,B\,a^3-B\,a\,b^2}{2\,(a^4+2\,a^2\,b^2+b^4)}-\frac{\cot(c+d\,x)\,(B\,b^3-B\,a^2\,b)}{a^4+2\,a^2\,b^2+b^4}}{d\,a^2+2\,d\,a\,b\cot\left(c+d\,x\right)+d\,b^2\cot\left(c+d\,x\right)^2}+\ln\left(a+b\cot(c+d\,x)\right)\left(\frac{B\,a}{d\,{(a^2+b^2)}^2}-\frac{4\,B\,a\,b^2}{d\,{(a^2+b^2)}^3}\right)+\frac{A\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)\,1{}\mathrm{i}}{2\,\left(d\,a^3+3{}\mathrm{i}\,d\,a^2\,b-3\,d\,a\,b^2-1{}\mathrm{i}\,d\,b^3\right)}+\frac{A\,\ln\left(\cot(c+d\,x)+1{}\mathrm{i}\right)}{2\,\left(1{}\mathrm{i}\,d\,a^3+3\,d\,a^2\,b-3{}\mathrm{i}\,d\,a\,b^2-d\,b^3\right)}-\frac{B\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)\,1{}\mathrm{i}}{2\,\left(1{}\mathrm{i}\,d\,a^3-3\,d\,a^2\,b-3{}\mathrm{i}\,d\,a\,b^2+d\,b^3\right)}-\frac{B\,\ln\left(\cot(c+d\,x)+1{}\mathrm{i}\right)}{2\,\left(d\,a^3-3{}\mathrm{i}\,d\,a^2\,b-3\,d\,a\,b^2+1{}\mathrm{i}\,d\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^3,x)
```

```
[Out] ((A*b^3 + 5*A*a^2*b)/(2*(a^4 + b^4 + 2*a^2*b^2)) + (2*A*a*b^2*cot(c + d*x))
/(a^4 + b^4 + 2*a^2*b^2))/(a^2*d + b^2*d*cot(c + d*x)^2 + 2*a*b*d*cot(c + d
*x)) - log(a + b*cot(c + d*x))*((3*A*b)/(d*(a^2 + b^2)^2) - (4*A*b^3)/(d*(a
^2 + b^2)^3)) - ((3*B*a^3 - B*a*b^2)/(2*(a^4 + b^4 + 2*a^2*b^2)) - (cot(c +
 d*x)*(B*b^3 - B*a^2*b))/(a^4 + b^4 + 2*a^2*b^2))/(a^2*d + b^2*d*cot(c + d*
x)^2 + 2*a*b*d*cot(c + d*x)) + log(a + b*cot(c + d*x))*((B*a)/(d*(a^2 + b^2
)^2) - (4*B*a*b^2)/(d*(a^2 + b^2)^3)) + (A*log(cot(c + d*x) - 1i)*1i)/(2*(a
^3*d - b^3*d*1i - 3*a*b^2*d + a^2*b*d*3i)) + (A*log(cot(c + d*x) + 1i))/(2*
(a^3*d*1i - b^3*d - a*b^2*d*3i + 3*a^2*b*d)) - (B*log(cot(c + d*x) - 1i)*1i
)/(2*(a^3*d*1i + b^3*d - a*b^2*d*3i - 3*a^2*b*d)) - (B*log(cot(c + d*x) + 1
i))/(2*(a^3*d + b^3*d*1i - 3*a*b^2*d - a^2*b*d*3i))
```

3.95 $\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx$

Optimal. Leaf size=188

$$\frac{(a-ib)^{5/2}(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{5/2}(iA-B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d}-$$

```
[Out] (a-I*b)^(5/2)*(I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I*
b)^(5/2)*(I*A-B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2/3*(A*b+B
*a)*(a+b*cot(d*x+c))^(3/2)/d-2/5*B*(a+b*cot(d*x+c))^(5/2)/d-2*(2*A*a*b+B*a^
2-B*b^2)*(a+b*cot(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.32, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3609, 3620, 3618, 65, 214}

$$-\frac{2(a^2B+2aAb-b^2B)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2(aB+Ab)(a+b\cot(c+dx))^{3/2}}{3d}+\frac{(a-ib)^{5/2}(B+iA)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{5/2}(-B+iA)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])^(5/2)*(A + B*Cot[c + d*x]),x]
```

```
[Out] ((a - I*b)^(5/2)*(I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])
/d - ((a + I*b)^(5/2)*(I*A - B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I
*b]])/d - (2*(2*a*A*b + a^2*B - b^2*B)*Sqrt[a + b*Cot[c + d*x]])/d - (2*(A*
b + a*B)*(a + b*Cot[c + d*x])^(3/2))/(3*d) - (2*B*(a + b*Cot[c + d*x])^(5/2
))/(5*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = -\frac{2B(a+b\cot(c+dx))^{5/2}}{5d} + \int (a+b\cot(c+dx))^{3/2}(aA$$

$$= -\frac{2(Ab+aB)(a+b\cot(c+dx))^{3/2}}{3d} - \frac{2B(a+b\cot(c+dx$$

$$= -\frac{2(2aAb+a^2B-b^2B)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2(Ab+aB$$

$$= -\frac{2(2aAb+a^2B-b^2B)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2(Ab+aB$$

$$= -\frac{2(2aAb+a^2B-b^2B)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2(Ab+aB$$

$$= -\frac{2(2aAb+a^2B-b^2B)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2(Ab+aB$$

$$= \frac{(a-ib)^{5/2}(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 379 vs. 2(188) = 376.

time = 1.89, size = 379, normalized size = 2.02

$$-\frac{2\left(\frac{\sqrt{a-\sqrt{-b^2}}\left(-3a^2b\left(A\sqrt{-b^2}+bB\right)+b^3\left(A\sqrt{-b^2}+bB\right)+a^3\left(Ab-\sqrt{-b^2}B\right)+3ab^2\left(-Ab+\sqrt{-b^2}B\right)\right)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{2\left(b^2+a\sqrt{-b^2}\right)}+\frac{\left(b^3\left(A\sqrt{-b^2}-bB\right)+3a^2b\left(-A\sqrt{-b^2}+bB\right)-a^3\left(Ab+\sqrt{-b^2}B\right)+3ab^2\left(Ab+\sqrt{-b^2}B\right)\right)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{2\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}}+(2aAb+a^2B-b^2B)\sqrt{a+b\cot(c+dx)}+\frac{1}{3}(Ab+aB)(a+b\cot(c+dx))^{3/2}+\frac{1}{5}B(a+b\cot(c+dx))^{5/2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])^(5/2)*(A + B*Cot[c + d*x]),x]
```

```
[Out] (-2*((Sqrt[a - Sqrt[-b^2]]*(-3*a^2*b*(A*Sqrt[-b^2] + b*B) + b^3*(A*Sqrt[-b^
2] + b*B) + a^3*(A*b - Sqrt[-b^2]*B) + 3*a*b^2*(-(A*b) + Sqrt[-b^2]*B))*Arc
Tanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(2*(b^2 + a*Sqrt[-b^2]
)) + ((b^3*(A*Sqrt[-b^2] - b*B) + 3*a^2*b*(-(A*Sqrt[-b^2]) + b*B) - a^3*(A*
b + Sqrt[-b^2]*B) + 3*a*b^2*(A*b + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a + b*Cot[c
+ d*x]]/Sqrt[a + Sqrt[-b^2]]])/(2*Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + (2*a*A
*b + a^2*B - b^2*B)*Sqrt[a + b*Cot[c + d*x]] + ((A*b + a*B)*(a + b*Cot[c +
d*x])^(3/2))/3 + (B*(a + b*Cot[c + d*x])^(5/2))/5))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(160) = 320$.
time = 0.68, size = 1249, normalized size = 6.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))^(5/2)*(A+B*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/5*B*(a+b*cot(d*x+c))^(5/2)-2/3*A*b*(a+b*cot(d*x+c))^(3/2)-2/3*B*a*(
a+b*cot(d*x+c))^(3/2)-4*A*a*b*(a+b*cot(d*x+c))^(1/2)-2*B*a^2*(a+b*cot(d*x+c
))^(1/2)+2*B*b^2*(a+b*cot(d*x+c))^(1/2)-1/2/b*(1/2*(-A*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/
2)*b^2+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*a*b^2+2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b-3*B*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*a^2*b+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^3)*ln(b*cot(d*
x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2)
)+2*(-4*A*(a^2+b^2)^(1/2)*a*b^2-2*B*(a^2+b^2)^(1/2)*a^2*b+2*B*(a^2+b^2)^(1/
2)*b^3-1/2*(-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+A*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a
^3-3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2+2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*(a^2+b^2)^(1/2)*a*b-3*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b+B*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*b^3)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(
2*(a^2+b^2)^(1/2)-2*a)^(1/2)))-1/2/b*(-1/2*(-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*(a^2+b^2)^(1/2)*a^2+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2+A
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2+
2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b-3*B*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)*a^2*b+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^3)*ln((a+b*cot(d*x+c))
^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))+2*(4*A
*(a^2+b^2)^(1/2)*a*b^2+2*B*(a^2+b^2)^(1/2)*a^2*b-2*B*(a^2+b^2)^(1/2)*b^3+1/
```

```
2*(-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+A*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-3*A*(2
*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2+2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^
2)^(1/2)*a*b-3*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b+B*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*b^3)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^(5/2)*(A+B*cot(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*cot(d*x + c) + A)*(b*cot(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^(5/2)*(A+B*cot(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cot(c + dx))\,(a + b\cot(c + dx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))**(5/2)*(A+B*cot(d*x+c)),x)
[Out] Integral((A + B*cot(c + d*x))*(a + b*cot(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^(5/2)*(A+B*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cot(d*x + c) + A)*(b*cot(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 31.59, size = 2500, normalized size = 13.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cot(c + d*x))*(a + b*cot(c + d*x))^(5/2),x)
```

```
[Out] log((8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 - ((((((-B^4*b^2*d^4*(5*a
^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*
a*b^4*d^2)/d^4)^(1/2)*(32*B*a^4*b^2 - 32*B*b^6 + 32*a*b^2*d*(((-B^4*b^2*d^4
*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5
*B^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) - (16*B^2*b^2
*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(((
-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3
*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2))/2)*((20*B^4*a^2*b^8*d^4 - B^4*b^10*
d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2*d^4)^(1/2)
/(4*d^4) + (B^2*a^5)/(4*d^2) - (5*B^2*a^3*b^2)/(2*d^2) + (5*B^2*a*b^4)/(4*d
^2))^(1/2) - log((((((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B
^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(32*B*b^6 - 3
2*B*a^4*b^2 + 32*a*b^2*d*(((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2
) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*c
ot(c + d*x))^(1/2)))/(2*d) - (16*B^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 -
b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*
b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1
/2))/2 + (8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)*(((20*B^4*a^2*b^8*d
^4 - B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*
b^2*d^4)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/(4*d^4
))^(1/2) - log(((((-((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - B^
2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(32*B*b^6 - 32
*B*a^4*b^2 + 32*a*b^2*d*(-((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2
) - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*c
ot(c + d*x))^(1/2)))/(2*d) - (16*B^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 -
b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(-((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2
*b^2)^2)^(1/2) - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^(
1/2))/2 + (8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)*(-((20*B^4*a^2*b^8
*d^4 - B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^
8*b^2*d^4)^(1/2) - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/(4*d
^4))^(1/2) + log((8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 - ((((-((-B^
4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - B^2*a^5*d^2 + 10*B^2*a^3*b^
2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(32*B*a^4*b^2 - 32*B*b^6 + 32*a*b^2*d*(
-((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - B^2*a^5*d^2 + 10*B^2*
a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d
```

) - (16*B^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4
b^2))/d^2)(-((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - B^2*a^5*
d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^(1/2))/2)*((B^2*a^5)/(4*d^
2) - (20*B^4*a^2*b^8*d^4 - B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6
*b^4*d^4 - 25*B^4*a^8*b^2*d^4)^(1/2)/(4*d^4) - (5*B^2*a^3*b^2)/(2*d^2) + (5
*B^2*a*b^4)/(4*d^2))^(1/2) - ((4*B*a^2)/d - (2*B*(a^2 + b^2))/d)*(a + b*cot
(c + d*x))^(1/2) - log((8*A^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3 - ((-((-
A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*
b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(((-((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*
a^2*b^2)^2)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4
)^(1/2)*(64*A*a^3*b^3 + 64*A*a*b^5 - 32*a*b^2*d*(-((-A^4*b^2*d^4*(5*a^4 + b
^4 - 10*a^2*b^2)^2)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*
d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) - (16*A^2*b^2*(a + b*cot
(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2)*(-((20*A^4
*a^2*b^8*d^4 - A^4*b^10*d^4 - 110*A^4*a^4*b^6*d^4 + 100*A^4*a^6*b^4*d^4 - 2
5*A^4*a^8*b^2*d^4)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d
^2)/(4*d^4))^(1/2) - log((8*A^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3 - ((((
-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - A^2*a^5*d^2 + 10*A^2*a^3
*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(((((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*
a^2*b^2)^2)^(1/2) - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4
)^(1/2)*(64*A*a^3*b^3 + 64*A*a*b^5 - 32*a*b^2*d*(((-A^4*b^2*d^4*(5*a^4 + b^
4 - 10*a^2*b^2)^2)^(1/2) - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d
^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) - (16*A^2*b^2*(a + b*cot(
c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2)*(((20*A^4*a
^2*b^8*d^4 - A^4*b^10*d^4 - 110*A^4*a^4*b^6*d^4 + 100*A^4*a^6*b^4*d^4 - 25*
A^4*a^8*b^2*d^4)^(1/2) - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2
)/(4*d^4))^(1/2) + log((8*A^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3 - ((((-A
^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - A^2*a^5*d^2 + 10*A^2*a^3*b
^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(((((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^
2*b^2)^2)^(1/2) - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^
(1/2)*(64*A*a^3*b^3 + 64*A*a*b^5 + 32*a*b^2*d*(((-A^4*b^2*d^4*(5*a^4 + b^4
- 10*a^2*b^2)^2)^(1/2) - A^2*a^5*d^2 + 10*A^2*a...

3.96 $\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx$

Optimal. Leaf size=150

$$\frac{(a-ib)^{3/2}(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{3/2}(iA-B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d}-$$

```
[Out] (a-I*b)^(3/2)*(I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I*
b)^(3/2)*(I*A-B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2/3*B*(a+b
*cot(d*x+c))^(3/2)/d-2*(A*b+B*a)*(a+b*cot(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.22, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3609, 3620, 3618, 65, 214}

$$-\frac{2(aB+Ab)\sqrt{a+b\cot(c+dx)}}{d}+\frac{(a-ib)^{3/2}(B+iA)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{3/2}(-B+iA)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])^(3/2)*(A + B*Cot[c + d*x]),x]
```

```
[Out] ((a - I*b)^(3/2)*(I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])
/d - ((a + I*b)^(3/2)*(I*A - B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I
*b]])/d - (2*(A*b + a*B)*Sqrt[a + b*Cot[c + d*x]])/d - (2*B*(a + b*Cot[c +
d*x])^(3/2))/(3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx &= -\frac{2B(a+b\cot(c+dx))^{3/2}}{3d}+\int \sqrt{a+b\cot(c+dx)}\,(aA - \\
&= -\frac{2(Ab+aB)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2B(a+b\cot(c+dx)}{3d} \\
&= -\frac{2(Ab+aB)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2B(a+b\cot(c+dx)}{3d} \\
&= -\frac{2(Ab+aB)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2B(a+b\cot(c+dx)}{3d} \\
&= -\frac{2(Ab+aB)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2B(a+b\cot(c+dx)}{3d} \\
&= \frac{(a-ib)^{3/2}(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a}{}
\end{aligned}$$

Mathematica [A]
time = 1.01, size = 294, normalized size = 1.96

$$-\frac{\frac{3\sqrt{a-\sqrt{-b^2}}\left(-2ab\left(A\sqrt{-b^2}+bB\right)+a^2\left(Ab-\sqrt{-b^2}B\right)+b^2\left(-Ab+\sqrt{-b^2}B\right)\right)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{b^2+a\sqrt{-b^2}}+\frac{3\left(2ab\left(-A\sqrt{-b^2}+bB\right)-a^2\left(Ab+\sqrt{-b^2}B\right)+b^2\left(Ab+\sqrt{-b^2}B\right)\right)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}}+6(Ab+aB)\sqrt{a+b\cot(c+dx)}+2B(a+b\cot(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x])^(3/2)*(A + B*Cot[c + d*x]),x]
```

```
[Out] -1/3*((3*Sqrt[a - Sqrt[-b^2]]*(-2*a*b*(A*Sqrt[-b^2] + b*B) + a^2*(A*b - Sqr
t[-b^2]*B) + b^2*(-(A*b) + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/
Sqrt[a - Sqrt[-b^2]]])/(b^2 + a*Sqrt[-b^2]) + (3*(2*a*b*(-(A*Sqrt[-b^2]) +
b*B) - a^2*(A*b + Sqrt[-b^2]*B) + b^2*(A*b + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a
+ b*Cot[c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]])
+ 6*(A*b + a*B)*Sqrt[a + b*Cot[c + d*x]] + 2*B*(a + b*Cot[c + d*x])^(3/2))/
d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(126) = 252$.

time = 0.66, size = 920, normalized size = 6.13

method	result
derivativedivides	$-\frac{2B(a+b\cot(dx+c))^{\frac{3}{2}}}{3}-2Ab\sqrt{a+b\cot(dx+c)}-2Ba\sqrt{a+b\cot(dx+c)}-\frac{\left(-A\sqrt{2\sqrt{a^2+b^2}+\cdots}\right.}{\cdots}$
default	$-\frac{2B(a+b\cot(dx+c))^{\frac{3}{2}}}{3}-2Ab\sqrt{a+b\cot(dx+c)}-2Ba\sqrt{a+b\cot(dx+c)}-\frac{\left(-A\sqrt{2\sqrt{a^2+b^2}+\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))^(3/2)*(A+B*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/3*B*(a+b*cot(d*x+c))^(3/2)-2*A*b*(a+b*cot(d*x+c))^(1/2)-2*B*a*(a+b*
cot(d*x+c))^(1/2)-1/2/b*(1/2*(-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1
/2)*a+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b
^2+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b-2*B*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*a*b)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-2*A*(a^2+b^2)^(1/2)*b^2-2*B*(a^2+b^2)^(1/
2)*a*b-1/2*(-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a+A*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a^2-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2+B*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b-2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b)*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*
cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2)))-1/2/b*(-1/2*(-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a+A*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2+B*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b-2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
```

```
*a*b)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-
a-(a^2+b^2)^(1/2))+2*(2*A*(a^2+b^2)^(1/2)*b^2+2*B*(a^2+b^2)^(1/2)*a*b+1/2*(
-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a+A*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*a^2-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^2+B*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)*(a^2+b^2)^(1/2)*b-2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b)*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+
2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^(3/2)*(A+B*cot(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*cot(d*x + c) + A)*(b*cot(d*x + c) + a)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^(3/2)*(A+B*cot(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cot(c + dx))\,(a + b\cot(c + dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))**(3/2)*(A+B*cot(d*x+c)),x)
[Out] Integral((A + B*cot(c + d*x))*(a + b*cot(c + d*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^(3/2)*(A+B*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cot(d*x + c) + A)*(b*cot(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 13.76, size = 2823, normalized size = 18.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cot(c + d*x))*(a + b*cot(c + d*x))^(3/2),x)
```

```
[Out] log((16*A^3*a*b^3*(a^2 + b^2)^2)/d^3 - (((16*b^2*(((-A^4*b^2*d^4*(3*a^2 - b
^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b +
 a*d*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2
)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d + (16*A^2*b^2*(a + b*cot(c + d*
x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2)*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1
/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2))/2)*((6*A^4*a^2*b^4*d^4 - A
^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^(1/2)/(4*d^4) - (A^2*a^3)/(4*d^2) + (3*A^2*
a*b^2)/(4*d^2))^(1/2) - log((16*A^3*a*b^3*(a^2 + b^2)^2)/d^3 - (((16*b^2*((
(-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^
(1/2)*(A*b^3 + A*a^2*b - a*d*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a
^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d - (16*A
^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2)*(((-A^4*b^2
*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2))/2)
*(((6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^(1/2) - A^2*a^3*d^
2 + 3*A^2*a*b^2*d^2)/(4*d^4))^(1/2) - log((16*A^3*a*b^3*(a^2 + b^2)^2)/d^3
- (((16*b^2*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3*d^2 - 3*A^2*a
*b^2*d^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b - a*d*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^
2)^(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(
1/2)))/d - (16*A^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/
d^2)*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^
2)/d^4)^(1/2))/2)*(-((6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^
(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/(4*d^4))^(1/2) + log((16*A^3*a*b^3*(
a^2 + b^2)^2)/d^3 - (((16*b^2*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2
*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b + a*d*(-((-A^4*b^2*
d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(a +
 b*cot(c + d*x))^(1/2)))/d + (16*A^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 +
b^4 - 6*a^2*b^2))/d^2)*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3*d^
2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2))/2)*((3*A^2*a*b^2)/(4*d^2) - (A^2*a^3)/(4*d
^2) - (6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^(1/2)/(4*d^4))^
(1/2) - log((8*B^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3 - (((16*B^2*b^2*(a +
b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (16*a*b^2*(((-B^4*b^2*
d^4*(3*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(B*a
^2 + B*b^2 - d*(((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2
*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d)*(((-B^4*b^2*d^4*(3*a
^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^(1/2))/2)*(((6*B^4
```

```
*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^(1/2) + B^2*a^3*d^2 - 3*B^2
*a*b^2*d^2)/(4*d^4))^(1/2) - log((8*B^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3
- (((16*B^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (
16*a*b^2*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^2*a*b^
2*d^2)/d^4)^(1/2)*(B*a^2 + B*b^2 - d*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2
) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/
d)*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)
/d^4)^(1/2))/2)*(-((6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^(1
/2) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/(4*d^4))^(1/2) + log((((16*B^2*b^2*(a
+ b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 - (16*a*b^2*(((-B^4*b^
2*d^4*(3*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(B
*a^2 + B*b^2 + d*(((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B
^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d)*(((-B^4*b^2*d^4*(3
*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^(1/2))/2 + (8*B^
3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3)*((6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9
*B^4*a^4*b^2*d^4)^(1/2)/(4*d^4) + (B^2*a^3)/(4*d^2) - (3*B^2*a*b^2)/(4*d^2)
)^(1/2) + log((((16*B^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b
^2))/d^2 - (16*a*b^2*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2
+ 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(B*a^2 + B*b^2 + d*(-((-B^4*b^2*d^4*(3*a^2 -
b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*
x))^(1/2)))/d)*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^
2*a*b^2*d^2)/d^4)^(1/2))/2 + (8*B^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3)*((B
^2*a^3)/(4*d^2) - (6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^(1/
2)/(4*d^4) - (3*B^2*a*b^2)/(4*d^2))^(1/2) - (2*B*(a + b*cot(c + d*x))^(3/2)
)/(3*d) - (2*A*b*(a + b*cot(c + d*x))^(1/2))/d - (2*B*a*(a + b*cot(c + d*x)
)^(1/2))/d
```

3.97 $\int \sqrt{a+b\cot(c+dx)}\,(A+B\cot(c+dx))\,dx$

Optimal. Leaf size=122

$$\frac{\sqrt{a-ib}\,(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{\sqrt{a+ib}\,(iA-B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{2A}{}$$

```
[Out] (I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))*(a-I*b)^(1/2)/d-(I*A-
B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))*(a+I*b)^(1/2)/d-2*B*(a+b*c
ot(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3609, 3620, 3618, 65, 214}

$$\frac{\sqrt{a-ib}\,(B+iA)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{\sqrt{a+ib}\,(-B+iA)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{2B\sqrt{a+b\cot(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cot[c + d*x]]*(A + B*Cot[c + d*x]),x]
```

```
[Out] (Sqrt[a - I*b]*(I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/d
 - (Sqrt[a + I*b]*(I*A - B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]]
)/d - (2*B*Sqrt[a + b*Cot[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \sqrt{a+b\cot(c+dx)}\,(A+B\cot(c+dx))\,dx = -\frac{2B\sqrt{a+b\cot(c+dx)}}{d} + \int \frac{aA-bB+(Ab+aB)\cot(c\ldots}{\sqrt{a+b\cot(c+dx)}}$$

$$= -\frac{2B\sqrt{a+b\cot(c+dx)}}{d} + \frac{1}{2}((a-ib)(A-iB)) \int \frac{1+\ldots}{\sqrt{a+\ldots}}$$

$$= -\frac{2B\sqrt{a+b\cot(c+dx)}}{d} - \frac{(i(a-ib)(A-iB))\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{2B\sqrt{a+b\cot(c+dx)}}{d} + \frac{((a-ib)(A-iB))\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{\sqrt{a-ib}\,(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{\sqrt{a+\ldots}}{\ldots}$$

Mathematica [A]
time = 0.58, size = 212, normalized size = 1.74

$$-\frac{\frac{\left(aAb-Ab\sqrt{-b^2}-b^2B-a\sqrt{-b^2}\,B\right)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\,\sqrt{a-\sqrt{-b^2}}} - \frac{\left(aAb+Ab\sqrt{-b^2}-b^2B+a\sqrt{-b^2}\,B\right)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\,\sqrt{a+\sqrt{-b^2}}} + 2B\sqrt{a+b\cot(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cot[c + d*x]]*(A + B*Cot[c + d*x]),x]
```

```
[Out] -((((a*A*b - A*b*Sqrt[-b^2] - b^2*B - a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Co
t[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) - ((a*
A*b + A*b*Sqrt[-b^2] - b^2*B + a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Cot[c + d
*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + 2*B*Sqrt[a
+ b*Cot[c + d*x]])/d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(102) = 204$.

time = 0.58, size = 632, normalized size = 5.18

method	result
derivativedivides	$-2B\sqrt{a+b\cot(dx+c)} - \frac{-\frac{\left(-A\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}+A\sqrt{2\sqrt{a^2+b^2}+2a}\,a-B\right.}{}}{}$
default	$-2B\sqrt{a+b\cot(dx+c)} - \frac{-\frac{\left(-A\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}+A\sqrt{2\sqrt{a^2+b^2}+2a}\,a-B\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c))^(1/2)*(A+B*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*B*(a+b*cot(d*x+c))^(1/2)-1/2/b*(-1/2*(-A*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*(a^2+b^2)^(1/2)+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-B*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*b)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot
(d*x+c)-a-(a^2+b^2)^(1/2))+2*(2*B*(a^2+b^2)^(1/2)*b+1/2*(-A*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-B*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*b)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/
(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))-1/2/b*(1/2*(-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*(a^2+b^2)^(1/2)+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-B*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*b)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)+(a^2+b^2)^(1/2))+2*(-2*B*(a^2+b^2)^(1/2)*b-1/2*(-A*(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-B*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*b)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(
2*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^(1/2)*(A+B*cot(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cot(d*x + c) + A)*sqrt(b*cot(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^(1/2)*(A+B*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + B\cot\left(c + dx\right)\right)\sqrt{a + b\cot\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))**(1/2)*(A+B*cot(d*x+c)),x)
```

```
[Out] Integral((A + B*cot(c + d*x))*sqrt(a + b*cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))^(1/2)*(A+B*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cot(d*x + c) + A)*sqrt(b*cot(d*x + c) + a), x)
```

Mupad [B]
time = 3.03, size = 843, normalized size = 6.91

$$\mathrm{atanh}\left(\frac{d^3\left(\frac{16\left(A^2b^4-A^2a^2b^2\right)\sqrt{a+b\cot\left(c+dx\right)}}{d^2}+\frac{16ab^2\left(\sqrt{-A^4b^2d^4}+A^2ad^2\right)\sqrt{a+b\cot\left(c+dx\right)}}{d^4}\right)\sqrt{-\frac{\sqrt{-A^4b^2d^4}+A^2ad^2}{d^4}}}{16\left(A^3a^2b^3+A^3b^5\right)}\right)\sqrt{-\frac{\sqrt{-A^4b^2d^4}+A^2ad^2}{d^4}}+\mathrm{atanh}\left(\frac{d^3\left(\frac{16\left(A^2b^4-A^2a^2b^2\right)\sqrt{a+b\cot\left(c+dx\right)}}{d^2}-\frac{16ab^2\left(\sqrt{-A^4b^2d^4}-A^2ad^2\right)\sqrt{a+b\cot\left(c+dx\right)}}{d^4}\right)\sqrt{\frac{\sqrt{-A^4b^2d^4}-A^2ad^2}{d^4}}}{16\left(A^3a^2b^3+A^3b^5\right)}\right)\sqrt{\frac{\sqrt{-A^4b^2d^4}-A^2ad^2}{d^4}}+2\,\mathrm{atanh}\left(\frac{32B^2b^4\sqrt{\frac{\sqrt{-B^4b^2d^4}}{4d^4}+\frac{B^2a}{4d^2}}\sqrt{a+b\cot\left(c+dx\right)}}{\frac{16Bb^4\sqrt{-B^4b^2d^4}}{d^3}+\frac{16Ba^2b^2\sqrt{-B^4b^2d^4}}{d^3}}+\frac{32ab^2\sqrt{\frac{\sqrt{-B^4b^2d^4}}{4d^4}+\frac{B^2a}{4d^2}}\sqrt{a+b\cot\left(c+dx\right)}\sqrt{-B^4b^2d^4}}{\frac{16Bb^4\sqrt{-B^4b^2d^4}}{d}+\frac{16Ba^2b^2\sqrt{-B^4b^2d^4}}{d}}\right)\sqrt{\frac{\sqrt{-B^4b^2d^4}+B^2ad^2}{4d^4}}-2\,\mathrm{atanh}\left(\frac{32B^2b^4\sqrt{\frac{B^2a}{4d^2}-\frac{\sqrt{-B^4b^2d^4}}{4d^4}}\sqrt{a+b\cot\left(c+dx\right)}}{\frac{16Bb^4\sqrt{-B^4b^2d^4}}{d^3}+\frac{16Ba^2b^2\sqrt{-B^4b^2d^4}}{d^3}}-\frac{32ab^2\sqrt{\frac{B^2a}{4d^2}-\frac{\sqrt{-B^4b^2d^4}}{4d^4}}\sqrt{a+b\cot\left(c+dx\right)}\sqrt{-B^4b^2d^4}}{\frac{16Bb^4\sqrt{-B^4b^2d^4}}{d}+\frac{16Ba^2b^2\sqrt{-B^4b^2d^4}}{d}}\right)\sqrt{-\frac{\sqrt{-B^4b^2d^4}-B^2ad^2}{4d^4}}-\frac{2B\sqrt{a+b\cot\left(c+dx\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cot(c + d*x))*(a + b*cot(c + d*x))^(1/2),x)
```

```
[Out] atanh((d^3*((16*(A^2*b^4 - A^2*a^2*b^2)*(a + b*cot(c + d*x))^(1/2))/d^2 + (
16*a*b^2*((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)*(a + b*cot(c + d*x))^(1/2))/d^4
)*(-((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)/d^4)^(1/2))/(16*(A^3*b^5 + A^3*a^2*b
^3)))*(-((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)/d^4)^(1/2) + atanh((d^3*((16*(A^
2*b^4 - A^2*a^2*b^2)*(a + b*cot(c + d*x))^(1/2))/d^2 - (16*a*b^2*((-A^4*b^2
*d^4)^(1/2) - A^2*a*d^2)*(a + b*cot(c + d*x))^(1/2))/d^4)*(((-A^4*b^2*d^4)^
(1/2) - A^2*a*d^2)/d^4)^(1/2))/(16*(A^3*b^5 + A^3*a^2*b^3)))*(((-A^4*b^2*d^
4)^(1/2) - A^2*a*d^2)/d^4)^(1/2) + 2*atanh((32*B^2*b^4*((-B^4*b^2*d^4)^(1/2
)/(4*d^4) + (B^2*a)/(4*d^2))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*B*b^4*(
-B^4*b^2*d^4)^(1/2))/d^3 + (16*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d^3) + (32*a
*b^2*((-B^4*b^2*d^4)^(1/2)/(4*d^4) + (B^2*a)/(4*d^2))^(1/2)*(a + b*cot(c +
d*x))^(1/2)*(-B^4*b^2*d^4)^(1/2))/((16*B*b^4*(-B^4*b^2*d^4)^(1/2))/d + (16*
B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d))*(((-B^4*b^2*d^4)^(1/2) + B^2*a*d^2)/(4*
d^4))^(1/2) - 2*atanh((32*B^2*b^4*((B^2*a)/(4*d^2) - (-B^4*b^2*d^4)^(1/2)/(
4*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*B*b^4*(-B^4*b^2*d^4)^(1/2))/
d^3 + (16*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d^3) - (32*a*b^2*((B^2*a)/(4*d^2)
 - (-B^4*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-B^4*b^2
*d^4)^(1/2))/((16*B*b^4*(-B^4*b^2*d^4)^(1/2))/d + (16*B*a^2*b^2*(-B^4*b^2*d
^4)^(1/2))/d))*(-((-B^4*b^2*d^4)^(1/2) - B^2*a*d^2)/(4*d^4))^(1/2) - (2*B*(
a + b*cot(c + d*x))^(1/2))/d
```

3.98 $\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx$

Optimal. Leaf size=151

$$-\frac{(ia-b)(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{5/2}(ia+b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d}+\frac{2}{}$$

```
[Out] -(I*a-b)*(a-I*b)^(5/2)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+I
*b)^(5/2)*(I*a+b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2/5*b*(a+
b*cot(d*x+c))^(5/2)/d+2*b*(a^2+b^2)*(a+b*cot(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3609, 12, 3563, 3620, 3618, 65, 214}

$$\frac{2b(a^2+b^2)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}-\frac{(-b+ia)(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{5/2}(b+ia)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])^(5/2),x]
```

```
[Out] -(((I*a - b)*(a - I*b)^(5/2)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]
])/d) + ((a + I*b)^(5/2)*(I*a + b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a
+ I*b]])/d + (2*b*(a^2 + b^2)*Sqrt[a + b*Cot[c + d*x]])/d - (2*b*(a + b*Cot
[c + d*x])^(5/2))/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx &= -\frac{2b(a+b\cot(c+dx))^{5/2}}{5d} + \int \left(-a^2-b^2\right)(a+b\cot(c+ \\
&= -\frac{2b(a+b\cot(c+dx))^{5/2}}{5d} + \left(-a^2-b^2\right)\int (a+b\cot(c+ \\
&= \frac{2b(a^2+b^2)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2b(a+b\cot(c+dx))^{5/} }{5d} \\
&= \frac{2b(a^2+b^2)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2b(a+b\cot(c+dx))^{5/} }{5d} \\
&= \frac{2b(a^2+b^2)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2b(a+b\cot(c+dx))^{5/} }{5d} \\
&= \frac{2b(a^2+b^2)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2b(a+b\cot(c+dx))^{5/} }{5d} \\
&= -\frac{(ia-b)(a-ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a}{}
\end{aligned}
$$

Mathematica [A]
time = 4.13, size = 253, normalized size = 1.68

$$
\frac{(-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\left(\frac{5i(a^2+b^2)\left((a-ib)^2\sqrt{a+ib}\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)-\sqrt{a-ib}\,(a+ib)^2\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)\right)}{\sqrt{a-ib}\sqrt{a+ib}\,(a+b\cot(c+dx))^{5/2}}+\frac{2b(-4a^2-6b^2+2ab\cot(c+dx)+b^2\csc^2(c+dx))}{(a+b\cot(c+dx))^2}\right)\sin(c+dx)}{5d(-b\cos(c+dx)+a\sin(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])^(5/2),x]
```

```
[Out] ((-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])^(5/2)*(((5*I)*(a^2 + b^2)*((a -
 I*b)^2*Sqrt[a + I*b]*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]] - Sqr
t[a - I*b]*(a + I*b)^2*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]]))/(S
qrt[a - I*b]*Sqrt[a + I*b]*(a + b*Cot[c + d*x])^(5/2)) + (2*b*(-4*a^2 - 6*b
^2 + 2*a*b*Cot[c + d*x] + b^2*Csc[c + d*x]^2))/(a + b*Cot[c + d*x])^2)*Sin[
c + d*x])/(5*d*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 681 vs. 2(127) = 254.
time = 0.61, size = 682, normalized size = 4.52

method	result

derivativedivides	$-\frac{2b\left(\frac{(a+b\cot(dx+c))^{\frac{5}{2}}}{5}-a^2\sqrt{a+b\cot\left(dx+c\right)}-b^2\sqrt{a+b\cot\left(dx+c\right)}+(a^2+b^2)\left(\frac{\left(\sqrt{2\sqrt{a^2+}\right.}{}\right.\right.}{}$
default	$-\frac{2b\left(\frac{(a+b\cot(dx+c))^{\frac{5}{2}}}{5}-a^2\sqrt{a+b\cot\left(dx+c\right)}-b^2\sqrt{a+b\cot\left(dx+c\right)}+(a^2+b^2)\left(\frac{\left(\sqrt{2\sqrt{a^2+}\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*(1/5*(a+b*cot(d*x+c))^(5/2)-a^2*(a+b*cot(d*x+c))^(1/2)-b^2*(a+b*cot(
d*x+c))^(1/2)+(a^2+b^2)*(1/4/b^2*(1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b
^2)^(1/2)*a-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
*b^2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)+(a^2+b^2)^(1/2))+2*(2*(a^2+b^2)^(1/2)*b^2-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*(a^2+b^2)^(1/2)*a-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2)))+1/4/b^2*(-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^
2)^(1/2)*a-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*
b^2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a
-(a^2+b^2)^(1/2))+2*(-2*(a^2+b^2)^(1/2)*b^2+1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*(a^2+b^2)^(1/2)*a-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cot(d*x + c) + a)^(5/2)*(b*cot(d*x + c) - a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int a^3\sqrt{a+b\cot(c+dx)}\,dx-\int\left(-b^3\sqrt{a+b\cot(c+dx)}\cot^3(c+dx)\right)dx-\int\left(-ab^2\sqrt{a+b\cot(c+dx)}\cot^2(c+dx)\right)dx-\int a^2b\sqrt{a+b\cot(c+dx)}\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))**(5/2),x)
```

```
[Out] -Integral(a**3*sqrt(a + b*cot(c + d*x)), x) - Integral(-b**3*sqrt(a + b*cot
(c + d*x))*cot(c + d*x)**3, x) - Integral(-a*b**2*sqrt(a + b*cot(c + d*x))*
cot(c + d*x)**2, x) - Integral(a**2*b*sqrt(a + b*cot(c + d*x))*cot(c + d*x)
, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) + a)^(5/2)*(b*cot(d*x + c) - a), x)
```

Mupad [**B**]
time = 26.56, size = 2500, normalized size = 16.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*cot(c + d*x))^(5/2)*(a - b*cot(c + d*x)),x)
```

```
[Out] log(((((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3*
b^4*d^2 + 10*a^5*b^2*d^2)/d^4)^(1/2)*(((((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^
2*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3*b^4*d^2 + 10*a^5*b^2*d^2)/d^4)^(1/2)*(64*
a^2*b^5 + 64*a^4*b^3 - 32*a*b^2*d*(((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2
)^2)^(1/2) - a^7*d^2 - 5*a^3*b^4*d^2 + 10*a^5*b^2*d^2)/d^4)^(1/2)*(a + b*co
t(c + d*x))^(1/2)))/(2*d) - (16*a^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 - b
^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2 - (8*a^3*b^3*(3*a^2 - b^2)*(a^2 + b^
2)^3)/d^3)*((20*a^6*b^8*d^4 - a^4*b^10*d^4 - 110*a^8*b^6*d^4 + 100*a^10*b^4
*d^4 - 25*a^12*b^2*d^4)^(1/2)/(4*d^4) - a^7/(4*d^2) - (5*a^3*b^4)/(4*d^2) +
 (5*a^5*b^2)/(2*d^2))^(1/2) - log(((-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b
^2)^2)^(1/2) + a^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2)*(((-((-
a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7*d^2 + 5*a^3*b^4*d^2 -
 10*a^5*b^2*d^2)/d^4)^(1/2)*(64*a^2*b^5 + 64*a^4*b^3 + 32*a*b^2*d*(-((-a^4*
b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7*d^2 + 5*a^3*b^4*d^2 - 10*
a^5*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) + (16*a^2*b^2*(a
 + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2 - (
8*a^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3)*(-(a^7*d^2 + (20*a^6*b^8*d^4 -
a^4*b^10*d^4 - 110*a^8*b^6*d^4 + 100*a^10*b^4*d^4 - 25*a^12*b^2*d^4)^(1/2)
+ 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/(4*d^4))^(1/2) + log(((-((-a^4*b^2*d^4*(5
*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^
2)/d^4)^(1/2)*(((-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7*
d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2)*(64*a^2*b^5 + 64*a^4*b^3 -
 32*a*b^2*d*(-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7*d^2
+ 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(
2*d) - (16*a^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*
a^4*b^2))/d^2))/2 - (8*a^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3)*((5*a^5*b^
2)/(2*d^2) - a^7/(4*d^2) - (5*a^3*b^4)/(4*d^2) - (20*a^6*b^8*d^4 - a^4*b^10
*d^4 - 110*a^8*b^6*d^4 + 100*a^10*b^4*d^4 - 25*a^12*b^2*d^4)^(1/2)/(4*d^4))
^(1/2) - ((4*a^2*b)/d - (2*b*(a^2 + b^2))/d)*(a + b*cot(c + d*x))^(1/2) - l
og((((((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + 5*a*b^6*d^2 - 10*a^
3*b^4*d^2 + a^5*b^2*d^2)/d^4)^(1/2)*(32*b^7 - 32*a^4*b^3 + 32*a*b^2*d*(((-b
^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 +
 a^5*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) + (16*(a + b*co
t(c + d*x))^(1/2)*(b^10 - 15*a^2*b^8 + 15*a^4*b^6 - a^6*b^4))/d^2)*(((-b^6*
d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^
5*b^2*d^2)/d^4)^(1/2))/2 + (8*a*b^5*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)*(((20
*a^2*b^12*d^4 - b^14*d^4 - 110*a^4*b^10*d^4 + 100*a^6*b^8*d^4 - 25*a^8*b^6*
d^4)^(1/2) + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/(4*d^4))^(1/2) + l
og((8*a*b^5*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 - ((((((-b^6*d^4*(5*a^4 + b^4
- 10*a^2*b^2)^2)^(1/2) + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^(
1/2)*(32*a^4*b^3 - 32*b^7 + 32*a*b^2*d*(((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^
2)^2)^(1/2) + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^(1/2)*(a + b
*cot(c + d*x))^(1/2)))/(2*d) + (16*(a + b*cot(c + d*x))^(1/2)*(b^10 - 15*a^
2*b^8 + 15*a^4*b^6 - a^6*b^4))/d^2)*(((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^
2)^(1/2) + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^(1/2))/2)*((20*
```

```
a^2*b^12*d^4 - b^14*d^4 - 110*a^4*b^10*d^4 + 100*a^6*b^8*d^4 - 25*a^8*b^6*d
^4)^(1/2)/(4*d^4) + (5*a*b^6)/(4*d^2) - (5*a^3*b^4)/(2*d^2) + (a^5*b^2)/(4*
d^2))^(1/2) - log(((((-((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - 5*a
*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2)/d^4)^(1/2)*(32*b^7 - 32*a^4*b^3 +
32*a*b^2*d*(-((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - 5*a*b^6*d^2 +
 10*a^3*b^4*d^2 - a^5*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d
) + (16*(a + b*cot(c + d*x))^(1/2)*(b^10 - 15*a^2*b^8 + 15*a^4*b^6 - a^6*b^
4))/d^2)*(-((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - 5*a*b^6*d^2 + 1
0*a^3*b^4*d^2 - a^5*b^2*d^2)/d^4)^(1/2))/2 + (8*a*b^5*(a^2 - 3*b^2)*(a^2 +
b^2)^3)/d^3)*(-((20*a^2*b^12*d^4 - b^14*d^4 - 110*a^4*b^10*d^4 + 100*a^6*b^
8*d^4 - 25*a^8*b^6*d^4)^(1/2) - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2)
/(4*d^4))^(1/2) + log((8*a*b^5*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 - ((((-((-b
^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 -
 a^5*b^2*d^2)/d^4)^(1/2)*(32*a^4*b^3 - 32*b^7 + 32*a*b^2*d*(-((-b^6*d^4*(5*
a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d
^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) + (16*(a + b*cot(c + d*x)
)^(1/2)*(b^10 - 15*a^2*b^8 + 15*a^4*b^6 - a^6*b^4))/d^2)*(-((-b^6*d^4*(5*a^
4 + b^4 - 10*a^2*b^2)^2)^(1/2) - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2
)/d^4)^(1/2))/2)*((5*a*b^6)/(4*d^2) - (20*a^2*b^12*d^4 - b^14*d^4 - 110*a^4
*b^10*d^4 + 100*a^6*b^8*d^4 - 25*a^8*b^6*d^4)^(1/2)/(4*d^4) - (5*a^3*b^4)/(
2*d^2) + (a^5*b^2)/(4*d^2))^(1/2) - log(((((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*
a^2*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3*b^4*d^2 + 1...
```

3.99 $\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx$

Optimal. Leaf size=408

$$\frac{b(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d}-\frac{b(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\cdots}{\sqrt{a-\cdots}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+\cdots}}}$$

```
[Out] -2/3*b*(a+b*cot(d*x+c))^(3/2)/d+1/2*b*(a^2+b^2)*arctanh((-2^(1/2)*(a+b*cot(
d*x+c))^(1/2)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/
2)/(a-(a^2+b^2)^(1/2))^(1/2)-1/2*b*(a^2+b^2)*arctanh((2^(1/2)*(a+b*cot(d*x+
c))^(1/2)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(
a-(a^2+b^2)^(1/2))^(1/2)+1/4*b*(a^2+b^2)*ln(a+b*cot(d*x+c)+(a^2+b^2)^(1/2)-
2^(1/2)*(a+b*cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a+(a^2
+b^2)^(1/2))^(1/2)-1/4*b*(a^2+b^2)*ln(a+b*cot(d*x+c)+(a^2+b^2)^(1/2)+2^(1/2
)*(a+b*cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a+(a^2+b^2)^
(1/2))^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3609, 12, 3566, 714, 1143, 648, 632, 212, 642}

$$\frac{b(a^2+b^2)\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\cot(c+dx)}+\sqrt{a^2+b^2}+a+b\cot(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}}-\frac{b(a^2+b^2)\log\left(\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\cot(c+dx)}+\sqrt{a^2+b^2}+a+b\cot(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}}+\frac{b(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}}-\frac{b(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}+\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}}-\frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])^(3/2),x]
```

```
[Out] (b*(a^2 + b^2)*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[a + b*Cot[
c + d*x]])/Sqrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]*d
) - (b*(a^2 + b^2)*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] + Sqrt[2]*Sqrt[a + b*
Cot[c + d*x]])/Sqrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2
]]*d) - (2*b*(a + b*Cot[c + d*x])^(3/2))/(3*d) + (b*(a^2 + b^2)*Log[a + Sqr
t[a^2 + b^2] + b*Cot[c + d*x] - Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a +
b*Cot[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) - (b*(a^2 + b^2)*
Log[a + Sqrt[a^2 + b^2] + b*Cot[c + d*x] + Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]
]*Sqrt[a + b*Cot[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1143

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \int \left(-a^2-b^2\right)\sqrt{a+b\cot(c$$

$$= -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \left(-a^2-b^2\right)\int \sqrt{a+b\cot(c$$

$$= -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \frac{(b(a^2+b^2))\,\mathrm{Subst}\left(\int \frac{\sqrt{a+}}{b^2+x}\right.}{d}$$

$$= -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \frac{(2b(a^2+b^2))\,\mathrm{Subst}\left(\int \frac{}{a^2+b^2}\right.}{}$$

$$= -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \frac{(b(a^2+b^2))\,\mathrm{Subst}\left(\int \frac{}{\sqrt{a^2}}\right.}{}$$

$$= -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \frac{(b(a^2+b^2))\,\mathrm{Subst}\left(\int \frac{}{\sqrt{a^2}}\right.}{}$$

$$= -\frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \frac{b(a^2+b^2)\log\left(a+\sqrt{a^2+b}\right.}{}$$

$$= \frac{b(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c}{\sqrt{a-\sqrt{a^2+b^2}}}\right.}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.06, size = 178, normalized size = 0.44

$$\frac{(-a+b\cot(c+dx))(a+b\cot(c+dx))\left(3i\sqrt{a-ib}\left(a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)-3i\sqrt{a+ib}\left(a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)+2b(a+b\cot(c+dx))^{3/2}\right)\sin^2(c+dx)}{-3b^2d\cos^2(c+dx)+3a^2d\sin^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])^(3/2),x]
```

```
[Out] ((-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])*((3*I)*Sqrt[a - I*b]*(a^2 + b^2
)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]] - (3*I)*Sqrt[a + I*b]*(a^
2 + b^2)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]] + 2*b*(a + b*Cot[c
 + d*x])^(3/2))*Sin[c + d*x]^2)/(-3*b^2*d*Cos[c + d*x]^2 + 3*a^2*d*Sin[c +
d*x]^2)
```

Maple [A]
time = 0.63, size = 390, normalized size = 0.96

method	result
derivativedivides	$-\frac{2b\left(\frac{(a+b\cot(dx+c))^{\frac{3}{2}}}{3}+(-a^2-b^2)\left(\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\left(\sqrt{a^2+b^2}-a\right)\left(\frac{\ln\left(\sqrt{a+b\cot(dx+c)}\right.}{}\right.}{}\right.\right.}{}$
default	$-\frac{2b\left(\frac{(a+b\cot(dx+c))^{\frac{3}{2}}}{3}+(-a^2-b^2)\left(\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\left(\sqrt{a^2+b^2}-a\right)\left(\frac{\ln\left(\sqrt{a+b\cot(dx+c)}\right.}{}\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*(1/3*(a+b*cot(d*x+c))^(3/2)+(-a^2-b^2)*(1/4*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)*((a^2+b^2)^(1/2)-a)/b^2*(1/2*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))-(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a
+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))-1/4*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)*((a^2+b^2)^(1/2)-a)/b^2*(1/2*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c)
)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))-(2*(a^2+b^2)^(1/2)+2
```

```
*a)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2
*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cot(d*x + c) + a)^(3/2)*(b*cot(d*x + c) - a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int a^{2}\sqrt{a+b\cot\left(c+dx\right)}\,dx-\int\left(-b^{2}\sqrt{a+b\cot\left(c+dx\right)}\cot^{2}\left(c+dx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))**(3/2),x)
[Out] -Integral(a**2*sqrt(a + b*cot(c + d*x)), x) - Integral(-b**2*sqrt(a + b*cot
(c + d*x))*cot(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cot(d*x + c) + a)^(3/2)*(b*cot(d*x + c) - a), x)
```

Mupad [B]
time = 11.96, size = 2529, normalized size = 6.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*cot(c + d*x))^(3/2)*(a - b*cot(c + d*x)),x)
```

```
[Out] log((((16*b^4*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 - (16
*a*b^2*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)
^(1/2)*(a^2*b + b^3 + d*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 +
a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d)*(((-b^6*d^4*(3*a^2
- b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^(1/2))/2 + (8*b^5*(a^2 -
b^2)*(a^2 + b^2)^2)/d^3)*((6*a^2*b^8*d^4 - b^10*d^4 - 9*a^4*b^6*d^4)^(1/2)/
(4*d^4) - (3*a*b^4)/(4*d^2) + (a^3*b^2)/(4*d^2))^(1/2) - log((8*b^5*(a^2 -
b^2)*(a^2 + b^2)^2)/d^3 - (((16*b^4*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 -
 6*a^2*b^2))/d^2 + (16*a*b^2*(-((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) + 3*a*b^4*
d^2 - a^3*b^2*d^2)/d^4)^(1/2)*(a^2*b + b^3 - d*(-((-b^6*d^4*(3*a^2 - b^2)^2
)^(1/2) + 3*a*b^4*d^2 - a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2))
)/d)*(-((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) + 3*a*b^4*d^2 - a^3*b^2*d^2)/d^4)^
(1/2))/2)*(-((6*a^2*b^8*d^4 - b^10*d^4 - 9*a^4*b^6*d^4)^(1/2) + 3*a*b^4*d^2
 - a^3*b^2*d^2)/(4*d^4))^(1/2) - log((8*b^5*(a^2 - b^2)*(a^2 + b^2)^2)/d^3
- (((16*b^4*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (16*a
*b^2*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^(
1/2)*(a^2*b + b^3 - d*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^
3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d)*(((-b^6*d^4*(3*a^2 -
b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^(1/2))/2)*(((6*a^2*b^8*d^4
- b^10*d^4 - 9*a^4*b^6*d^4)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/(4*d^4))^(1/
2) + log((((16*b^4*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2
- (16*a*b^2*(-((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) + 3*a*b^4*d^2 - a^3*b^2*d^2
)/d^4)^(1/2)*(a^2*b + b^3 + d*(-((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) + 3*a*b^4
*d^2 - a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d)*(-((-b^6*d^4
*(3*a^2 - b^2)^2)^(1/2) + 3*a*b^4*d^2 - a^3*b^2*d^2)/d^4)^(1/2))/2 + (8*b^5
*(a^2 - b^2)*(a^2 + b^2)^2)/d^3)*((a^3*b^2)/(4*d^2) - (3*a*b^4)/(4*d^2) - (
6*a^2*b^8*d^4 - b^10*d^4 - 9*a^4*b^6*d^4)^(1/2)/(4*d^4))^(1/2) - log(((-((-
a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + a^5*d^2 - 3*a^3*b^2*d^2)/d^4)^(1/2)*((
16*a^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (16*a*
b^2*(-((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + a^5*d^2 - 3*a^3*b^2*d^2)/d^4)
^(1/2)*(a^2*b + b^3 + d*(-((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + a^5*d^2 -
 3*a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d))/2 - (16*a^4*b^3
*(a^2 + b^2)^2)/d^3)*(-((6*a^6*b^4*d^4 - a^4*b^6*d^4 - 9*a^8*b^2*d^4)^(1/2)
 + a^5*d^2 - 3*a^3*b^2*d^2)/(4*d^4))^(1/2) - log(((((-a^4*b^2*d^4*(3*a^2 -
b^2)^2)^(1/2) - a^5*d^2 + 3*a^3*b^2*d^2)/d^4)^(1/2)*((16*a^2*b^2*(a + b*cot
(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (16*a*b^2*(((-a^4*b^2*d^4*(
3*a^2 - b^2)^2)^(1/2) - a^5*d^2 + 3*a^3*b^2*d^2)/d^4)^(1/2)*(a^2*b + b^3 +
d*(((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - a^5*d^2 + 3*a^3*b^2*d^2)/d^4)^(1
/2)*(a + b*cot(c + d*x))^(1/2)))/d))/2 - (16*a^4*b^3*(a^2 + b^2)^2)/d^3)*((
(6*a^6*b^4*d^4 - a^4*b^6*d^4 - 9*a^8*b^2*d^4)^(1/2) - a^5*d^2 + 3*a^3*b^2*d
^2)/(4*d^4))^(1/2) + log(- ((((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - a^5*d^
```

```
2 + 3*a^3*b^2*d^2)/d^4)^(1/2)*((16*a^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4
+ b^4 - 6*a^2*b^2))/d^2 - (16*a*b^2*(((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2)
- a^5*d^2 + 3*a^3*b^2*d^2)/d^4)^(1/2)*(a^2*b + b^3 - d*(((-a^4*b^2*d^4*(3*a
^2 - b^2)^2)^(1/2) - a^5*d^2 + 3*a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x
))^(1/2)))/d))/2 - (16*a^4*b^3*(a^2 + b^2)^2)/d^3)*((6*a^6*b^4*d^4 - a^4*b^
6*d^4 - 9*a^8*b^2*d^4)^(1/2)/(4*d^4) - a^5/(4*d^2) + (3*a^3*b^2)/(4*d^2))^(
1/2) + log(- ((-((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + a^5*d^2 - 3*a^3*b^2
*d^2)/d^4)^(1/2)*((16*a^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2
*b^2))/d^2 - (16*a*b^2*(-((-a^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + a^5*d^2 -
3*a^3*b^2*d^2)/d^4)^(1/2)*(a^2*b + b^3 - d*(-((-a^4*b^2*d^4*(3*a^2 - b^2)^2
)^(1/2) + a^5*d^2 - 3*a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/
d))/2 - (16*a^4*b^3*(a^2 + b^2)^2)/d^3)*((3*a^3*b^2)/(4*d^2) - a^5/(4*d^2)
- (6*a^6*b^4*d^4 - a^4*b^6*d^4 - 9*a^8*b^2*d^4)^(1/2)/(4*d^4))^(1/2) - (2*b
*(a + b*cot(c + d*x))^(3/2))/(3*d)
```

3.100 $\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx$

Optimal. Leaf size=422

$$\frac{b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}\,d}-\frac{b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\cdots}{\sqrt{a-\cdots}}\right.}{\sqrt{2}\sqrt{a-\sqrt{a^2+\cdots}}}$$

```
[Out] -2*b*(a+b*cot(d*x+c))^(1/2)/d+1/2*b*arctanh((-2^(1/2)*(a+b*cot(d*x+c))^(1/2
)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))*(a^2+b^2)^(1/2)/d*2
^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)-1/2*b*arctanh((2^(1/2)*(a+b*cot(d*x+c))^(1
/2)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))*(a^2+b^2)^(1/2)/d
*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)-1/4*b*ln(a+b*cot(d*x+c)+(a^2+b^2)^(1/2)-
2^(1/2)*(a+b*cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))*(a^2+b^2)^(1/2)/d
*2^(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)+1/4*b*ln(a+b*cot(d*x+c)+(a^2+b^2)^(1/2)+
2^(1/2)*(a+b*cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))*(a^2+b^2)^(1/2)/d
*2^(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3609, 12, 3566, 722, 1108, 648, 632, 212, 642}

$$-\frac{b\sqrt{a^2+b^2}\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\cot(c+dx)}+\sqrt{a^2+b^2}+a+b\cot(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}}+\frac{b\sqrt{a^2+b^2}\log\left(\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\cot(c+dx)}+\sqrt{a^2+b^2}+a+b\cot(c+dx)\right)}{2\sqrt{2}d\sqrt{\sqrt{a^2+b^2}+a}}+\frac{b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}}-\frac{b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+b^2}+a}+\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}}-\frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*Cot[c + d*x])*Sqrt[a + b*Cot[c + d*x]],x]
```

```
[Out] (b*Sqrt[a^2 + b^2]*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[a + b*
Cot[c + d*x]])/Sqrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2
]]*d) - (b*Sqrt[a^2 + b^2]*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] + Sqrt[2]*Sqr
t[a + b*Cot[c + d*x]])/Sqrt[a - Sqrt[a^2 + b^2]]])/(Sqrt[2]*Sqrt[a - Sqrt[a
^2 + b^2]]*d) - (2*b*Sqrt[a + b*Cot[c + d*x]])/d - (b*Sqrt[a^2 + b^2]*Log[a
 + Sqrt[a^2 + b^2] + b*Cot[c + d*x] - Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqr
t[a + b*Cot[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) + (b*Sqrt[a
^2 + b^2]*Log[a + Sqrt[a^2 + b^2] + b*Cot[c + d*x] + Sqrt[2]*Sqrt[a + Sqrt[
a^2 + b^2]]*Sqrt[a + b*Cot[c + d*x]]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx = -\frac{2b\sqrt{a+b\cot(c+dx)}}{d}+\int \frac{-a^2-b^2}{\sqrt{a+b\cot(c+dx)}}\,dx$$

$$= -\frac{2b\sqrt{a+b\cot(c+dx)}}{d}+\left(-a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cot(c+d}}$$

$$= -\frac{2b\sqrt{a+b\cot(c+dx)}}{d}+\frac{\left(b\left(a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+x}\,(}\right.}{d}$$

$$= -\frac{2b\sqrt{a+b\cot(c+dx)}}{d}+\frac{\left(2b\left(a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{a^2+b^2-2ax}\right.}{}$$

$$= -\frac{2b\sqrt{a+b\cot(c+dx)}}{d}+\frac{\left(b\sqrt{a^2+b^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{a^2+}}\right.}{}$$

$$= -\frac{2b\sqrt{a+b\cot(c+dx)}}{d}+\frac{\left(b\sqrt{a^2+b^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{a^2+}}\right.}{}$$

$$= -\frac{2b\sqrt{a+b\cot(c+dx)}}{d}-\frac{b\sqrt{a^2+b^2}\,\log\left(a+\sqrt{a^2+b^2}\right.}{}$$

$$= \frac{b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\,\sqrt{a+b\cot(c+}}{\sqrt{a-\sqrt{a^2+b^2}}}\right.}{\sqrt{2}\,\sqrt{a-\sqrt{a^2+b^2}}\,d}$$

Mathematica [C] Result contains complex when optimal does not.

time = 1.07, size = 158, normalized size = 0.37

$$\frac{(-a+b\cot(c+dx))\left(\frac{i(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}}-\frac{i(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}+2b\sqrt{a+b\cot(c+dx)}\right)\sin(c+dx)}{d(-b\cos(c+dx)+a\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*Cot[c + d*x])*Sqrt[a + b*Cot[c + d*x]],x]
```

```
[Out] ((-a + b*Cot[c + d*x])*((I*(a^2 + b^2)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqr
t[a - I*b]])/Sqrt[a - I*b] - (I*(a^2 + b^2)*ArcTanh[Sqrt[a + b*Cot[c + d*x]
]/Sqrt[a + I*b]])/Sqrt[a + I*b] + 2*b*Sqrt[a + b*Cot[c + d*x]])*Sin[c + d*x
])/(d*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 801 vs. $2(341) = 682$.
time = 0.64, size = 802, normalized size = 1.90

method	result
derivativedivides	$-\frac{2b\left(\sqrt{a+b\cot(dx+c)}+(-a^2-b^2)\left(-\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^2+\sqrt{2\sqrt{a^2+b^2}}\right.}{}\right.\right.}{}$
default	$-\frac{2b\left(\sqrt{a+b\cot(dx+c)}+(-a^2-b^2)\left(-\frac{\left(\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^2+\sqrt{2\sqrt{a^2+b^2}}\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*((a+b*cot(d*x+c))^(1/2)+(-a^2-b^2)*(1/4/b^2/(a^2+b^2)^(3/2)*(-1/2*((
2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-(2*(a^2+b^2)^(1/2
```

```
)+2*a)^(1/2)*a*b^2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))+2*(-2*a^2*b^2-2*b^4+1/2*((2*(a^2+b^2)^(1/2
)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1
/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b
^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2
*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2)))+1/4/b^2/(a^2+b^2)^(3/2)*(1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2
+b^2)^(1/2)*a^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*ln(b*cot(d*x+
c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+
2*(2*a^2*b^2+2*b^4-1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2
)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cot(d*x + c) + a)*(b*cot(d*x + c) - a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int a\sqrt{a+b\cot{(c+dx)}}\,dx-\int\left(-b\sqrt{a+b\cot{(c+dx)}}\cot{(c+dx)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))**(1/2),x)
[Out] -Integral(a*sqrt(a + b*cot(c + d*x)), x) - Integral(-b*sqrt(a + b*cot(c + d
*x))*cot(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cot(d*x + c) + a)*(b*cot(d*x + c) - a), x)
```

Mupad [B]
time = 2.57, size = 583, normalized size = 1.38

$$-\mathrm{atanh}\left(\frac{d^3\,\left(\frac{16\,\left(a^2\,b^4-a^4\,b^2\right)\,\sqrt{a+b\,\mathrm{cot}\left(c+d\,x\right)}}{d^2}+\frac{16\,a\,b^2\,\left(a^3+1{}\mathrm{i}\,b\,a^2\right)\,\sqrt{a+b\,\mathrm{cot}\left(c+d\,x\right)}}{d^2}\right)\,\sqrt{-\frac{a^3+1{}\mathrm{i}\,b\,a^2}{d^2}}}{16\,\left(a^5\,b^3+a^3\,b^5\right)}\right)\,\sqrt{-\frac{a^3+1{}\mathrm{i}\,b\,a^2}{d^2}}-\mathrm{atanh}\left(\frac{d^3\,\sqrt{\frac{-a^3+a^2\,b\,1{}\mathrm{i}}{d^2}}\,\left(\frac{16\,\left(a^2\,b^4-a^4\,b^2\right)\,\sqrt{a+b\,\mathrm{cot}\left(c+d\,x\right)}}{d^2}-\frac{16\,a\,b^2\,\left(-a^3+a^2\,b\,1{}\mathrm{i}\right)\,\sqrt{a+b\,\mathrm{cot}\left(c+d\,x\right)}}{d^2}\right)}{16\,\left(a^5\,b^3+a^3\,b^5\right)}\right)\,\sqrt{\frac{-a^3+a^2\,b\,1{}\mathrm{i}}{d^2}}-\frac{2\,b\,\sqrt{a+b\,\mathrm{cot}\left(c+d\,x\right)}}{d}+\mathrm{atan}\left(\frac{b^6\,\sqrt{\frac{a\,b^2}{4\,d^2}-\frac{b^3\,1{}\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\,\mathrm{cot}\left(c+d\,x\right)}\,32{}\mathrm{i}}{\frac{b^8\,16{}\mathrm{i}}{d}+\frac{a^2\,b^6\,16{}\mathrm{i}}{d}}+\frac{32\,a\,b^5\,\sqrt{\frac{a\,b^2}{4\,d^2}-\frac{b^3\,1{}\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\,\mathrm{cot}\left(c+d\,x\right)}}{\frac{b^8\,16{}\mathrm{i}}{d}+\frac{a^2\,b^6\,16{}\mathrm{i}}{d}}\right)\,\sqrt{\frac{a\,b^2-b^3\,1{}\mathrm{i}}{4\,d^2}}\,2{}\mathrm{i}-\mathrm{atan}\left(\frac{b^6\,\sqrt{\frac{a\,b^2}{4\,d^2}+\frac{b^3\,1{}\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\,\mathrm{cot}\left(c+d\,x\right)}\,32{}\mathrm{i}}{\frac{b^8\,16{}\mathrm{i}}{d}+\frac{a^2\,b^6\,16{}\mathrm{i}}{d}}-\frac{32\,a\,b^5\,\sqrt{\frac{a\,b^2}{4\,d^2}+\frac{b^3\,1{}\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\,\mathrm{cot}\left(c+d\,x\right)}}{\frac{b^8\,16{}\mathrm{i}}{d}+\frac{a^2\,b^6\,16{}\mathrm{i}}{d}}\right)\,\sqrt{\frac{b^3\,1{}\mathrm{i}+a\,b^2}{4\,d^2}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*cot(c + d*x))^(1/2)*(a - b*cot(c + d*x)),x)
```

```
[Out] atan((b^6*((a*b^2)/(4*d^2) - (b^3*1i)/(4*d^2))^(1/2)*(a + b*cot(c + d*x))^(
1/2)*32i)/((b^8*16i)/d + (a^2*b^6*16i)/d) + (32*a*b^5*((a*b^2)/(4*d^2) - (b
^3*1i)/(4*d^2))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((b^8*16i)/d + (a^2*b^6*1
6i)/d))*((a*b^2 - b^3*1i)/(4*d^2))^(1/2)*2i - atan((b^6*((b^3*1i)/(4*d^2) +
 (a*b^2)/(4*d^2))^(1/2)*(a + b*cot(c + d*x))^(1/2)*32i)/((b^8*16i)/d + (a^2
*b^6*16i)/d) - (32*a*b^5*((b^3*1i)/(4*d^2) + (a*b^2)/(4*d^2))^(1/2)*(a + b*
cot(c + d*x))^(1/2))/((b^8*16i)/d + (a^2*b^6*16i)/d))*((a*b^2 + b^3*1i)/(4*
d^2))^(1/2)*2i - atanh((d^3*((16*(a^2*b^4 - a^4*b^2)*(a + b*cot(c + d*x))^(
1/2))/d^2 + (16*a*b^2*(a^2*b*1i + a^3)*(a + b*cot(c + d*x))^(1/2))/d^2)*(-(
a^2*b*1i + a^3)/d^2)^(1/2))/(16*(a^3*b^5 + a^5*b^3)))*(-(a^2*b*1i + a^3)/d^
2)^(1/2) - atanh((d^3*((a^2*b*1i - a^3)/d^2)^(1/2)*((16*(a^2*b^4 - a^4*b^2)
*(a + b*cot(c + d*x))^(1/2))/d^2 - (16*a*b^2*(a^2*b*1i - a^3)*(a + b*cot(c
+ d*x))^(1/2))/d^2))/(16*(a^3*b^5 + a^5*b^3)))*((a^2*b*1i - a^3)/d^2)^(1/2)
 - (2*b*(a + b*cot(c + d*x))^(1/2))/d
```

3.101 $\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

Optimal. Leaf size=102

$$\frac{(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d}-\frac{(iA-B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d}$$

```
[Out] (I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)-(I*A-
B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d/(a+I*b)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3620, 3618, 65, 214}

$$\frac{(B+iA)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{(-B+iA)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

```
[Out] ((I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*
d) - ((I*A - B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/(Sqrt[a +
I*b]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
```

```
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{1}{2}(A-iB)\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx + \frac{1}{2}(A+iB)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,d$$

$$= \frac{(iA-B)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx, x, -i\cot(c+dx)\right)}{2d} - \frac{(iA+B)\text{Subst}\left(\right.}{}$$

$$= \frac{(A-iB)\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd} + \frac{(A+iB)\text{Subst}\left(\right.}{}$$

$$= \frac{(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d} - \frac{(iA-B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c}}{\sqrt{a+ib}}\right.}{\sqrt{a+ib}\,d}$$

Mathematica [A]
time = 0.54, size = 154, normalized size = 1.51

$$\frac{\left(\sqrt{a+ib}\,(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)+\sqrt{a-ib}\,(-iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)\right)(A+B\cot(c+dx))\sin(c+dx)}{\sqrt{a-ib}\,\sqrt{a+ib}\,d(B\cos(c+dx)+A\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

```
[Out] ((Sqrt[a + I*b]*(I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]] +
 Sqrt[a - I*b]*((-I)*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]]
)*(A + B*Cot[c + d*x])*Sin[c + d*x])/(Sqrt[a - I*b]*Sqrt[a + I*b]*d*(B*Cos[
c + d*x] + A*Sin[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1406 vs. 2(84) = 168.
time = 0.60, size = 1407, normalized size = 13.79 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a^2+b^2)^(3/2)/b^2*(1/2*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^
2)^(1/2)*a^2*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^3-A*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)*a^3*b-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^3+B*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*
(a^2+b^2)^(1/2)*a^3-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-B
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)
*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^
2+b^2)^(1/2))+2*(2*A*a^2*b^3+2*A*b^5-2*B*a^3*b^2-2*B*a*b^4-1/2*(A*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*
(a^2+b^2)^(1/2)*b^3-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b-A*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)*a*b^3+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a-B*(2
*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-B*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)*(a^2+b^2)^(1/2)*a*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-B*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/
2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))-1/2/(a^2+b^2)^(3/2)/b^2*(-1/2*(A*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*(a^2+b^2)^(1/2)*b^3-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b-A*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a*b^3+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a-B
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-B*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-B*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))+2*(-2*A*a^2*b^3-2*A*b^5+2*B*a^
3*b^2+2*B*a*b^4+1/2*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b+
A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^3-A*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*a^3*b-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^3+B*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*(a^2+b^2)^(3/2)*a-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)
*a^3-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-B*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)*a^2*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2
*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cot(d*x + c) + A)/sqrt(b*cot(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cot(c + dx)}{\sqrt{a + b\cot(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cot(c + d*x))/sqrt(a + b*cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cot(d*x + c) + A)/sqrt(b*cot(d*x + c) + a), x)
```

Mupad [B]
time = 2.29, size = 2909, normalized size = 28.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^(1/2),x)
```

```
[Out] 2*atanh((32*B^2*b^2*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^4
)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*B^
3*b^2)/d - (16*B^3*a^2*b^2*d^3)/(a^2*d^4 + b^2*d^4) + (4*B*a*b^2*d^2*(-16*B
^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*((B^2*a*d^2)/(4*(a^2*d^4
 + b^2*d^4)) - (-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2)*(a +
 b*cot(c + d*x))^(1/2)*(-16*B^4*b^2*d^4)^(1/2))/(16*B^3*b^4*d + 16*B^3*a^2*
b^2*d - (16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - (16*B^3*a^4*b^2*d^5)/(a^
2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2
*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*
B^2*a^2*b^2*d^2*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^4)^(1
/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/(16*B^3*b^4
*d + 16*B^3*a^2*b^2*d - (16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - (16*B^3*
```

a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^(1/2)
)/(a^2*d^5 + b^2*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 +
b^2*d^5)))*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^4)^(1/2)/(
16*(a^2*d^4 + b^2*d^4)))^(1/2) + 2*atanh((8*a*b^2*((-16*B^4*b^2*d^4)^(1/2)/
(16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a +
b*cot(c + d*x))^(1/2)*(-16*B^4*b^2*d^4)^(1/2))/((16*B^3*a^2*b^4*d^5)/(a^2*d
^4 + b^2*d^4) - 16*B^3*a^2*b^2*d - 16*B^3*b^4*d + (16*B^3*a^4*b^2*d^5)/(a^2
*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*
d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*B
^2*b^2*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(
a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*B^3*a^2*b^2*d^3
)/(a^2*d^4 + b^2*d^4) - (16*B^3*b^2)/d + (4*B*a*b^2*d^2*(-16*B^4*b^2*d^4)^(
1/2))/(a^2*d^5 + b^2*d^5)) + (32*B^2*a^2*b^2*d^2*((-16*B^4*b^2*d^4)^(1/2)/(
16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b
*cot(c + d*x))^(1/2))/((16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - 16*B^3*a^
2*b^2*d - 16*B^3*b^4*d + (16*B^3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^
3*b^2*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*B*a*b^4*d^4*(-1
6*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*((-16*B^4*b^2*d^4)^(1/2)/(16*(a
^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2) + 2*atanh((
32*A^2*b^2*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/
(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*A^3*a*b^3*d
^3)/(a^2*d^4 + b^2*d^4) - (4*A*b^3*d^2*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 +
b^2*d^5)) + (8*a*b^2*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A
^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-16*A^
4*b^2*d^4)^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*b^5*d^4*(-
16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4
+ b^2*d^4) - (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5))
 - (32*A^2*a^2*b^2*d^2*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) -
(A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16
*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*b^5*d^4*(-16*A^4*b^2*d^4)^(1/2))
/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*a^2*
b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*((-16*A^4*b^2*d^4)^(
1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)
- 2*atanh((8*a*b^2*(- (-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A
^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-16*A^
4*b^2*d^4)^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*b^5*d^4*(-
16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4
+ b^2*d^4) + (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5))
 - (32*A^2*b^2*(- (-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a
d^2)/(4(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*A^3*a
*b^3*d^3)/(a^2*d^4 + b^2*d^4) + (4*A*b^3*d^2*(-16*A^4*b^2*d^4)^(1/2))/(a^2*
d^5 + b^2*d^5)) + (32*A^2*a^2*b^2*d^2*(- (-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d
^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c +
d*x))^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*b^5*d^4*(-16*A^
4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 + b^2

```
*d^4) + (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*(-
(-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4
+ b^2*d^4)))^(1/2)
```

3.102 $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}-\frac{(iA-B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}+\frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}}$$

```
[Out] (I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d-(I*A-
B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d+2*(A*b-B*a
)/(a^2+b^2)/d/(a+b*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3610, 3620, 3618, 65, 214}

$$\frac{2(Ab-aB)}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}+\frac{(B+iA)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}}-\frac{(-B+iA)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^(3/2),x]
```

```
[Out] ((I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(3/2
)*d) - ((I*A - B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/((a + I*
b)^(3/2)*d) + (2*(A*b - a*B))/((a^2 + b^2)*d*Sqrt[a + b*Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
```

```
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}} + \frac{\int \frac{aA+bB-(Ab-aB)\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx}{a^2+b^2}$$

$$= \frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}} + \frac{(A-iB)\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx}{2(a-ib)} + \frac{(A$$

$$= \frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}} + \frac{(i(A+iB))\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}}\,dx,\right.}{2(a+ib)d}$$

$$= \frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}} + \frac{(A-iB)\text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}}\,dx, x, \sqrt{a+\right.}{(a-ib)bd}$$

$$= \frac{(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{(iA-B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(}{\sqrt{a+}\right.}{(a+ib)^{3/2}d}$$

Mathematica [A]

time = 1.77, size = 226, normalized size = 1.64

$$-\frac{\frac{\left(aAb+Ab\sqrt{-b^2}+b^2B-a\sqrt{-b^2}\,B\right)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\,\sqrt{a-\sqrt{-b^2}}} - \frac{\left(aAb-Ab\sqrt{-b^2}+b^2B+a\sqrt{-b^2}\,B\right)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\,\sqrt{a+\sqrt{-b^2}}} + \frac{2(-Ab+aB)}{\sqrt{a+b\cot(c+dx)}}}{(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^(3/2),x]
```

```
[Out] -((((a*A*b + A*b*Sqrt[-b^2] + b^2*B - a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Co
t[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) - ((a*
A*b - A*b*Sqrt[-b^2] + b^2*B + a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Cot[c + d
*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + (2*(-(A*b)
+ a*B))/Sqrt[a + b*Cot[c + d*x]])/((a^2 + b^2)*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2273 vs. $2(118) = 236$.
time = 0.62, size = 2274, normalized size = 16.48

method	result	size
derivativedivides	Expression too large to display	2274
default	Expression too large to display	2274

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(A*b-B*a)/(a^2+b^2)/(a+b*cot(d*x+c))^(1/2)-2/(a^2+b^2)*(1/4/b^2/(3*a
^2-b^2)/(a^2+b^2)^(3/2)*(1/2*(3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(
1/2)*a^5*b+2*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3*b^3-A*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^5-3*A*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*a^6*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4*b^3+3*A*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*a^2*b^5-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^7+B*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a^4-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^
(3/2)*b^4-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^6+2*B*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4*b^2+3*B*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*(a^2+b^2)^(1/2)*a^2*b^4-6*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5*b^2-4*B*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^4+2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^
6)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(
a^2+b^2)^(1/2))+2*(12*A*a^5*b^3+8*A*a^3*b^5-4*A*a*b^7-6*B*a^6*b^2+2*B*a^4*b
^4+6*B*a^2*b^6-2*B*b^8-1/2*(3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/
2)*a^5*b+2*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3*b^3-A*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^5-3*A*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)*a^6*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4*b^3+3*A*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*a^2*b^5-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^7+B*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*(a^2+b^2)^(3/2)*a^4-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3
/2)*b^4-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^6+2*B*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4*b^2+3*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*(a^2+b^2)^(1/2)*a^2*b^4-6*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5*b^2-4*B*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^4+2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^6)
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+
```

```
b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2)))+1/4/b^2/(3*a^2-b^2)/(a^2+b^2)^(3/2)*(-1/2*(3*A*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*(a^2+b^2)^(1/2)*a^5*b+2*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)
^(1/2)*a^3*b^3-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^5-3*A*(2
*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^6*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4*b^3+3
*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^5-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^
7+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a^4-B*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*(a^2+b^2)^(3/2)*b^4-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1
/2)*a^6+2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4*b^2+3*B*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b^4-6*B*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*a^5*b^2-4*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^4+2*B*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*a*b^6)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))+2*(-12*A*a^5*b^3-8*A*a^3*b^5+4*A*a*b^7
+6*B*a^6*b^2-2*B*a^4*b^4-6*B*a^2*b^6+2*B*b^8+1/2*(3*A*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*(a^2+b^2)^(1/2)*a^5*b+2*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^
(1/2)*a^3*b^3-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^5-3*A*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*a^6*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4*b^3+3*
A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^5-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^7
+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a^4-B*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*(a^2+b^2)^(3/2)*b^4-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/
2)*a^6+2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4*b^2+3*B*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b^4-6*B*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*a^5*b^2-4*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^4+2*B*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)*a*b^6)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2
*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((B*cot(d*x + c) + A)/(b*cot(d*x + c) + a)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cot(c + dx)}{(a + b\cot(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cot(c + d*x))/(a + b*cot(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*cot(d*x + c) + A)/(b*cot(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 6.47, size = 2500, normalized size = 18.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^(3/2),x)
```

```
[Out] (log((((a + b*cot(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 - 3
2*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3) + ((((96*A^4*a^2*b^4*d^4 - 16*A^4*b
^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^
6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(64*A*a*b^11*d^4 -
((((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^
2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^
2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5
+ 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4
 + 256*A*a^3*b^9*d^4 + 384*A*a^5*b^7*d^4 + 256*A*a^7*b^5*d^4 + 64*A*a^9*b^3
*d^4))/4)*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/
2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 +
 3*a^4*b^2*d^4))^(1/2))/4 - 8*A^3*b^9*d^2 - 24*A^3*a^2*b^7*d^2 - 24*A^3*a^4
*b^5*d^2 - 8*A^3*a^6*b^3*d^2)*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*
A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d
^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 + (log((((a + b*cot(c + d*x))
^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3 - 16*A^2*
```

```
a^8*b^2*d^3) + ((-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d
^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^
4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(64*A*a*b^11*d^4 - ((-((96*A^4*a^2*b^4*d^4 -
16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*
d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a + b*cot(
c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a
^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 + 256*A*a^3*b^9*d^4 + 38
4*A*a^5*b^7*d^4 + 256*A*a^7*b^5*d^4 + 64*A*a^9*b^3*d^4))/4)*(-((96*A^4*a^2*
b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*
A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/
4 - 8*A^3*b^9*d^2 - 24*A^3*a^2*b^7*d^2 - 24*A^3*a^4*b^5*d^2 - 8*A^3*a^6*b^3
*d^2)*(-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2)
+ 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*
a^4*b^2*d^4))^(1/2))/4 - log(- ((a + b*cot(c + d*x))^(1/2)*(16*A^2*b^10*d^3
 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3) - (((96*A^
4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2
 + 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2
*d^4))^(1/2)*((((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)
^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^
2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^
5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5
+ 64*a^11*b^2*d^5) + 64*A*a*b^11*d^4 + 256*A*a^3*b^9*d^4 + 384*A*a^5*b^7*d^
4 + 256*A*a^7*b^5*d^4 + 64*A*a^9*b^3*d^4))*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b
^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(16
*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 8*A^3*b^9
*d^2 - 24*A^3*a^2*b^7*d^2 - 24*A^3*a^4*b^5*d^2 - 8*A^3*a^6*b^3*d^2)*(((96*A
^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^
2 + 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^
2*d^4))^(1/2) - log(- ((a + b*cot(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2
*a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3) - (-((96*A^4*a^2*b^
4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^
2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(
1/2)*((-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2)
+ 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d
^4 + 48*a^4*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320
*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^
11*b^2*d^5) + 64*A*a*b^11*d^4 + 256*A*a^3*b^9*d^4 + 384*A*a^5*b^7*d^4 + 256
*A*a^7*b^5*d^4 + 64*A*a^9*b^3*d^4))*(-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4
 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(16*a^6*d
^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 8*A^3*b^9*d^2 -
 24*A^3*a^2*b^7*d^2 - 24*A^3*a^4*b^5*d^2 - 8*A^3*a^6*b^3*d^2)*(-((96*A^4*a^
2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 1
2*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4
))^(1/2) + (log(24*B^3*a^3*b^6*d^2 - ((((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^
4 - 144*B^4*a^4*b^2*d^4)^(1/2) + 4*B^2*a^3*d^2 - 12*B^2*a*b^2*d^2)/(a^6*d^4
```

```
 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(((((96*B^4*a^2*b^4*d^4
- 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) + 4*B^2*a^3*d^2 - 12*B^2*a*b^
2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(((((96*B
^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) + 4*B^2*a^3*d^
2 - 12*B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^
(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5...
```

3.103 $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}-\frac{(iA-B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d}+\frac{2(Ab-a}{3\left(a^2+b^2\right)d(a+b\,c}$$

[Out] (I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d-(I*A-B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d+2/3*(A*b-B*a)/(a^2+b^2)/d/(a+b*cot(d*x+c))^(3/2)+2*(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2/d/(a+b*cot(d*x+c))^(1/2)

Rubi [A]
time = 0.28, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3610, 3620, 3618, 65, 214}

$$\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\cot(c+dx))^{3/2}}+\frac{2\left(a^2(-B)+2aAb+b^2B\right)}{d\left(a^2+b^2\right)^2\sqrt{a+b\cot(c+dx)}}+\frac{(B+iA)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}}-\frac{(-B+iA)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^(5/2),x]

[Out] ((I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(5/2)*d) - ((I*A - B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/((a + I*b)^(5/2)*d) + (2*(A*b - a*B))/(3*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^(3/2)) + (2*(2*a*A*b - a^2*B + b^2*B))/((a^2 + b^2)^2*d*Sqrt[a + b*Cot[c + d*x]])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
```

```
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx = \frac{2(Ab-aB)}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}}+\frac{\int \frac{aA+bB-(Ab-aB)\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx}{a^2+b^2}$$

$$= \frac{2(Ab-aB)}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}}+\frac{2\left(2aAb-a^2B+b^2B\right)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\cot(c+dx)}}+\frac{\int \frac{a^2A}{}}{}$$

$$= \frac{2(Ab-aB)}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}}+\frac{2\left(2aAb-a^2B+b^2B\right)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\cot(c+dx)}}+\frac{(A-}{}$$

$$= \frac{2(Ab-aB)}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}}+\frac{2\left(2aAb-a^2B+b^2B\right)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\cot(c+dx)}}+\frac{(iA-}{}$$

$$= \frac{2(Ab-aB)}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}}+\frac{2\left(2aAb-a^2B+b^2B\right)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\cot(c+dx)}}-\frac{(A-}{}$$

$$= \frac{(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}-\frac{(iA-B)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c}}{\sqrt{a+ib}}\right.}{(a+ib)^{5/2}d}$$

Mathematica [A]

time = 3.66, size = 319, normalized size = 1.72

$$-\frac{\frac{3\left(2ab\left(A\sqrt{-b^2}+bB\right)+a^2\left(Ab-\sqrt{-b^2}\,B\right)+b^2\left(-Ab+\sqrt{-b^2}\,B\right)\right)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}}+\frac{3\left(2ab\left(A\sqrt{-b^2}-bB\right)-a^2\left(Ab+\sqrt{-b^2}\,B\right)+b^2\left(Ab+\sqrt{-b^2}\,B\right)\right)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}}+\frac{2(a^2+b^2)(-Ab+aB)}{(a+b\cot(c+dx))^{3/2}}+\frac{6(-2aAb+a^2B-b^2B)}{\sqrt{a+b\cot(c+dx)}}}{3\left(a^2+b^2\right)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^(5/2),x]
```

```
[Out] -1/3*((3*(2*a*b*(A*Sqrt[-b^2] + b*B) + a^2*(A*b - Sqrt[-b^2]*B) + b^2*(-(A*
b) + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])
/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) + (3*(2*a*b*(A*Sqrt[-b^2] - b*B) - a^2*(
A*b + Sqrt[-b^2]*B) + b^2*(A*b + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a + b*Cot[c +
d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + (2*(a^2 +
b^2)*(-(A*b) + a*B))/(a + b*Cot[c + d*x])^(3/2) + (6*(-2*a*A*b + a^2*B - b^
2*B))/Sqrt[a + b*Cot[c + d*x]])/((a^2 + b^2)^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3235 vs. $2(161) = 322$.
time = 0.66, size = 3236, normalized size = 17.49

method	result	size
derivativedivides	Expression too large to display	3236
default	Expression too large to display	3236

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/3*(A*b-B*a)/(a^2+b^2)/(a+b*cot(d*x+c))^(3/2)+2*(2*A*a*b-B*a^2+B*b^2)
/(a^2+b^2)^2/(a+b*cot(d*x+c))^(1/2)-2/(a^2+b^2)^2*(1/4/b/(5*a^4-10*a^2*b^2+
b^4)/(a^2+b^2)^(3/2)*(1/2*(3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2
)*a^6+5*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a^4*b^2+A*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a^2*b^4-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*(a^2+b^2)^(3/2)*b^6+2*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^8
-18*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^6*b^2-10*A*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4*b^4+10*A*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*(a^2+b^2)^(1/2)*a^2*b^6-5*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^9+20*A*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*a^7*b^2-6*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5*b^4
-28*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^6+3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*a*b^8+10*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^7*b-10*B*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^5*b^3-18*B*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*(a^2+b^2)^(1/2)*a^3*b^5+2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^
2)^(1/2)*a*b^7-15*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^8*b+20*B*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)*a^6*b^3+22*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4*b^5-12*B*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^7+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^9)*ln(
```

b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^
2)^(1/2))+2*(30*A*a^8*b^2-40*A*a^6*b^4-44*A*a^4*b^6+24*A*a^2*b^8-2*A*b^10-1
0*B*a^9*b+40*B*a^7*b^3-12*B*a^5*b^5-56*B*a^3*b^7+6*B*a*b^9-1/2*(3*A*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a^6+5*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*(a^2+b^2)^(3/2)*a^4*b^2+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a
^2*b^4-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*b^6+2*A*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^8-18*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a
^2+b^2)^(1/2)*a^6*b^2-10*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^
4*b^4+10*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b^6-5*A*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)*a^9+20*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^7*b^2-6*A*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5*b^4-28*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^
3*b^6+3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^8+10*B*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)*(a^2+b^2)^(1/2)*a^7*b-10*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1
/2)*a^5*b^3-18*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3*b^5+2*B*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^7-15*B*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*a^8*b+20*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^6*b^3+22*B*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*a^4*b^5-12*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^7+B*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^9)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a
)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b/(5*a^4-10*a^2*b^2+b^4)/(a^2+
b^2)^(3/2)*(-1/2*(3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a^6+5*A
(2(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a^4*b^2+A*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*(a^2+b^2)^(3/2)*a^2*b^4-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^
2)^(3/2)*b^6+2*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^8-18*A*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^6*b^2-10*A*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4*b^4+10*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+
b^2)^(1/2)*a^2*b^6-5*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^9+20*A*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*a^7*b^2-6*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5*b^4-28*A*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^6+3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^8+
10*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^7*b-10*B*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^5*b^3-18*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
*(a^2+b^2)^(1/2)*a^3*b^5+2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*
a*b^7-15*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^8*b+20*B*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*a^6*b^3+22*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4*b^5-12*B*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a^2*b^7+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^9)*ln((a+b*cot(
d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))
+2*(-30*A*a^8*b^2+40*A*a^6*b^4+44*A*a^4*b^6-24*A*a^2*b^8+2*A*b^10+10*B*a^9*
b-40*B*a^7*b^3+12*B*a^5*b^5+56*B*a^3*b^7-6*B*a*b^9+1/2*(3*A*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a^6+5*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b
^2)^(3/2)*a^4*b^2+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a^2*b^4-A
(2(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*b^6+2*A*(2*(a^2+b^2)^(1/2)+2
a)^(1/2)(a^2+b^2)^(1/2)*a^8-18*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^
(1/2)*a^6*b^2-10*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4*b^4+10
A(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b^6-5*A*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)*a^9+20*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^7*b^2-6*A*(2*(a^2+

```
b^2)^(1/2)+2*a)^(1/2)*a^5*b^4-28*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^6+3*
A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^8+10*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a
^2+b^2)^(1/2)*a^7*b-10*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^5*
b^3-18*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3*b^5+2*B*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^7-15*...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((B*cot(d*x + c) + A)/(b*cot(d*x + c) + a)^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cot\left(c + dx\right)}{\left(a + b\cot\left(c + dx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**(5/2),x)
[Out] Integral((A + B*cot(c + d*x))/(a + b*cot(c + d*x))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((B*cot(d*x + c) + A)/(b*cot(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 17.93, size = 2500, normalized size = 13.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^(5/2),x)
```

```
[Out] (log((((a + b*cot(c + d*x))^(1/2)*(320*A^2*a^4*b^14*d^3 - 16*A^2*b^18*d^3 +
 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8*b^10*d^3 + 1024*A^2*a^10*b^8*d^3 + 32
0*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*d^3) + ((((320*A^4*a^2*b^8*d^4 - 16*A^
4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*
d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d
^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2
*d^4))^(1/2)*(896*A*a^6*b^15*d^4 - ((((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^
4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2
) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d^4 + b^10
*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1
/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5
*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 +
13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*
d^5 + 64*a^21*b^2*d^5))/4 - 160*A*a^2*b^19*d^4 - 128*A*a^4*b^17*d^4 - 32*A*
b^21*d^4 + 3136*A*a^8*b^13*d^4 + 4928*A*a^10*b^11*d^4 + 4480*A*a^12*b^9*d^4
 + 2432*A*a^14*b^7*d^4 + 736*A*a^16*b^5*d^4 + 96*A*a^18*b^3*d^4))/4)*(((320
*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^
4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 2
0*A^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10
*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 - 96*A^3*a^3*b^13*d^2 - 240*A^3*a^5
*b^11*d^2 - 320*A^3*a^7*b^9*d^2 - 240*A^3*a^9*b^7*d^2 - 96*A^3*a^11*b^5*d^2
 - 16*A^3*a^13*b^3*d^2 - 16*A^3*a*b^15*d^2)*(((320*A^4*a^2*b^8*d^4 - 16*A^4
*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d
^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d^
4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*
d^4))^(1/2))/4 + (log((((a + b*cot(c + d*x))^(1/2)*(320*A^2*a^4*b^14*d^3 -
16*A^2*b^18*d^3 + 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8*b^10*d^3 + 1024*A^2*
a^10*b^8*d^3 + 320*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*d^3) + ((-((320*A^4*a
^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4
- 400*A^4*a^8*b^2*d^4)^(1/2) + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*
a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b
^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(896*A*a^6*b^15*d^4 - ((-((320*A^4*a^2*b^8*d
^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^
4*a^8*b^2*d^4)^(1/2) + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^
2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 +
 5*a^8*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*
b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 161
```

28*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*
d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 - 160*A*a^2*b^19*d^4 - 128*A*a
^4*b^17*d^4 - 32*A*b^21*d^4 + 3136*A*a^8*b^13*d^4 + 4928*A*a^10*b^11*d^4 +
4480*A*a^12*b^9*d^4 + 2432*A*a^14*b^7*d^4 + 736*A*a^16*b^5*d^4 + 96*A*a^18*
b^3*d^4))/4)*(-((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d
^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) + 4*A^2*a^5*d^2 - 40
*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 +
10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 - 96*A^3*a^3*b^
13*d^2 - 240*A^3*a^5*b^11*d^2 - 320*A^3*a^7*b^9*d^2 - 240*A^3*a^9*b^7*d^2 -
96*A^3*a^11*b^5*d^2 - 16*A^3*a^13*b^3*d^2 - 16*A^3*a*b^15*d^2)*(-((320*A^4
*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^
4 - 400*A^4*a^8*b^2*d^4)^(1/2) + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^
2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6
*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 - log(- ((a + b*cot(c + d*x))^(1/2)*(32
0*A^2*a^4*b^14*d^3 - 16*A^2*b^18*d^3 + 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8
*b^10*d^3 + 1024*A^2*a^10*b^8*d^3 + 320*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*
d^3) - (((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 16
00*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^
3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 +
160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*((((320*A^4*a^2
*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 -
400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*
b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 16
0*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^2
2*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^
9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5
+ 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5) - 32*A*b^21*d^4 -
160*A*a^2*b^19*d^4 - 128*A*a^4*b^17*d^4 + 896*A*a^6*b^15*d^4 + 3136*A*a^8*
b^13*d^4 + 4928*A*a^10*b^11*d^4 + 4480*A*a^12*b^9*d^4 + 2432*A*a^14*b^7*d^4
+ 736*A*a^16*b^5*d^4 + 96*A*a^18*b^3*d^4))*(((...

3.104 $\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

Optimal. Leaf size=102

$$-\frac{(ia-b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d}+\frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d}$$

```
[Out] -(I*a-b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)+(I*a
+b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d/(a+I*b)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3620, 3618, 65, 214}

$$\frac{(b+ia)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}-\frac{(-b+ia)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

```
[Out] -(((I*a - b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/(Sqrt[a - I*b
]*d)) + ((I*a + b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/(Sqrt[a
 + I*b]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
```

```
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{1}{2}(-a-ib)\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx + \frac{1}{2}(-a+ib)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,d$$

$$= \frac{(ia-b)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx, x, i\cot(c+dx)\right)}{2d} - \frac{(ia+b)\text{Subst}\left(\int \frac{}{(-}\right.}{}$$

$$= -\frac{(a-ib)\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx, x, \sqrt{a+b\cot(c+dx)}\right)}{bd} - \frac{(a+ib)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(ia-b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d} + \frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c-}}{\sqrt{a+ib}}\right.}{\sqrt{a+ib}\,d}$$

Mathematica [A]
time = 0.48, size = 156, normalized size = 1.53

$$\frac{\left(\sqrt{a+ib}\,(-ia+b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)+\sqrt{a-ib}\,(ia+b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)\right)(a-b\cot(c+dx))\sin(c+dx)}{\sqrt{a-ib}\,\sqrt{a+ib}\,d(-b\cos(c+dx)+a\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

```
[Out] ((Sqrt[a + I*b]*((-I)*a + b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]
] + Sqrt[a - I*b]*(I*a + b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]]
)*(a - b*Cot[c + d*x])*Sin[c + d*x])/(Sqrt[a - I*b]*Sqrt[a + I*b]*d*(-(b*Co
s[c + d*x]) + a*Sin[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(84) = 168$.
time = 0.66, size = 783, normalized size = 7.68

method	result
derivativedivides	$-\frac{2b\left(\frac{\left(-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a\,b^2+\sqrt{2\sqrt{a^2+b^2}}\right.}{}\right.}{}$
default	$-\frac{2b\left(\frac{\left(-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^3-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a\,b^2+\sqrt{2\sqrt{a^2+b^2}}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*(1/4/b^2/(a^2+b^2)^(3/2)*(1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b
^2)^(1/2)*a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2+(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*a^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(b*cot(d*x+c)
+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*
(-4*a^3*b^2-4*a*b^4-1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3
-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2+(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*a^4-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b
^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2/(a^2+b^2)^(3/
2)*(-1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))+2*(4*a^3*b^2+4*a*b^4+1/2*(-(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*(a^2+b^2)^(1/2)*a*b^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cot(d*x + c) - a)/sqrt(b*cot(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{\sqrt{a+b\cot(c+dx)}}\,dx-\int\left(-\frac{b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))**(1/2),x)
```

```
[Out] -Integral(a/sqrt(a + b*cot(c + d*x)), x) - Integral(-b*cot(c + d*x)/sqrt(a
+ b*cot(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) - a)/sqrt(b*cot(d*x + c) + a), x)
```

Mupad [**B**]
time = 2.20, size = 2731, normalized size = 26.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a - b*cot(c + d*x))/(a + b*cot(c + d*x))^(1/2),x)
```

```
[Out] 2*atanh((32*a^4*b^2*d^2*(- (-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4))
 - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((1
6*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) +
 (4*a^3*b^3*d^4*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*a*b^5*d^4
*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*a^2*b^2*(- (-16*a^4*b^
2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^
(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*a^4*b^3*d^3)/(a^2*d^4 + b^2*d^4) + (
4*a*b^3*d^2*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*(- (-1
6*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2
*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-16*a^4*b^2*d^4)^(1/2))/((16*a^4*
b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) + (4*a^
3*b^3*d^4*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*a*b^5*d^4*(-16*
a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*(- (-16*a^4*b^2*d^4)^(1/2)/(16*(a
^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2) - 2*atanh((32
*a^2*b^2*((-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(
a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*a^4*b^3*d^3)/(a
^2*d^4 + b^2*d^4) - (4*a*b^3*d^2*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^
5)) - (32*a^4*b^2*d^2*((-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (
a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*a^
4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*
a^3*b^3*d^4*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) - (4*a*b^5*d^4*(-1
6*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*((-16*a^4*b^2*d^4)^(1
/2)/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a
+ b*cot(c + d*x))^(1/2)*(-16*a^4*b^2*d^4)^(1/2))/((16*a^4*b^5*d^5)/(a^2*d^4
 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*a^3*b^3*d^4*(-16*a^
4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) - (4*a*b^5*d^4*(-16*a^4*b^2*d^4)^(1/2
))/(a^2*d^5 + b^2*d^5)))*((-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4))
- (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2) + 2*atanh((32*b^4*((a*b^2*d^2)/(
4*(a^2*d^4 + b^2*d^4)) - (-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2
)*(a + b*cot(c + d*x))^(1/2))/((16*b^5)/d - (16*a^2*b^5*d^3)/(a^2*d^4 + b^2
*d^4) + (4*a*b^3*d^2*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*(
(a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^
2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-16*b^6*d^4)^(1/2))/(16*b^7*d +
16*a^2*b^5*d - (16*a^2*b^7*d^5)/(a^2*d^4 + b^2*d^4) - (16*a^4*b^5*d^5)/(a^2
*d^4 + b^2*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (
4*a^3*b^3*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*a^2*b^4*d^2*(
(a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^
2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/(16*b^7*d + 16*a^2*b^5*d - (16*a
^2*b^7*d^5)/(a^2*d^4 + b^2*d^4) - (16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4
*a*b^5*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*a^3*b^3*d^4*(-16*b
^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*((a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)) -
```

(-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2) + 2*atanh((8*a*b^2*((-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-16*b^6*d^4)^(1/2))/((16*a^2*b^7*d^5)/(a^2*d^4 + b^2*d^4) - 16*a^2*b^5*d - 16*b^7*d + (16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*a^3*b^3*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*b^4*((-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*a^2*b^5*d^3)/(a^2*d^4 + b^2*d^4) - (16*b^5)/d + (4*a*b^3*d^2*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (32*a^2*b^4*d^2*((-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*a^2*b^7*d^5)/(a^2*d^4 + b^2*d^4) - 16*a^2*b^5*d - 16*b^7*d + (16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*a^3*b^3*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*((-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)

3.105 $\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{(ia-b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}+\frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}-\frac{4ab}{(a^2+b^2)\,d\sqrt{a+b\cot(}$$

```
[Out] -(I*a-b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d+(I*a
+b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d-4*a*b/(a^
2+b^2)/d/(a+b*cot(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3610, 3620, 3618, 65, 214}

$$-\frac{4ab}{d\,(a^2+b^2)\,\sqrt{a+b\cot(c+dx)}}-\frac{(-b+ia)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}}+\frac{(b+ia)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*Cot[c + d*x])/(a + b*Cot[c + d*x])^(3/2),x]
```

```
[Out] -(((I*a - b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(3
/2)*d)) + ((I*a + b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/((a +
 I*b)^(3/2)*d) - (4*a*b)/((a^2 + b^2)*d*Sqrt[a + b*Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
```

```
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = -\frac{4ab}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}}+\frac{\int \frac{-a^2+b^2+2ab\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx}{a^2+b^2}$$

$$= -\frac{4ab}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}}-\frac{(a-ib)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx}{2(a+ib)}-\frac{(a$$

$$= -\frac{4ab}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}}-\frac{(a+ib)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\,dx,x,\right.}{2(ia+b)d}$$

$$= -\frac{4ab}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}}-\frac{(a-ib)\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}}\,dx,x,\sqrt{a+\right.}{(a+ib)bd}$$

$$= -\frac{(ia-b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}+\frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c}{\sqrt{a+ib}}\right.}{(a+ib)^{3/2}d}$$

Mathematica [A]
time = 1.58, size = 216, normalized size = 1.64

$$\frac{(a-b\cot(c+dx))\left(-i(a+ib)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)\sqrt{a+b\cot(c+dx)}+\sqrt{a-ib}\left(-4a\sqrt{a+ib}\,b+i(a-ib)^2\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)\sqrt{a+b\cot(c+dx)}\right)\right)\sin(c+dx)}{(a-ib)^{3/2}(a+ib)^{3/2}d\sqrt{a+b\cot(c+dx)}\,(-b\cos(c+dx)+a\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*Cot[c + d*x])/(a + b*Cot[c + d*x])^(3/2),x]
```

```
[Out] ((a - b*Cot[c + d*x])*((-I)*(a + I*b)^(5/2)*ArcTanh[Sqrt[a + b*Cot[c + d*x]
]/Sqrt[a - I*b]]*Sqrt[a + b*Cot[c + d*x]] + Sqrt[a - I*b]*(-4*a*Sqrt[a + I*
b]*b + I*(a - I*b)^2*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]]*Sqrt[a
 + b*Cot[c + d*x]]))*Sin[c + d*x])/((a - I*b)^(3/2)*(a + I*b)^(3/2)*d*Sqrt[
a + b*Cot[c + d*x]]*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 927 vs. $2(112) = 224$.
time = 0.63, size = 928, normalized size = 7.03

method	result
derivativedivides	$-\frac{2b\left(-\frac{\left(-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^4+\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,b^4+\sqrt{2\sqrt{a^2+b^2}}\right.}{}\right.}{}$
default	$-\frac{2b\left(-\frac{\left(-\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,a^4+\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\,b^4+\sqrt{2\sqrt{a^2+b^2}}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*(1/(a^2+b^2)*(1/4/b^2/(a^2+b^2)^(3/2)*(-1/2*(-(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*(a^2+b^2)^(1/2)*a^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b
^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^
2-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^4)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+
```

```
b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))+2*(6*a^4*b^2+4*a^2*b^
4-2*b^6+1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4+(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5-2*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^2-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^4
)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(
a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2)))+1/4/b^2/(a^2+b^2)^(3/2)*(1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+
b^2)^(1/2)*a^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^4+(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a^5-2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b^2-3*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*a*b^4)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-6*a^4*b^2-4*a^2*b^4+2*b^6-1/2*
(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4+(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*(a^2+b^2)^(1/2)*b^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5-2*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a^3*b^2-3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^4)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c
))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))+2*
a/(a^2+b^2)/(a+b*cot(d*x+c))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cot(d*x + c) - a)/(b*cot(d*x + c) + a)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{a\sqrt{a+b\cot\left(c+dx\right)}+b\sqrt{a+b\cot\left(c+dx\right)}\cot\left(c+dx\right)}\,dx-\int\left(-\frac{b\cot\left(c+dx\right)}{a\sqrt{a+b\cot\left(c+dx\right)}+b\sqrt{a+b\cot\left(c+dx\right)}\cot\left(c+dx\right)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))**(3/2),x)
```

```
[Out] -Integral(a/(a*sqrt(a + b*cot(c + d*x)) + b*sqrt(a + b*cot(c + d*x))*cot(c
+ d*x)), x) - Integral(-b*cot(c + d*x)/(a*sqrt(a + b*cot(c + d*x)) + b*sqrt
(a + b*cot(c + d*x))*cot(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) - a)/(b*cot(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 5.96, size = 2500, normalized size = 18.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a - b*cot(c + d*x))/(a + b*cot(c + d*x))^(3/2),x)
```

```
[Out] log(8*a*b^11*d^2 - (((((24*a*b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4
 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 12*a*b^4*d^2 + 4*
a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2
)*(64*a^6*b^7*d^4 - 96*a^2*b^11*d^4 - 64*a^4*b^9*d^4 - 32*b^13*d^4 + 96*a^8
*b^5*d^4 + 32*a^10*b^3*d^4 + (a + b*cot(c + d*x))^(1/2)*((((24*a*b^4*d^2 -
8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4
*b^2*d^4))^(1/2) - 12*a*b^4*d^2 + 4*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3
*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 6
40*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5)) + (a
 + b*cot(c + d*x))^(1/2)*(16*b^12*d^3 + 32*a^2*b^10*d^3 - 32*a^6*b^6*d^3 -
16*a^8*b^4*d^3))*((((24*a*b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 +
16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 12*a*b^4*d^2 + 4*a^3
*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) +
 24*a^3*b^9*d^2 + 24*a^5*b^7*d^2 + 8*a^7*b^5*d^2)*((((24*a*b^4*d^2 - 8*a^3*
b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d
^4))^(1/2) - 12*a*b^4*d^2 + 4*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b
^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + log(8*a*b^11*d^2 - ((-(((24*a*b^4*d^2 - 8
*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*
b^2*d^4))^(1/2) + 12*a*b^4*d^2 - 4*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*
a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(64*a^6*b^7*d^4 - 96*a^2*b^11*d^4 - 64
*a^4*b^9*d^4 - 32*b^13*d^4 + 96*a^8*b^5*d^4 + 32*a^10*b^3*d^4 + (a + b*cot(
c + d*x))^(1/2)*(-(((24*a*b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 +
16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) + 12*a*b^4*d^2 - 4*a^3
*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(
```

```
64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*
a^9*b^4*d^5 + 64*a^11*b^2*d^5)) + (a + b*cot(c + d*x))^(1/2)*(16*b^12*d^3 +
 32*a^2*b^10*d^3 - 32*a^6*b^6*d^3 - 16*a^8*b^4*d^3))*(-(((24*a*b^4*d^2 - 8*
a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b
^2*d^4))^(1/2) + 12*a*b^4*d^2 - 4*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a
^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + 24*a^3*b^9*d^2 + 24*a^5*b^7*d^2 + 8*a
^7*b^5*d^2)*(-(((24*a*b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b
^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) + 12*a*b^4*d^2 - 4*a^3*b^2
*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + (lo
g((((a + b*cot(c + d*x))^(1/2)*(16*a^2*b^10*d^3 + 32*a^4*b^8*d^3 - 32*a^8*b
^4*d^3 - 16*a^10*b^2*d^3) - ((((96*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b
^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^3*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*
d^4 + 3*a^4*b^2*d^4))^(1/2)*(((((96*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*
b^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^3*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4
*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 32
0*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a
^11*b^2*d^5))/4 + 64*a^2*b^11*d^4 + 256*a^4*b^9*d^4 + 384*a^6*b^7*d^4 + 256
*a^8*b^5*d^4 + 64*a^10*b^3*d^4))/4)*(((96*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 14
4*a^8*b^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^3*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a
^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 + 8*a^3*b^9*d^2 + 24*a^5*b^7*d^2 + 24
*a^7*b^5*d^2 + 8*a^9*b^3*d^2)*(((96*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*
b^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^3*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4
*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 + (log((((a + b*cot(c + d*x))^(1/2)*(16*a^2
*b^10*d^3 + 32*a^4*b^8*d^3 - 32*a^8*b^4*d^3 - 16*a^10*b^2*d^3) - ((-((96*a^
6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b^2*d^4)^(1/2) + 4*a^5*d^2 - 12*a^3*b^
2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(((-((96*
a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b^2*d^4)^(1/2) + 4*a^5*d^2 - 12*a^3*
b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a + b*
cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 6
40*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 + 64*a^2*b^11*d^4 +
256*a^4*b^9*d^4 + 384*a^6*b^7*d^4 + 256*a^8*b^5*d^4 + 64*a^10*b^3*d^4))/4)*
(-((96*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b^2*d^4)^(1/2) + 4*a^5*d^2 -
12*a^3*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))
/4 + 8*a^3*b^9*d^2 + 24*a^5*b^7*d^2 + 24*a^7*b^5*d^2 + 8*a^9*b^3*d^2)*(-((9
6*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b^2*d^4)^(1/2) + 4*a^5*d^2 - 12*a^
3*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 -
log(8*a^3*b^9*d^2 - ((a + b*cot(c + d*x))^(1/2)*(16*a^2*b^10*d^3 + 32*a^4*b
^8*d^3 - 32*a^8*b^4*d^3 - 16*a^10*b^2*d^3) + (((96*a^6*b^4*d^4 - 16*a^4*b^6
*d^4 - 144*a^8*b^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^3*b^2*d^2)/(16*a^6*d^4 + 1
6*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2)*(64*a^2*b^11*d^4 - (((9
6*a^6*b^4*d^4 - 16*a^4*b^6*d^4 - 144*a^8*b^2*d^4)^(1/2) - 4*a^5*d^2 + 12*a^
3*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/
2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + ...
```

3.106 $\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{(ia-b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}+\frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d}-\frac{4ab}{3(a^2+b^2)d(a+b\cot($$

```
[Out] -(I*a-b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d+(I*a
+b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d-4/3*a*b/(
a^2+b^2)/d/(a+b*cot(d*x+c))^(3/2)-2*b*(3*a^2-b^2)/(a^2+b^2)^2/d/(a+b*cot(d*
x+c))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3610, 3620, 3618, 65, 214}

$$-\frac{2b(3a^2-b^2)}{d(a^2+b^2)^2\sqrt{a+b\cot(c+dx)}}-\frac{4ab}{3d(a^2+b^2)(a+b\cot(c+dx))^{3/2}}-\frac{(-b+ia)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}}+\frac{(b+ia)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*Cot[c + d*x])/(a + b*Cot[c + d*x])^(5/2),x]
```

```
[Out] -(((I*a - b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(5
/2)*d)) + ((I*a + b)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/((a +
 I*b)^(5/2)*d) - (4*a*b)/(3*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^(3/2)) - (2*
b*(3*a^2 - b^2))/((a^2 + b^2)^2*d*Sqrt[a + b*Cot[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
```

```
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx = -\frac{4ab}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}}+\frac{\int \frac{-a^2+b^2+2ab\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx}{a^2+b^2}$$

$$= -\frac{4ab}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}}-\frac{2b(3a^2-b^2)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\cot(c+dx)}}+ \cdots$$

$$= -\frac{4ab}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}}-\frac{2b(3a^2-b^2)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\cot(c+dx)}}- \cdots$$

$$= -\frac{4ab}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}}-\frac{2b(3a^2-b^2)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\cot(c+dx)}}+ \cdots$$

$$= -\frac{4ab}{3\left(a^2+b^2\right)d(a+b\cot(c+dx))^{3/2}}-\frac{2b(3a^2-b^2)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\cot(c+dx)}}+ \cdots$$

$$= -\frac{(ia-b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}+\frac{(ia+b)\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c \cdots}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} \cdots$$

Mathematica [A]

time = 6.19, size = 232, normalized size = 1.33

$$\frac{(-a+b\cot(c+dx))\left(\frac{3i\left((a+ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)-(a-ib)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)\right)(a+b\cot(c+dx))^{5/2}}{(a-ib)^{5/2}(a+ib)^{5/2}}-\frac{2b(a+b\cot(c+dx))(-11a^3+ab^2+(-9a^2b+3b^3)\cot(c+dx))}{(a^2+b^2)^2}\right)\sin(c+dx)}{3d(a+b\cot(c+dx))^{5/2}(-b\cos(c+dx)+a\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*Cot[c + d*x])/(a + b*Cot[c + d*x])^(5/2),x]
```

```
[Out] ((-a + b*Cot[c + d*x])*(((3*I)*((a + I*b)^(7/2)*ArcTanh[Sqrt[a + b*Cot[c +
d*x]]/Sqrt[a - I*b]] - (a - I*b)^(7/2)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqr
t[a + I*b]])*(a + b*Cot[c + d*x])^(5/2))/((a - I*b)^(5/2)*(a + I*b)^(5/2))
- (2*b*(a + b*Cot[c + d*x])*(-11*a^3 + a*b^2 + (-9*a^2*b + 3*b^3)*Cot[c + d
*x]))/(a^2 + b^2)^2)*Sin[c + d*x])/(3*d*(a + b*Cot[c + d*x])^(5/2)*(-(b*Cos
[c + d*x]) + a*Sin[c + d*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1181 vs. $2(150) = 300$.
time = 0.65, size = 1182, normalized size = 6.79 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*(1/(a^2+b^2)^2*(1/4/(a^2+b^2)^(3/2)/b^2*(1/2*(-(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*(a^2+b^2)^(1/2)*a^5+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2
)*a^3*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^4+(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*a^6-5*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4*b^2-5*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a^2*b^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^6)*ln(b*cot(d*x
+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))
+2*(-8*a^5*b^2+8*a*b^6-1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*
a^5+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3*b^2+3*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^6-5*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4*b^2-5*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b
^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^6)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)
+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/(a^2+b^2)^(3/2)/b^2*(-1/2*
(-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^5+2*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*(a^2+b^2)^(1/2)*a^3*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^
(1/2)*a*b^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^6-5*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*a^4*b^2-5*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^4+(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)*b^6)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*
x+c)-a-(a^2+b^2)^(1/2))+2*(8*a^5*b^2-8*a*b^6+1/2*(-(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)*(a^2+b^2)^(1/2)*a^5+2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a
^3*b^2+3*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^4+(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)*a^6-5*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4*b^2-5*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*a^2*b^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^6)*(2*(a^2+b^2)^(1
```

```
/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*
a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))-(-3*a^2
+b^2)/(a^2+b^2)^2/(a+b*cot(d*x+c))^(1/2)+2/3/(a^2+b^2)*a/(a+b*cot(d*x+c))^(
3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cot(d*x + c) - a)/(b*cot(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{a^{2}\sqrt{a+b\cot{\left(c+dx \right)}}+2ab\sqrt{a+b\cot{\left(c+dx \right)}}\cot{\left(c+dx \right)}+b^{2}\sqrt{a+b\cot{\left(c+dx \right)}}\cot^{2}{\left(c+dx \right)}}\, dx-\int \left(-\frac{b\cot{\left(c+dx \right)}}{a^{2}\sqrt{a+b\cot{\left(c+dx \right)}}+2ab\sqrt{a+b\cot{\left(c+dx \right)}}\cot{\left(c+dx \right)}+b^{2}\sqrt{a+b\cot{\left(c+dx \right)}}\cot^{2}{\left(c+dx \right)}}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))**(5/2),x)
```

```
[Out] -Integral(a/(a**2*sqrt(a + b*cot(c + d*x)) + 2*a*b*sqrt(a + b*cot(c + d*x))
*cot(c + d*x) + b**2*sqrt(a + b*cot(c + d*x))*cot(c + d*x)**2), x) - Integr
al(-b*cot(c + d*x)/(a**2*sqrt(a + b*cot(c + d*x)) + 2*a*b*sqrt(a + b*cot(c
+ d*x))*cot(c + d*x) + b**2*sqrt(a + b*cot(c + d*x))*cot(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c) - a)/(b*cot(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 16.17, size = 2500, normalized size = 14.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a - b*cot(c + d*x))/(a + b*cot(c + d*x))^(5/2),x)
```

```
[Out] (log(((-(4*a^7*d^2 - (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4
+ 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2
*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^
4 + 5*a^8*b^2*d^4))^(1/2)*((a + b*cot(c + d*x))^(1/2)*(320*a^6*b^14*d^3 - 1
6*a^2*b^18*d^3 + 1024*a^8*b^12*d^3 + 1440*a^10*b^10*d^3 + 1024*a^12*b^8*d^3
 + 320*a^14*b^6*d^3 - 16*a^18*b^2*d^3) - ((-(4*a^7*d^2 - (320*a^6*b^8*d^4 -
 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)
^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*
d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(((-(4*a^7*d^
2 - (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d
^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^10*d^4 +
 b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4
))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 288
0*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d
^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19
*b^4*d^5 + 64*a^21*b^2*d^5))/4 - 32*a*b^21*d^4 - 160*a^3*b^19*d^4 - 128*a^5
*b^17*d^4 + 896*a^7*b^15*d^4 + 3136*a^9*b^13*d^4 + 4928*a^11*b^11*d^4 + 448
0*a^13*b^9*d^4 + 2432*a^15*b^7*d^4 + 736*a^17*b^5*d^4 + 96*a^19*b^3*d^4))/4
))/4 + 16*a^4*b^15*d^2 + 96*a^6*b^13*d^2 + 240*a^8*b^11*d^2 + 320*a^10*b^9*
d^2 + 240*a^12*b^7*d^2 + 96*a^14*b^5*d^2 + 16*a^16*b^3*d^2)*(-(4*a^7*d^2 -
(320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 -
 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^10*d^4 + b^1
0*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(
1/2))/4 - log(16*a^4*b^15*d^2 - (-(4*a^7*d^2 - (320*a^6*b^8*d^4 - 16*a^4*b^
10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 2
0*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4
 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*((a + b*cot(c
 + d*x))^(1/2)*(320*a^6*b^14*d^3 - 16*a^2*b^18*d^3 + 1024*a^8*b^12*d^3 + 14
40*a^10*b^10*d^3 + 1024*a^12*b^8*d^3 + 320*a^14*b^6*d^3 - 16*a^18*b^2*d^3)
+ (-(4*a^7*d^2 - (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 16
00*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2
)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b
^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*(896*a^7*b^15*d^4 - (-(4*a^7*d^2 - (320*a^6
*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^1
2*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a^10*d^4 + 16*b^10*
```

```
d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))
^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*
a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5
 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b
^4*d^5 + 64*a^21*b^2*d^5) - 160*a^3*b^19*d^4 - 128*a^5*b^17*d^4 - 32*a*b^21
*d^4 + 3136*a^9*b^13*d^4 + 4928*a^11*b^11*d^4 + 4480*a^13*b^9*d^4 + 2432*a^
15*b^7*d^4 + 736*a^17*b^5*d^4 + 96*a^19*b^3*d^4)) + 96*a^6*b^13*d^2 + 240*a
^8*b^11*d^2 + 320*a^10*b^9*d^2 + 240*a^12*b^7*d^2 + 96*a^14*b^5*d^2 + 16*a^
16*b^3*d^2)*(-(4*a^7*d^2 - (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^
6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a
^5*b^2*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 +
 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2) + (log(((((320*a^2*b^12*d^4 - 16*
b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^(1/2) +
20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2
*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(((((320
*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^
8*b^6*d^4)^(1/2) + 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b^2*d^2)/(a^10*d^4
 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d
^4))^(1/2)*(96*a*b^21*d^4 + 736*a^3*b^19*d^4 + 2432*a^5*b^17*d^4 + 4480*a^7
*b^15*d^4 + 4928*a^9*b^13*d^4 + 3136*a^11*b^11*d^4 + 896*a^13*b^9*d^4 - 128
*a^15*b^7*d^4 - 160*a^17*b^5*d^4 - 32*a^19*b^3*d^4 - ((((320*a^2*b^12*d^4 -
 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^(1/2
) + 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5
*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(a +
 b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^
5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^
13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64
*a^21*b^2*d^5))/4))/4 - (a + b*cot(c + d*x))^(1/2)*(320*a^4*b^16*d^3 - 16*b
^20*d^3 + 1024*a^6*b^14*d^3 + 1440*a^8*b^12*d^3 + 1024*a^10*b^10*d^3 + 320*
a^12*b^8*d^3 - 16*a^16*b^4*d^3)))/4 - 8*b^19*d^2 - 40*a^2*b^17*d^2 - 72*a^4
*b^15*d^2 - 40*a^6*b^13*d^2 + 40*a^8*b^11*d^2 + 72*a^10*b^9*d^2 + 40*a^12*b
^7*d^2 + 8*a^14*b^5*d^2)*(((320*a^2*b^12*d^4 - ...
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```